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FREQUENCY- AND TIME-DOMAIN METHODS RELATED TO FLUTTER INSTABILITY PROBLEM

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Summary: *Bridge flutter phenomenon presents an important criterion of instability, which has to be considered in the bridge design phase. This paper presents different bridge flutter methods which can be used to solve the flutter problem, such as most commonly used frequency-domain approach, then equivalent approuch in time-domain, and the approximation based on the quasi-steady theory. A numerical example related to the tipical bridge cross-section follows presented approaches.*

Keywords: *Flutter, flutter solution, unsteady load models*

1. FLUTTER

Classic flutter is a type of the flutter, in which two degrees of freedom of the structure, namely rotation and vertical translation, couple in a flow-driven unstable oscillation. The motion is characterized by the fluid forces feeding energy into the system during one cycle of its oscillation. This energy counteracts the energy absorptions by structural damping. The critical condition is reached by the certain wind speed, called critical wind velocity, related to the total zero damping, i.e. structural and aerodynamic damping together. In addition, the structure oscillates with the same frequency in bending and torsion (critical frequency).

1.1 Frequency Approach

Most commonly used formulation of the motion-induced forces (also called self-excited or aeroelastic forces) is proposed by Scanlan and Tomko (1971). The method takes into account aerodynamic parameters called flutter derivatives to define a linear aeroelastic subsystem expressed through aeroelastic forces as:

$$L_{ae} = \frac{1}{2}\rho U^2 B\left[KH_1^* \frac{\dot{z}}{U} + KH_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha + K^2 H_4^* \frac{z}{B} \right] \quad (1)$$

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$$M_{ae} = \frac{1}{2}\rho U^2 B^2 \left[K A_1^* \frac{\dot{z}}{U} + K A_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha + K^2 A_4^* \frac{z}{B} \right] \quad (2)$$

In these equations, $K = B\omega / U$ is the reduced frequency and $H_i^*, A_i^* (i = 1...4)$ are the flutter derivatives, $q_0 = 1/2\, \rho U^2$ is the kinetic pressure, ρ represents the air density, U the mean wind velocity, B is the bridge deck width. Speciality of the proposed method is that these coefficients are functions of reduced frequency and this dependency is usually evaluated experimentally using the wind tunnel tests for a specific cross-sectional shape of a bridge deck.

1.2 Solution of Flutter Equations

Once the aeroelastic forces are established (Eq. 1 and Eq. 2), the critical conditions, i.e. critical velocity for the onset of flutter can be calculated. The simplest way to establish critical wind velocity is to consider a rigid section model of the bridge deck with two degrees of freedom (2DOF model), namely vertical z (heave) and torsional α (pitch). The 2DOF equation of motion per unit length can be written as follows:

$$m\ddot{z} + c_z \dot{z} + k_z z = L_{ae}, \qquad I\ddot{\alpha} + c_\alpha \dot{\alpha} + k_\alpha \alpha = M_{ae} \quad (3)$$

where L_{ae} and M_{ae} are self-excited forces presented in Eq. 1 and Eq. 2, m is mass and I mass moment of inertia per unit length of the bridge cross-section and k_z and k_α are stiffnesses and c_z and c_α damping coefficients, for respective degrees of freedom.

For the case of coupled flutter critical condition, heave and pitch can be considered as harmonic motion with the same circular frequency:

$$z(t) = \bar{z} e^{i\omega t}, \alpha(t) = \bar{\alpha} e^{i\omega t} \quad (4)$$

After the substitution of Eqs. 1, 2 and 4 in Eq. 3 eigenvalue problem of stability of motion is formulated with flutter frequency and the critical wind speed as unknowns. Details related to the implementation of this solution could be found in Mannini (2006).

1.3 Time-Domain Approach

The flutter derivatives are not well suited for the time domain simulations, as being expressed as a function of frequency. As a counterpart to flutter derivatives in time domain, non-analytical functions can be estimated. They describe the development of the forces due to the sudden infinitesimal structural motions and these functions are called the indicial functions. Then the history of motion can be seen as a series of these infinitesimal step-wise increments. Under the assumption of linearity of load, the self-excited forces in time domain (counterparts of Eq. 1 and 2) can be expressed as convolutions of these indicial functions.

Usual practice to determine these functions is from the corresponding (measured) flutter derivatives (Borri and Höffer, 2000, Salvatori, 2007) taking the typical approximation of representing the indicial function as a sum of exponential filters:

$$\bar{\Phi}_{Rr}(s) = 1 - \sum_{n=1}^{N_{Rr}} \bar{a}_n^{Rr} \exp(-\bar{b}_n^{Rr} s) \quad (5)$$

where R=L or M and r= $\dot{z}$ /U and α , $s = 2Ut/B$ is the non-dimensional time, $\overline{a}_n^{Rr}$ and $\overline{b}_n^{Rr}$ are non-dimensional coefficients, and N_{Rr} is the number of terms chosen to approximate the indicial function $\overline{\Phi}_{Rr}$. Substituting harmonic motions into the previously mentioned convolution integrals, the aeroelastic load is again expressed in frequency domain, and in this form can be compared to the load based on flutter derivatives (Eq. 1 and 2). In this way the relationships between indicial function coefficients and flutter derivatives are obtained. Due to the nature of these relationships, the indicial functions (with non-dimensional coefficients $\overline{a}_n^{Rr}$ and $\overline{b}_n^{Rr}$ as unknowns) are then identified by means of nonlinear least square optimisation. Details of the method, which is followed within this work, are described in Salvatori and Borri (2007).

1.4 Quasi-Steady Approximation

When the reduced frequency of oscillation is small the time needed by the fluid particles to travel the bridge width (*B/U*) is small with respect to the period of oscillation of the structure (*2π/ω*). Consequently the fluid memory effects tend to become small and the quasi-steady theory can be used instead of the unsteady theory. In the quasi-steady approach at each time the forces do not depend on what happened before and the structure is seen by the flow as it is stationary with its instantaneous values of displacements and velocities. Expressions for self-excited forces using quasi-steady approximation are:

$$L_{ae}^{qs} = q_0 B \left[-\left(\frac{dC_L}{d\alpha} - C_D \right) \frac{\dot{z}}{U} + \frac{dC_L}{d\alpha} \alpha + \left(\frac{dC_L}{d\alpha} - C_D \right) \beta_z \frac{B}{U} \dot{\alpha} \right] \tag{6}$$

$$M_{ae}^{qs} = q_0 B^2 \left[-\frac{dC_M}{d\alpha} \frac{\dot{z}}{U} + \frac{dC_M}{d\alpha} \alpha + \frac{dC_M}{d\alpha} \beta_\alpha \frac{B}{U} \dot{\alpha} \right] \tag{7}$$

C_i are quasi-stationary coefficients which can be obtained normalizing mean along-wind drag force D, an across-wind lift force L and a pitch moment M as:

$$C_D(\alpha) = \frac{D}{q_0 B L_B}, \qquad C_L(\alpha) = \frac{L}{q_0 B L_B}, \qquad C_M(\alpha) = \frac{M}{q_0 B^2 L_B} \tag{8}$$

with respect to the mean angle of flow attack and $dC_i/d\alpha$ are its first derivatives. Since these coefficients depend on geometry of the cross-section, they are usually obtained experimentally from standard wind tunnel tests as a function of angle of attack α . The dimensionless parameters β_i represents the eccentricity parameter (Salvatori and Borri (2007)).

2. NUMERICAL EXAMPLE

Experiments are performed in the boundary layer wind tunnel of Ruhr University Bochum. The flutter derivatives are identified based on the forced vibration method. The experimental rig also provides a set up for the investigation of a fixed bridge deck placed under a certain angle of attack. The bare symmetric box section is considered. Detailed explanation of the measurements and the bridge cross-section can be found in Šarkić et al. (2012).

The obtained quasi-stationary coefficients are plotted in Figure 1 as a function of the angle of the flow attack. The oncoming wind velocity is around 4 m/s.

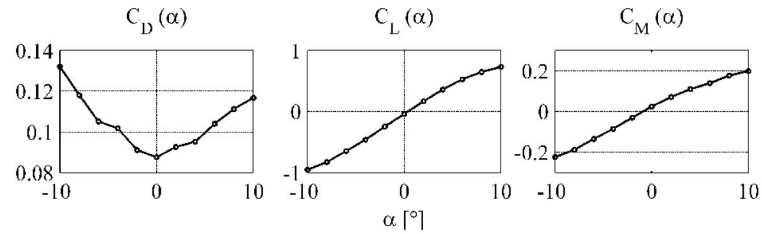

Figure 1. Quasi-stationary force coefficients

All eight flutter derivatives used in Eq. 1 and 2 are presented in Figure 2. They are presented for the range of reduced velocities till $U_{red}=30$ (where $U_{red}=U/Bf=2\pi/K$). Based on these functions, aeroelastic loads can be evaluated by the Eq. 1 and 2. Using the 2DOF model described in Section 2.2 critical velocity can be estimated. In this case structural properties of the used bridge deck are given in Table 1. The critical velocity is obtained around *Ucr=70.46m/s.*

In addition to flutter derivatives evaluated from the wind tunnel tests, Figure 2 also shows quasi-steady approximations of derivatives. They are evaluated comparing the coefficients which stand beside considered displacements and its first derivatives in two aeroelastic formulations: aeroelastic forces based on the derivatives (Eq. 1 and 2) and quasi-steady approximation (Eq. 6 and 7). As it can be observed, not all flutter derivatives have their counterparts in quasi-steady approximation. The missing ones are H_4* and A_4* which are not decisive related practical examples of bridge aerodynamics. It can be remarked that the approximations are following the same trend. Another unknown in case of quasi-steady approximation is related to the choice of eccentricity parameters β_i . They have strong influence on most important damping derivatives H_2^* and A_2^*. Namely, parameters β_i describe the position of the neutral points for the respective force components. In general case of the bridge section a common point does not exist (Salvatori, 2007, Neuhaus and Höffer (2011)). Therefore, β_i parameters must be evaluated from dynamic tests. Here following the procedure described in Neuhaus and Höffer (2011) parameters are evaluated from H_2^* and A_2^* curves which leads to $\beta_z=1.761$ and $\beta_\alpha=-1.378$. Based on the quasi-steady flutter derivatives from Figure 2 critical velocity is evaluated as more conservative value *Ucr=66.97m/s*.

Based on these measured flutter derivatives, indicial functions are evaluated and selected ones are presented in Figure 3. As it is already mentioned, as an outcome of the nonlinear optimisation for each indicial function non-dimensional coefficients $\overline{a}_n^{Rr}$ and $\overline{b}_n^{Rr}$ are identified. Based on the relationships between indicial function and flutter derivatives, flutter derivatives can be approximated also using these non-dimensional coefficients. This represents a quality check for identified indicial functions. Therefore in Figure 2 corresponding flutter derivatives evaluated based on identified non-dimensional coefficients $\overline{a}_n^{Rr}$ and $\overline{b}_n^{Rr}$ (related to indicial functions in Figure 3) are also included, showing satisfying agreement. These functions, in the form of convolution integrals can also be used to estimate critical wind velocity. Namely, the equations of motion presented in Section 2.2 can be solved for different velocities only with the aeroelastic forces expressed in time domain. By increasing the velocity, the critical velocity can be estimated as one which is causing unstable oscillations. Nevertheless, due to the equivalency of two approaches, namely frequency and time, both solutions should converge.

Table 1: Structural properties of considered bridge[4]

B[m]	m_z[kg/m]	m_α[kg/m]	f_z[Hz]	f_α[Hz]	ζ_z[-]	ζ_α[-]
18.3	12820	426000	0.142	0.355	0.006	0.005

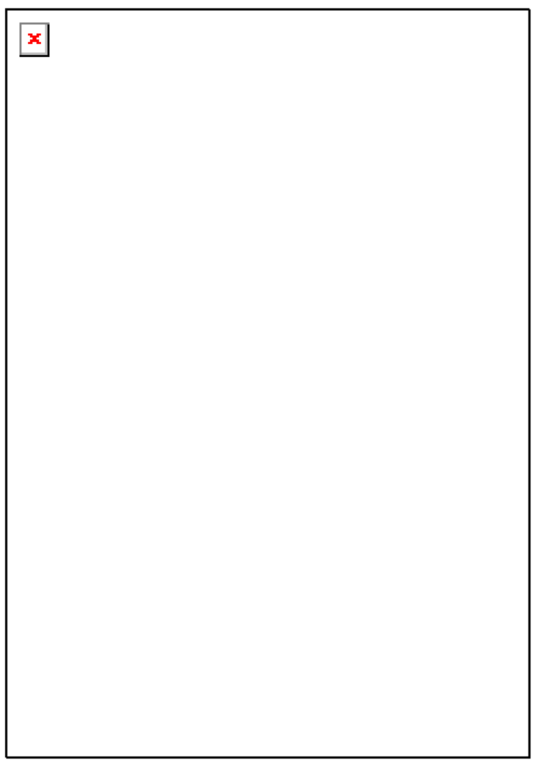

Figure 2. Flutter derivatives obtained directly from the measurements, using quasi-steady approximation and using optimized values from identified indicial functions

[4] Values are taken from Øiseth at al. (2010), where the similar bridge deck section is considered with the use of multi-modal analisys. Two heave and pitch modes are corresponding to the presented main coupled modes from the related paper.

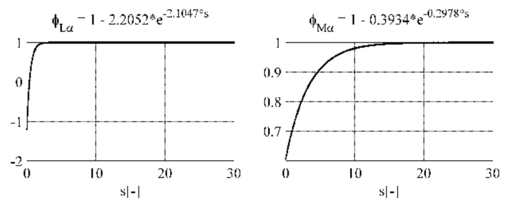

Figure 3. Selected indicial functions

3. CONCLUSIONS

The main concern of this paper is to present different bridge flutter approaches. Three different methods are presented: approach related to the frequency domain, approach related to the time domain and an approximation based on the quasi-steady coefficients.

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АНАЛИЗА ПРОБЛЕМА ФЛАТЕРА У ВРЕМЕНСКОМ И ФРЕКВЕНТНОМ ДОМЕНУ

Резиме: *Феномен флатера мостова представља вазан критеријум стабилности, који мора бити узет у обзир током процеса пројектовања. У овом раду су приказане различите методе за решавање проблема флатера, међу којима су приступ у фреквентном домену, еквивалентни приступ у временском домену, као и апроксимација на бази квази-стационарне теорије. Нумерички пример типичног попрепчног пресека моста прати приказане приступе.*

Кључне речи: *Флатер, решење флатера, модели нестационарног оптерећења*