4th International Scientific Conference on Geometry and Graphics

moNGeometrija 2014

PROCEEDINGS VOLUME 2

moNGeometrija 2014

June 20th - 22nd 2014

Vlasina, Serbia

ISBN 978-86-88601-14-6

4th international scientific conference
moNGeometrija 2014

Publisher:
Faculty of Civil engineering and Architecture in Niš
Serbian Society for Geometry and Graphics SUGIG

Title of Publication
PROCEEDINGS - Volume 2:
- Theoretical geometry, exposed by synthetical or analytical methodology
- Geometry applied in Visual Arts and Design
- Education and didactics

Editor-in-Chief
Sonja Krasić, PhD

Co-Editor
Petar Pejić

Text formating
Petar Pejić

ISBN 978-86-88601-14-6

Number of copies printed 50
Printing: Galaksija Niš

4th International Scientific Conference on Geometry and Graphics
moNGeometrija 2014

Is organized by:
University of Niš, Faculty of Civil Engineering and Architecture
Serbian Society for Geometry and Graphics
University of Niš, Faculty of Mechanical Engineering
University of Niš, Faculty of Occupational Safety
College of Applied Technical Sciences Niš

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THE STRUCTURAL TRANSFORMATION OF CONCAVE CUPOLAE OF FOURTH SORT USING DIFFERENT VARIANTS OF CONSTRUCTIVE PROCEDURE

Slobodan Mišić [1]
Marija Obradović [2]
Branislav Popkonstantinović [3]

Abstract

Concave Cupolae of fourth sort (CC IV) are polyhedra which lateral surface is created by folding a quadruple strip of equilateral triangles, while the bases are regular polygons. The unit cell that forms the deltahedral lateral surface by radial sequenceing around the axis of the solid is composed of two spatial hexahedrals. These unit cells are linked together in its upper zone by spatial quadrihedals, which consist of four equilateral triangles grouped around a common vertex. Over the same polygonal base, there can be formed four diverse Concave Cupolae of IV sort, using different variants of constructive procedure. In this paper, the application of the program for modeling the lateral surface of concave cupolae of IV sort, using the software package MATLAB, allows visual monitoring of the structural transformation of the concave cupolae of IV sort, for different polygonal bases and different variants of constructive procedure.

Key words: *Concave cupola, polyhedron, basis, deltahedral surface, transformation*

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1. INTRODUCTION

Concave Cupolae (***CC***) are polyhedra which follow the method of generating the Johnson's cupolae (Johnson's solids J3, J4 and J5), whereat the convexity criterion is omitted, while in the lateral surface appear two or more rows of equilateral triangles [1], [2], [3], [4]. The lateral surface of ***CC*** is, as the name suggests, a concave polyhedral surface, so the planes of the ***CC***'s faces may pass through the interior of the solid.

Concave cupola is formed over the regular polygonal bases Ω_1 and Ω_2 which are set in parallel planes. The base Ω_1 is the initial ***n***-sided regular polygon around which the deltahedral lateral surface of the cupola is formed, while the base Ω_2 is ***2n***-sided regular polygon coaxial to the base Ω_1 (Fig. 1a). The central axis of ***CC*** is perpendicular to the planes of the bases Ω_1 and Ω_2, and is determined by their centroids. Concave Cupola is required to meet certain criteria that are felicitous to the engineering profession [1], [5], such as: ***CC*** does not allow cases of degeneration, neither faces, edges nor vertices. Each face of the Cupola is visible from the exterior, i.e. there cannot exist interior faces of the solid.

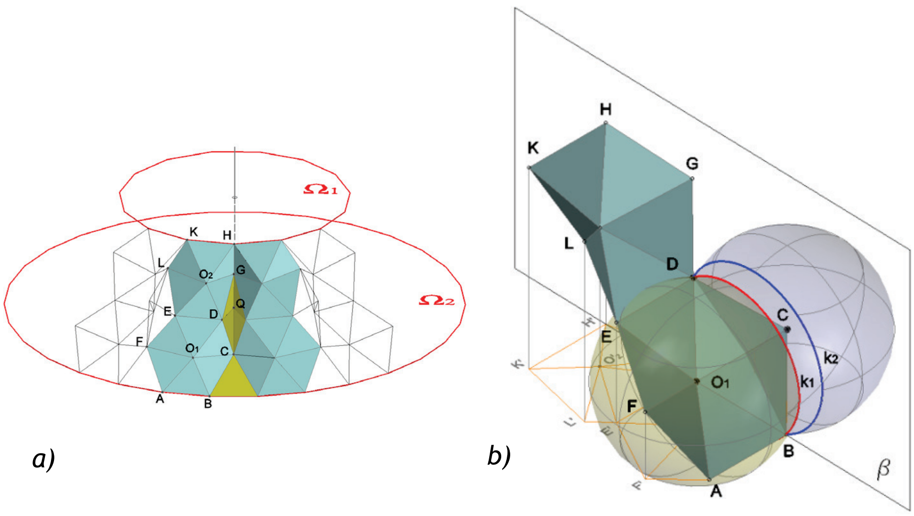


Figure 1. *a) The formation of deltahedral lateral surface by the polar arrangement of CC IV unit cells,* ***b)*** *3D model of the CC IV unit cell, the construction of the vertices* ***B*** *and* ***D*** *positions for the spatial hexahedral* $ABCDEFO_1$ *[1]* ***(Fig. 15)***

The sort of Cupola is determined by the number of rows of equilateral triangles in the lateral surface's net. The equilateral triangles are grouped into the spatial hexahedrals, unit cells of ***CC*** (Fig. 1b), by which polar array around the central axis of the polyhedron the deltahedral lateral surface is obtained. From the above, we can conclude that ***CC*** is always of even sort (second, fourth, sixth, etc.). If we intend to determine the greatest possible representative within one sort of Cupolae, we need to induce that the perpendicular distance from the edge of the ***n***-sided base Ω_1 and ***2n***-sided base Ω_2 must be lesser than the product of the height of unit equilateral triangle, and the number ***x*** of the triangles' rows in the net (the number ***x*** represents the sort of the Cupola): $d < \frac{ax\sqrt{3}}{2}$ [1]. The value (n_{max}) for the maximal number of sides of the polygon Ω_1, from which there can be developed concave polyhedral surface that meets the above mentioned properties of the ***CC***'s lateral surface is expressed as:

$$n_{max} < \frac{\pi}{arcsin\frac{1}{x\sqrt{3}}} \qquad (1)$$

If we adopt the first lesser integer for n_{max}, by the analysis of the obtained values, we may conclude that for each sort of ***CC*** the deltahedral lateral surface (with even number of rows of equilateral triangles) can be formed over 11 (eleven) additional polygonal bases, in relation to the ***CC*** of the lower sort[1].

The number of vertices (V), edges (E) and faces (F) for any ***CC*** of the sort ***x*** over ***n***-sided polygonal base are calculated by formulae:

$$V = \frac{5x}{2} \cdot n \qquad (2)$$

$$E = \left(\frac{15x}{2} - 3\right) \cdot n \qquad (3)$$

$$F = (5x - 3) \cdot n + 2 \qquad (4)$$

[1] The only exception are ***CC II***, because they can be generated over seven different polygonal bases (4≤***n***≤10), as described in [3], [5].

2. CONCAVE CUPOLAE OF FOURTH SORT

The unit cell of ***CC IV*** consists of two spatial hexahedrals, formed from six equilateral triangles grouped around a common vertex. The spatial hexahedral element $ABCDEFO_1$ which participate in the composition of the lateral surface's lower belt, closer to the Ω_2, adjoints the hexahedral element $EDGHKLO_2$, closer to the Ω_1 (Fig.1b). Equilateral triangles which form the spatial hexahedrals are grouped around the common vertex O_1 in the lower belt, and around the vertex O_2 in the upper belt of the ***CC IV*** lateral surface. In the net, it is necessary to insert additional spatial elements between the spatial hexahedrals, composed of four equilateral triangles, grouped around a common vertex ***Q*** [1].

There are four types of ***CC IV*** with the same polygonal base, depending on the positions of the vertices O_1 and O_2 (as whached from the exterior):

- ***CC IV-Mm,*** retracted vertex O_2, and extracted vertex O_1.
- ***CC IV -mm,*** both vertice O_2 and O_1 extracted.
- ***CC IV -mM,*** extraxted vertex O_2, and retracted vertex O_1.
- ***CC IV -MM,*** both vertice O_2 and O_1 retracted.

The constructive procedures for the generation of each of the above types of ***CC IV*** are described in [1]. The positions and heights of the vertices of ***CC IV*** are obtained by the intersection of the vertical plane ***B*** containing the vertices ***B*** and ***D***, and spheres of the radii $r=a$ (Fig.1b). Centers of the spheres are set in the neighboring vertices ***C***, ***Q*** and O_2 of the spatial hexahedral. In the construction process, we are able to choose one of two possible positions of these vertices (to be of smaller/minor or greater/major height), and of which depends the obtained type of ***CC IV***. The result of the analysis described in [1] are:

- ***CCIV-Mm,*** obtained by using variants of constructive procedure with adopted minor heights of the vertices ***C***, ***Q*** and O_2.
- ***CCIV-mm,*** obtained by using variants of constructive procedure with adopted major heights of the vertex O_2.
- ***CCIV-mM,*** obtained by using variants of constructive procedure with adopted major heights of the vertices ***C*** and O_2.
- ***CCIV-MM,*** obtained by using variants of constructive procedure with adopted major heights of the vertices ***C*** and ***Q***.

Deltahedral lateral surface of ***CC IV*** can be generated over an ***n***-sided poligonal base, if ***n*** belongs to the range of $11 \leq n \leq 21$.

3. MODELING OF CONCAVE CUPOLAE OF FOURTH SORT

Using the constructive procedure described in [1], we have found the positions and the heights of the vertices of ***CC IV***, using the intersection of the vertical planes (ray projected as straight lines) and the curves of the higher order – the trajectories of the vertices (***D, H***), induced by changing the posititon of the vertex O_1.

The procedure was the basis for the creation of the algorithm, solving of which (by the application of iterations in *Microsoft Excel*) it has been enabled to calculate all the metric relations and parameters for the direct reading of the measurements of ***CC IV***.

Input data:

n - number of vertices in the base polygon Ω_1

a - side lenght of the base polygon Ω_1

Δ - expected error after iteration procedure performed.

The height h_1 (of the vertex O_1) is nivelated by the iterative procedure (and consequently - all the other parameters) untill the expected error is achieved, after iteration procedure is performed. More about the algorithm, see in [1].

Based on the iteratively obtained parameters in the software package *MATLAB*, a program for modeling the lateral surface of ***CC IV*** has been created. For each vertex, there were determined cylindrical coordinates in relation to the origin adopted in the center of the basic polygon Ω_2. The values of radii, angles and heights of the vertices of ***CC IV*** unit cell, i.e. hexahedrals $ABCDEFO_1$ and $EDGHKLO_2$, have been obtained by the iterative procedure. The lateral surface of ***CC IV*** is generated by polar array of the unit cells around the central axis of the cupola, whereat every point with the same denotation is distant from the previous one (e.g. D_1 and D_2) for the angle:

$$\beta = \frac{2\pi}{n} \tag{5}$$

which is the amount of the growth angle in cylindrical coordinates for each vertex. The structural transformation of ***CC IV*** using different variants of constructive procedure are tracked in **Tab. 1-4** where the 3D models and the front views of the characteristic Cupolae are presented for all the four types of ***CC IV*** $(a=1,\ \Delta=1^{-10})$ together with the number of vertices (***V***), edges (***E***) and faces (***F***) of the cupola, maximal height of the cupola (h_m) and the elevation (h_Ω) of the base Ω_1.

Table 1 *- CC IV-Mm, 3D model and front view for n=11, n=12, n=13 and n=21*

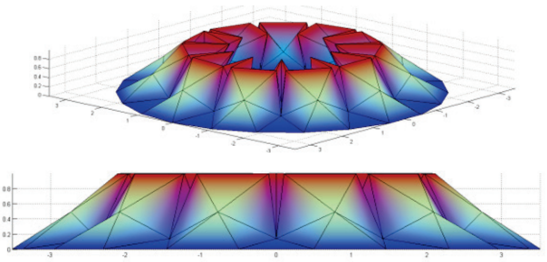
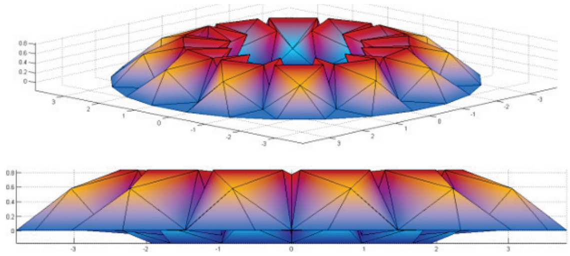
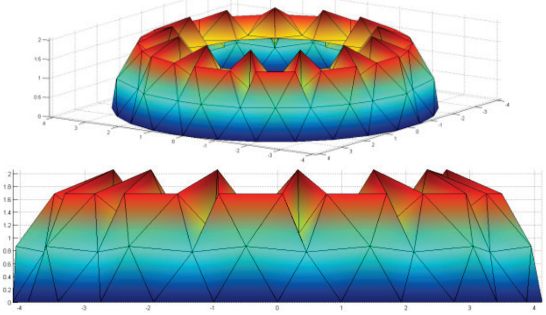
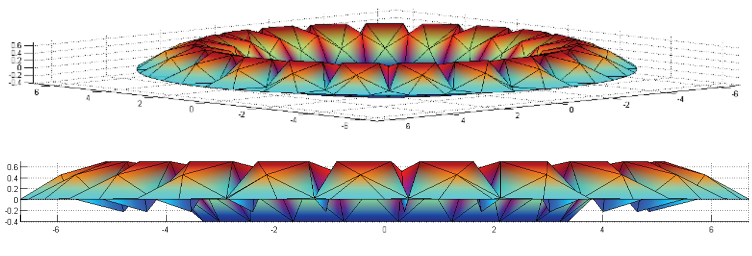
	**CC IV-11Mm** V = 110 E = 297 F = 189 a = 1 h_m = 0,99511 h_Ω = 0,05618
	**CC IV-12Mm** samo omotač V = 120 E = 324 F = 204 a = 1 h_m = 0,82585 h_Ω = -0,17354
	**CC IV-13Mm** V = 130 E = 351 F = 223 a = 1 h_m = 2,05827 h_Ω = 1,37493
	**CC IV-21Mm** samo omotač V = 210 E = 567 F = 357 a = 1 h_m = 0,69119 h_Ω = -0,41323

Table 2 *- CC IV-mm, 3D model and front view for n=13, n=14 and n=16*

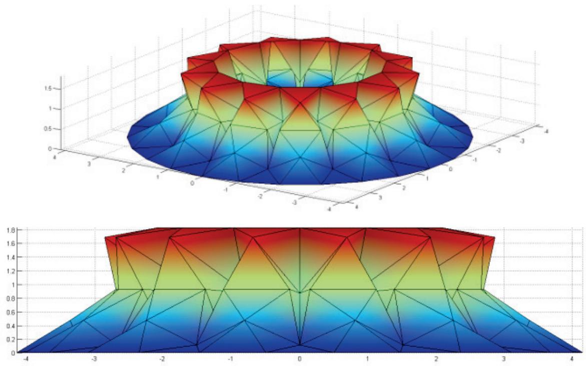
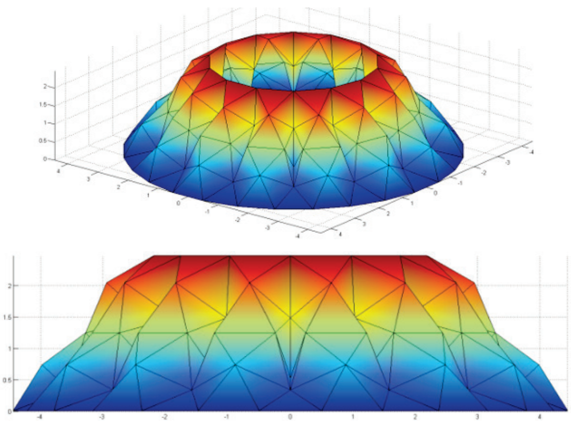
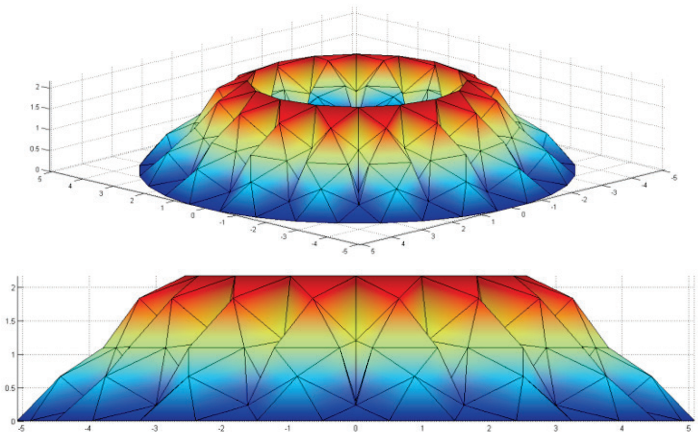
	**CC IV-13mm** V = 130 E = 351 F = 223 a = 1 h_m = 1,82578 $h_\Omega = h_m$
	**CC IV-14mm** V = 140 E = 378 F = 240 a = 1 h_m = 2,47259 $h_\Omega = h_m$
	**CC IV-16mm** V = 160 E = 432 F = 274 a = 1 h_m = 2,15860 $h_\Omega = h_m$

Table 3 - *CC IV-mM, 3D model and front view for n=14, n=16 and n=20*

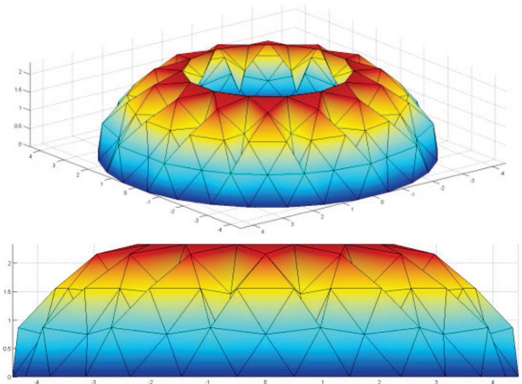
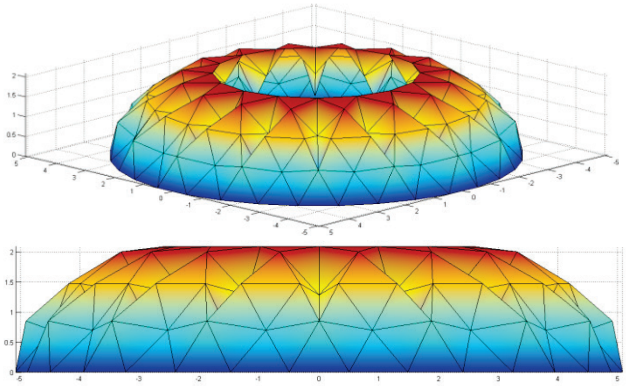
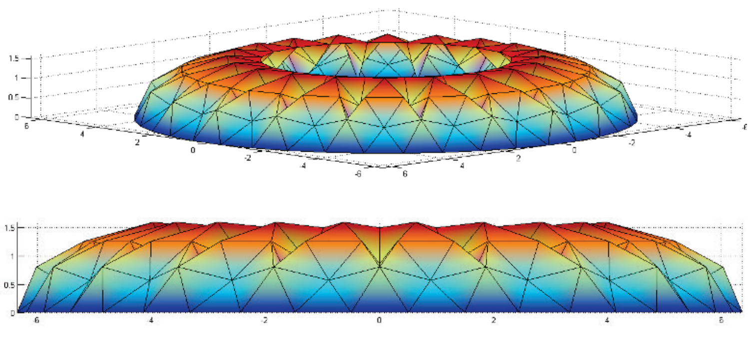
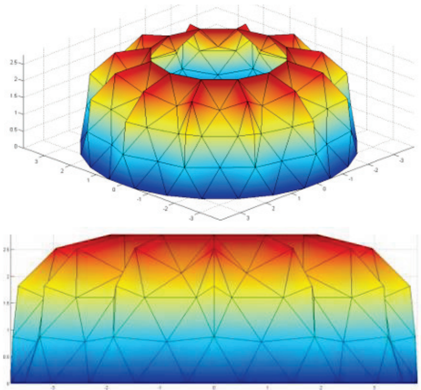
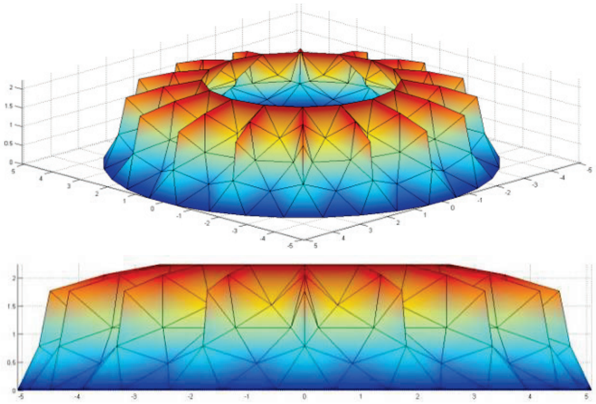
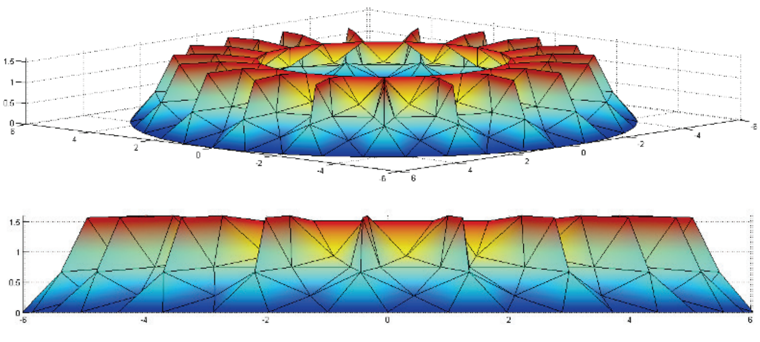
	**CC IV-14mM** V = 140 E = 378 F = 240 a = 1 h_m = 2,32289 $h_\Omega = h_m$
	**CC IV-16mM** V = 160 E = 432 F = 274 a = 1 h_m = 2,08579 $h_\Omega = h_m$
	**CC IV-20mM** V = 200 E = 540 F = 342 a = 1 h_m = 1,58520 h_Ω = 1,48585

Table 4 *- CC IV-MM, 3D model and front view for n=12, n=16 and n=19*

	**CC IV-12MM** V = 120 E = 324 F = 206 a = 1 h_m = 2,77025 $h_\Omega = h_m$
	**CC IV-16MM** V = 160 E = 432 F = 274 a = 1 h_m = 2,23057 $h_\Omega = h_m$
	**CC IV-19MM** V = 190 E = 513 F = 325 a = 1 h_m = 1,58957 h_Ω = 1,50344

4. CONCLUSIONS

Using the created program for modeling of ***CC IV*** in the software package MATLAB, we explored the possibility of their generation, whereby the formed polyhedra retain all the features of ***CC***, defined in the introductory section of this paper. The results are shown in **Tab.5**.

Table 5. *The possibilities of generating CC IV over n-sided polygonal base*

n	11	12	13	14	15	16	17	18	19	20	21
CC IV-Mm	•	x	•	•	•	•	•	x	x	x	x
CC IV-mm	•	•	•	•	•	•	x	x	x		
CC IV-mM			•	•	•	•	•	•	•	•	
CC IV-MM		•	•	•	•	•	•	•	•		

For the adopted values $a=1$, $\Delta=1^{-10}$ we can notice that possibility of generating the cupola, marked by the symbol (•), depends on ***n*** (number of the base Ω_1 polygon's sides, and the type of the applied constructive procedure. From the point of ***CC IV*** implementation in engineering practice, we also explored the examples, marked with the symbol (x), where the plane of the base Ω_1 is below the plane of the base Ω_2, in which case we can consider generation of just deltahedral surface based on the geometry of Concave Cupolae of the fourth sort.

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