GEOMETRICALLY NONLINEAR TRANSIENT ANALYSIS OF DELAMINATED COMPOSITE PLATES

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Summary: *This paper analyses geometrically nonlinear transient response of laminated composite plates with delaminations, using FEM. Numerical model is based on Reddy's Generalized Laminated Plate Theory. Geometric nonlinearity is accounted using von Karman strain field. Delamination openings in three orthogonal directions are implemented using Heaviside step functions. Equations of motion are derived using Hamilton's principle. Partial differential equations are reduced to a set of ordinary differential equations in time using Newmark's integration schemes. Nonlinear equations of motion are solved using constant average acceleration method and Picard's algorithm. Effects of crack size and position on transient response are shown.*

Keywords: *Composites, nonlinearity, transient analysis, delamination, FEM*

1. INTRODUCTION

Laminar composites are used as main load carrying members for different engineering purposes. They are composed from several orthotropic layers, which orthotropy comes from the high-strength fibers oriented in the arbitrary direction for each lamina individually. Lower-order plate theories are not adequate for the analysis of thick plates, because of the neglection of transverse shear deformation. While exposed to different transient loadings, laminar composites suffer high transverse shear deformation, so higher-order plate theories (such as layerwise plate theory) must be used. It is of the great importance that perfect bonding between the layers in laminar composites remains intact during the service life of the structure, to provide the composite panel to perform correctly. This is not always satisfied, so delamination between the layers often occurs, usually in the production phase or due to impact forces.
In this paper, extended version of Reddy's Generalized Laminated Plate Theory (GLPT) served as a basis for the development of enriched finite elements [1-3]. GLPT allows independent interpolation of in-plane and transverse displacement components, and includes possible jump discontinuities at delaminated layer interfaces. Piece-wise linear variation of in-plane displacements and constant transverse displacement are imposed.

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Using these assumptions, cross-sectional warping is taken into the calculation. Using the proposed model, transient response of delaminated plate is calculated using originally coded MATLAB® program. **The goal of the paper** is to compare transient responses of intact and delaminated composite plates, and to illustrate how the included geometric nonlinearity influences the plate response under different forcing functions.

2. GENERALIZED LAMINATED PLATE THEORY

We will analyze the laminated plate composed from *n* orthotropic material layers. *N* is the number of mathematical layers, while *ND* denotes the number of delaminated interfaces. GLPT is based on these assumptions: (1) all layers are perfectly bonded together, except in the delaminated area, (2) material is orthotropic, linearly elastic and follows Hooke's law, (3) von Karman kinematic relations are imposed, and (4) inextensibility of transverse normal is assumed. Detailed explanation of the theory is given in Refs. [2, 4]. Displacement field of the GLPT can be written as follows:

$$\begin{aligned}
u_1(x,y,z) &= u(x,y) + \sum_{I=1}^{N} u^I(x,y)\Phi^I(z) + \sum_{I=1}^{ND} U^I(x,y)H^I(z) \\
u_2(x,y,z) &= v(x,y) + \sum_{I=1}^{N} v^I(x,y)\Phi^I(z) + \sum_{I=1}^{ND} V^I(x,y)H^I(z) \\
u_3(x,y,z) &= w(x,y) + \sum_{I=1}^{ND} W^I(x,y)H^I(z)
\end{aligned} \tag{1}$$

In Eq. (1), (u,v,w) are mid-plane displacement components, (u^I,v^I) are undetermined coefficients which describe the layerwise expansion of the displacements, while (U^I,V^I,W^I) are displacement jumps of the I^{th} delamination. The delamination front is defined by enforcing the essential boundary condition $U^I=V^I=W^I=0$ on the crack boundary. $\Phi^I(z)$ are layerwise continuous functions of z-coordinate, while $H^I(z)$ are Heaviside step functions (see Ref. [1]). In this paper, linear layerwise variation of in-plane displacements is assumed, so in-plane displacements are piece-wise continuous through the plate thickness in the intact region. On the other hand, all displacement components are discontinuous at delaminated interfaces in delaminated area.

Geometrically nonlinear strain field is derived using von Karman kinematic relations and it is given in Eqs. (2). $\varepsilon_z=0$ because of the inextensibility of transverse normal. Constitutive equations for the k^{th} orthotropic lamina, for linearly elastic material are firstly derived in material coordinate system. We derive the constitutive relations for k^{th} lamina in the global coordinate system (see Ref. [4]) by using the transformation matrices for each layer. When deriving the dynamic equilibrium of the virtual strain energy (U), virtual work of external forces (V) and virtual kinetic energy (K), it is assumed that transverse loading q acts in the middle plane of the plate [5]. Dynamic relations for U, V and K are given in Eqs. (3). Dynamic version of virtual work statement is then derived using Hamilton’s principle. Using the integration of stresses through the thickness of the plate, we derive stress resultants and inertia terms (see Ref. [1]).

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u_1}{\partial x} + \frac{1}{2}\left(\frac{\partial u_3}{\partial x}\right)^2 = \frac{\partial u}{\partial x} + \sum_{I=1}^{N} \frac{\partial u^I}{\partial x}\Phi^I + \sum_{I=1}^{ND} \frac{\partial U^I}{\partial x} H^I + \\
&\quad + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\sum_{I=1}^{ND}\frac{\partial W^I}{\partial x}H^I + \frac{1}{2}\sum_{I=1}^{ND}\sum_{J=1}^{ND}\frac{\partial W^I}{\partial x}\frac{\partial W^J}{\partial x}H^I H^J \\
\varepsilon_y &= \frac{\partial u_2}{\partial y} + \frac{1}{2}\left(\frac{\partial u_3}{\partial y}\right)^2 = \frac{\partial v}{\partial y} + \sum_{I=1}^{N} \frac{\partial v^I}{\partial y}\Phi^I + \sum_{I=1}^{ND} \frac{\partial V^I}{\partial y} H^I + \\
&\quad + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\sum_{I=1}^{ND}\frac{\partial W^I}{\partial y}H^I + \frac{1}{2}\sum_{I=1}^{ND}\sum_{J=1}^{ND}\frac{\partial W^I}{\partial y}\frac{\partial W^J}{\partial y}H^I H^J \\
\gamma_{yz} &= \frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y} = \frac{\partial w}{\partial y} + \sum_{I=1}^{N} v^I \frac{d\Phi^I}{dz} + \sum_{I=1}^{ND}\frac{\partial W^I}{\partial y}H^I \\
\gamma_{xz} &= \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} = \frac{\partial w}{\partial x} + \sum_{I=1}^{N} u^I \frac{d\Phi^I}{dz} + \sum_{I=1}^{ND}\frac{\partial W^I}{\partial x}H^I \\
\gamma_{xy} &= \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} + \frac{\partial u_3}{\partial x}\frac{\partial u_3}{\partial y} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \sum_{I=1}^{N}\left(\frac{\partial u^I}{\partial y} + \frac{\partial v^I}{\partial x}\right)\Phi^I + \sum_{I=1}^{ND}\left(\frac{\partial U^I}{\partial y} + \frac{\partial V^I}{\partial x}\right)H^I + \\
&\quad + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \sum_{I=1}^{ND}\sum_{J=1}^{ND}\frac{\partial W^I}{\partial x}\frac{\partial W^J}{\partial y}H^I H^J
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\delta U &= \int_0^t \int_V \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}\right) dV dt \\
\delta V &= -\int_0^t \int_V \left(q(x,y,t)\delta w\right) dV dt \\
\delta K &= -\int_0^t \int_V \rho\left(\ddot{u}_1 \delta u_1 + \ddot{u}_2 \delta u_2 + \ddot{u}_3 \delta u_3\right) dV dt
\end{aligned}
\tag{3}
$$

3. FINITE ELEMENT MODEL

FE model based consists of the middle plane, *N* mathematical layers through the plate thickness (excepting the middle plane) and finally N*D* numerical layers in which debonding can occur. Isoparametric quadrilateral FE with 9 nodes is used. Proposed FE require only C^0 continuity of generalized displacements on element boundaries, and it is in detail explained in [1]. Using of different combinations of in-plane interpolations ψ_i and $\Phi^I(z)$ for through the thickness interpolation allows us to derive the variety of LW FE. All generalized displacements are interpolated using the same functions ψ_i:

$$(u, v, w, u^I, v^I, U^I, V^I, W^I) = \sum_{i=1}^{9} \left(u_i, v_i, w_i, u_i^I, v_i^I, U_i^I, V_i^I, W_i^I\right)\psi_i \qquad (4)$$

By substitution of assumed displacement field into virtual work principle, we obtain:

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{C}\dot{\boldsymbol{d}} + \left(\boldsymbol{K}^L + \boldsymbol{K}^{NL}\right)\boldsymbol{d} = \boldsymbol{F} \qquad (5)$$

where $\boldsymbol{K}^L$, $\boldsymbol{K}^{NL}$, $\boldsymbol{C}$ and $\boldsymbol{M}$ are element stiffness, nonlinear stiffness, damping and consistent mass matrices, respectively; $\boldsymbol{F}$ is the element force vector, while $\boldsymbol{d}$ is the displacement vector. Note that nonlinear element stiffness matrix $\boldsymbol{K}^{NL}$ is unsymmetrical in this formulation. Dots above the vector $\boldsymbol{d}$ denote the differentiation in time. Linear and nonlinear element stiffness matrices are given in Ref. [3]. In further calculations we will assume only undamped structural response, so Eq. (5) will be used in reduced form. Element mass matrix is given in Ref. [1]. Integration over element domain is performed numerically using Gauss-Legendre quadrature. For elimination of shear locking, selective integration should be used for thin plate situations.

The governing partial differential equations of the problem are solved numerically using Newmark's integration [6]. Accelerations and velocities are approximated using truncated Taylor's series. Constant average acceleration method is chosen because of its numerical stability. We obtain the solution by solving the algebraic system (6), where Δt is time increment. Once $\boldsymbol{d}_{n+1}$ is derived, velocities and accelerations in t_{n+1} are obtained.

$$\hat{\boldsymbol{K}}\boldsymbol{d}_{n+1} = \hat{\boldsymbol{F}}, \quad \hat{\boldsymbol{K}} = \left(\boldsymbol{K}^L + \boldsymbol{K}^{NL}\right) + \boldsymbol{M}\frac{4}{(\Delta t)^2}, \quad \hat{\boldsymbol{F}} = \boldsymbol{F}_{n+1} + \boldsymbol{M}\left(\ddot{\boldsymbol{d}}_n + \frac{4}{\Delta t}\dot{\boldsymbol{d}}_n + \frac{4}{(\Delta t)^2}\boldsymbol{d}_n\right) \qquad (6)$$

Because $[\boldsymbol{K}^{NL}]$ depends on the unknown solution $\{\boldsymbol{d}\}_{n+1}$, the assembled equation must be solved iteratively until the convergence criterion is satisfied. Picard's direct integration procedure is employed until the error is less than or equal to some value (say $\varepsilon \leq 1\%$).

4. NUMERICAL EXAMPLE, DISCUSSION AND CONSLUSIONS

It is assumed that transient loading lasts until $t=T_1$, and after that structure is allowed to freely oscillate during $T_1<t<T_2$. Forcing functions used in this example, as well as different crack sizes or positions, are shown in Figure 1. Mesh of 10×10 9-node elements is used. Uniformly distributed loading $F_0=1$ over whole plate area is prescribed, and motion of the plate center is plotted. Here, Δt=0.5ms, T_1=25ms, T_2=40ms.

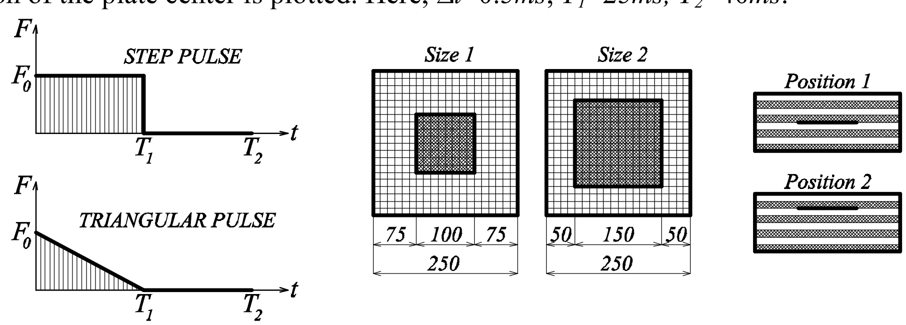

Figure 1. Different forcing functions, sizes and positions of delamination

An 8-layer simply supported composite plate with $(0/90/45/90)_s$ stacking sequence is considered. Side length of the plate is $a=b=250$mm, while overall plate height is $h=8\times h_k=2.12 mm$. Each layer is made of orthotropic material with following properties:

$$E_1 = 132\ GPa \qquad \upsilon_{12} = \upsilon_{13} = 0.291 \qquad G_{12} = G_{13} = 2.79\ GPa$$
$$E_2 = 5.35\ GPa \qquad \upsilon_{23} = 0.30 \qquad \rho = 1446.2\ kg/m^3$$

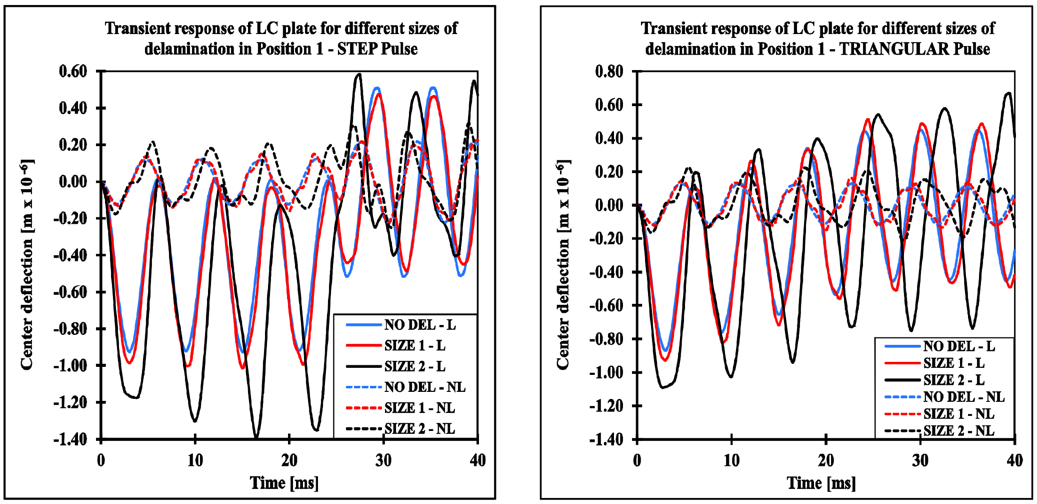

Figure 2. Influence of delamination size on linear and nonlinear transient response

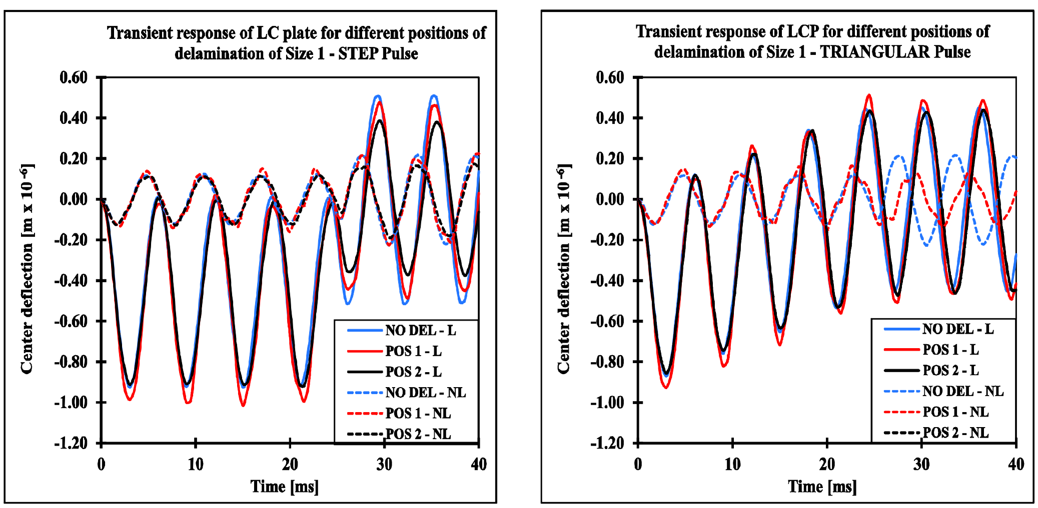

Figure 3. Influence of delamination position on linear and nonlinear transient response

The proposed FE model is capable to accurately catch the relative displacements of adjacent laminas in damaged area. Authors have verified the model for intact plates using the available data from the literature, in their preliminary investigations. Here, the set of new results for damaged plates is given, and comparison between linear and nonlinear structural response is shown in Figures 2-3. It is obvious that until delamination size is small enough, linear/nonlinear transient responses are not influenced severely, but the plate motion changes after increasing the delamination area above 35%. The presence of geometric nonlinearity influences the response by adding the bending stiffness to the model, which is resulted by reduction of the amplitudes and increasing of the frequency, both under step and triangular pulse. On the other hand, changing the position of delamination doesn't affect the frequency of oscillations in linear analysis, but in nonlinear case this influence is more pronounced, especially for triangular pulse, and it is shown that response becomes softer because of the reduction of the stiffness.

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ГЕОМЕТРИЈСКИ НЕЛИНЕАРНА ДИНАМИЧКА АНАЛИЗА ОШТЕЋЕНИХ КОМПОЗИТНИХ ПЛОЧА

Резиме: *Овај рад анализира геометријски нелинеаран динамички одговор ламинатних композитних плоча са деламинацијама, применом МКЕ. Нумерички модел је заснован на Reddy-евој слојевитој теорији плоча. Применом von Karman-овог поља деформације уведена је геометријска нелинеарност. Отварања прслине у три ортогонална правца су узета у обзир применом Heaviside-ових функција.*

Једначине кретања су изведене Hamilton-овим принципом. Применом Newmark-ове интеграције су парцијалне диференцијалне једначине редуковане на систем обичних диференцијалних једначина по времену. Нелинеарне једначине кретања су решене методом константног убрзања и Picard-овим поступком. Приказани су ефекти величине и положаја деламинације на динамички одговор.

Кључне речи: *Композити, нелинеарност, динамичка анализа, деламинација, МКЕ*