Project level pavement management optimization procedure combining optimal control theory and HDM-4 models

Jelena Cirilovic[a*], Goran Mladenovic[b] and Cesar Queiroz[b]

[a] *Intitute IMS, Belgrade, Serbia*
[b] *University of Belgrade, Belgrade, Serbia*

Abstract

The paper presents an optimal control theory-based procedure for finding the optimal timing and intensity of pavement maintenance treatments, which was adjusted based on the models for pavement deterioration and road user costs from the HDM-4 and RUCKS models. The model for improvement in pavement condition after a maintenance treatment was calibrated according to Paterson's bilinear model. The closed-form solution is then compared to the solution obtained by using genetic algorithms (GAs). In both methodologies special attention was given to the quality of the "optimal" solution in terms of evaluating: (i) the time between the maintenance treatments; (ii) minimal/maximal thicknesses of overlays calculated in the optimal maintenance plan; and (iii) parameters defining pavement condition before and after the maintenance treatment. The comparison between the two methodologies allowed analyzing limitations in each one of them and led to improvements in the "optimal" solution.

Keywords: Optimal pavement maintenance plan; Optimal control theory; Genetic algorithms; HDM-4; RUCKS

Résumé

Cette analyse présente une procédure fondée sur la théorie du contrôle optimal afin de trouver la fréquence et l'intensité optimale des traitements d'entretien des chaussées, qui ont été ajustés en fonction des modèles de détérioration de la chaussée et des coûts pour les usagers de la route en fonction des modèles HDM-4 et RUCKS. Le modèle d'amélioration de l'état de la chaussée après un traitement de maintenance a été calibré selon le modèle bilinéaire de Paterson. La solution de forme fermée est ensuite comparée à la solution obtenue en utilisant les algorithmes génétiques (GAs). Dans les deux méthodes une attention particulière a été accordée à la qualité de la solution "optimale" en termes d'évaluation: (i) du temps entre les traitements d'entretien; (ii) de l'épaisseur minimale/maximale des superpositions calculée dans le plan optimal de maintenance; et (iii) des paramètres définissant l'état de la chaussée avant et après le traitement de maintenance. La comparaison entre les deux méthodes employées a permis l'analyse des limitations de chacune d'entre elles et a conduit à des améliorations de la solution "optimale".

Mots-clés: plan optimal d'entretien des chaussées ; théorie du contrôle optimal; algorithmes génétiques; HDM-4; RUCKS

[*] Corresponding author
Tel.: +381 60 0772951; fax: +381 11 3692772
E-mail address: jelena.cirilovic@institutims.rs.

Nomenclature

s	Road condition
IRI	International Roughness Index [m/km]
s_0	Initial road condition-pavement roughness at the start of the analysis period [m/km]
s_{1i}	Roughness after maintenance treatment “i” is applied [m/km]
s_{2i}	Roughness at threshold value for maintenance treatment “i” [m/km]
t_0	Time before first maintenance treatment [years]
t_i	Time between maintenance treatments “i+1” and “i” [years]
τ	Time between maintenance treatments in a steady-state solution [years]
r	Discount rate [%]
C	Road User Costs [million US$/km]
M	Road Agency Costs [million US$/km]
w_i	Asphalt overlay thickness [mm] of treatment “i”
G	Improvement in condition after maintenance treatment is applied [m/km]
a_0	HDM-4 coefficient (value of 134)
a_1	HDM-4 coefficient (value of 0.7947)
a_2	HDM-4 coefficient (value of 0.0054)
K_{gm}	Calibration coefficient for environment (takes value 1)
K_{gp}	Calibration coefficient for deterioration rate (takes value 1)
m	Climatic factor, takes value 0.035 for Serbia
RDB	Roads Data Base
b_0-b_3	RUCKS coefficients for estimating user costs
c_1,c_2	Road Users costs calibration coefficients
m_1,m_2	Improvement function calibration coefficients

1. Introduction

Finding optimal pavement maintenance strategy on a project level, that includes timing and intensity of rehabilitation treatments over the analysis period, is one of the major problems dealt with in Pavement Management. Time spent on calculation holds the researchers back from searching for the “true” solution of the problem. Consequently various optimization techniques are developed that can be used for finding the “optimal” solution that is the closest to the “true” optima.

One of definitions of the “optimal” solution can be formulated as follows: if the deterioration model is known as well as the model describing the implications of applying certain type of maintenance activity, what is the timing and intensity of maintenance treatments for which the total society cost is minimal. Society cost is defined as the sum of the costs of road users and costs of maintenance treatments.

Optimal control theory was introduced in Pavement Management in the early 1990s; since then it has been employed in numerous methodologies to find the above-mentioned “optimal” solution. Among first researchers who solved the problem of finding the optimal maintenance plan as an optimal control problem were Friesz and Enrique Fernandez (1979), although in their work they accepted the generalization that pavement condition changes smoothly over time. This limitation was overcome by Tsunokawa and Schofer (1994) who used an approximation of saw-like curve, which represents current pavement condition. That was far more realistic solution procedure since the deterioration curve is approximated with a smooth line, and rehabilitation activities are shown as instantaneous improvements in pavement condition. The approach for solving the problem of optimal maintenance plan was to approximate condition saw-like curve with a continual curve which connects the middles of the spikes.

This methodology was later applied by many researchers, who improved solution by adding other parameters to the problem. Ouyang and Madanat (2006) found the optimal solution in case of constrained budget, for limited (Ouyang and Madanat, 2004) and unlimited (Li and Madanat, 2002) time period. Their findings (Tsunokawa and Schofer, 1994 and Li and Madanat, 2002) suggested that after the first few resurfacing pavement enters the “steady-state”, which significantly eases the calculations. Li and Ouyang (2006) optimized problem of multiple resurfacing activities in a finite horizon with realistic pavement performance models.

One of the applications of the total control theory suggests finding the "optimal" maintenance program in terms of minimal total cost as a closed-form solution, given as a combination of equations that represent models for: (i) pavement deterioration; (ii) improvement of pavement condition after a maintenance treatment; (iii) road user costs; and (iv) costs of maintenance treatments, which would be further explored within this paper.

Different tools were used in the past for solving the above mentioned optimization problems (Flintsch and Chen, 2004; Harvey, 2012), including linear, dynamic, and integer programming and genetic algorithms. Genetic algorithms (GAs) were discovered by Holland (1975) and later developed by Goldberg (1989), and is based on Darwin's theory of evolution. Since 1990, GAs are used in a number of methodologies to solve more complex optimization problems (Fwa et at., 1994; Morcous and Lounis, 2005; Santos and Ferreira, 2013). GA optimization begins with the development of a set of randomly selected "parent" of possible solutions. The characteristics that describe each solution are encoded in the chromosome. A solution is reached through an iterative process that involves copying, mutations and crossovers of the genes of chromosomes from a set of possible solutions, while choosing better solutions and removing bad solutions from the set, so the set evolves towards an optimal solution.

2. Research objective

The objective of this paper is to present and to compare two methodologies for developing optimal maintenance solution using:

a) Optimal control theory, and
b) Genetic algorithms,

and to assess the quality of the "optimal" solution in terms of (i) the time between the maintenance treatments; (ii) minimal/maximal thicknesses of overlays calculated in the optimal maintenance plan; and (iii) parameters defining pavement condition before and after the maintenance treatment. The application of the methodology is illustrated on case study with five representative road sections in Serbia.

3. Problem formulation and methodology

The pavement deterioration and road user costs models are based on corresponding models used in HDM-4 and RUCKS (Odoki and Kerali, 2000; The World Bank, 2011). The model currently includes asphalt concrete (AC) overlays as maintenance treatments and intensity of treatment is defined as thickness of AC overlay. Paterson's bilinear model (Paterson, 1990) is used for improvement in pavement condition after a maintenance treatment.

Road condition is expressed through roughness IRI that follows saw-tooth trajectory through time, meaning that pavement deteriorates to a point when maintenance treatment is applied (Figure 1). The application of maintenance treatment is represented with instantaneous improvement in pavement conditions (vertical line that links s_{2i}- threshold value and s_{1i} - roughness after treatment). Level of improvement depends on intensity of the treatment which is defined as the thickness of asphalt overlay, w_n and the pavement condition prior to the treatment s_{2n}.. After application of maintenance treatment, pavement continues to deteriorate which is represented with a line connecting point s_{1i} with $s_{2(i+1)}$. The goal in finding the "optimal" maintenance plan is to minimize the total society cost, discounted to present value, during the analysis period. Total society cost represents the sum of user costs and the cost of maintenance treatments that bears the company that manages the transport infrastructure (Road Agency costs) as shown by equation (1).

$$\min J = \sum_{n=1}^{\infty} \left\{ \int_{t_{n-1}}^{t_n} C\big(s(t)\big) \times e^{-rt} \times dt + M(w_n) \times e^{-rt_n} \right\} \tag{1}$$

$$\frac{ds(t)}{dt} = F\big(s(t)\big) \tag{2}$$

$$s_{2n} - s_{1n} = G(w_n, s_{2n}) \tag{3}$$

$$s(0) = s_0 \tag{4}$$

The decision variables are time, t_n and intensity of the maintenance treatment, w_n. Equation (2) states that the deterioration rate depends only on the current condition of the pavement, while Equation (3) states that the reduction in roughness depends only on the maintenance treatment (resurfacing thickness) and the condition of the pavement just before the resurfacing. The initial condition is given by Equation (4).

In this paper this problem was solved using two different approaches. In the first approach, problem was solved with the use of GAs for:

- Limited analysis period of 30 years, and
- Different trigger values and maintenance treatments are applied through time.

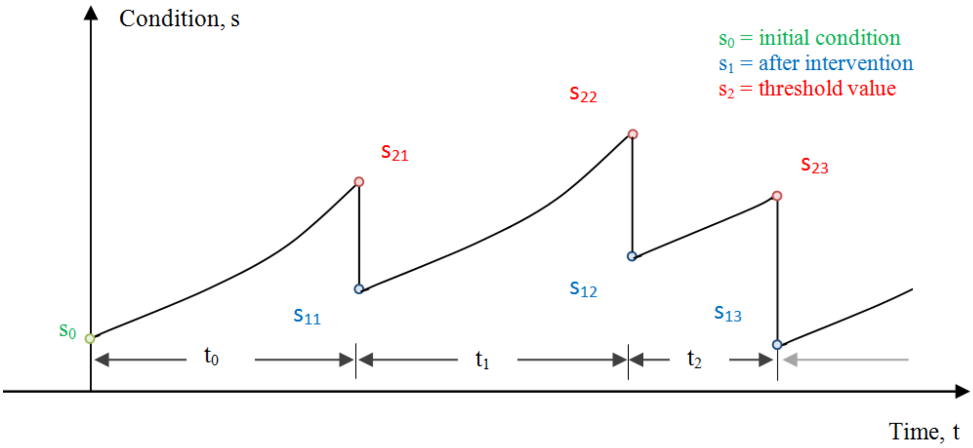

Fig. 1. Model formulation - condition curve

In the second approach, problem was solved as a closed form solution, as shown in Figure 2, with the following assumptions:

- Analysis period was unlimited;
- Pavement is already in the steady-state (same type of maintenance treatment is applied at the same trigger value through time). In other words, the decision whether to perform a certain type of maintenance treatment depends solely on the current condition of the pavement. The effect of this assumption is that the same maintenance treatment is applied whenever the system is in a certain condition.
- Condition at the beginning of the analysis period (s_0) in general can be better (s_0') or worse (s_0'') than the condition after maintenance treatment (s_1), but for reasons of simplicity of calculation is adopted that s_0 equals s_1.

Both approaches use the same set of calibrated equations that describe pavement deterioration (5), improvement function (6), road user cost function (7) and Road Agency cost function (8) and that were calibrated using HDM-4 model (pavement deterioration), RUCKS model (user costs), and Paterson's model (road effects) .

$$F(s_n) = f_0 \times s_n = f_0 \times e^{(f_1 \times t)} \quad (5)$$

$$\mathrm{G}(\mathrm{w}_n, \mathrm{s}_n) = \mathrm{s}_{2n} - \mathrm{s}_{1n} = \mathrm{g}_1 \times \sqrt{w_n} + g_2 \times s_{2n} + g_3 \quad (6)$$

$$C(s_n) = c_{1n} \times s_n + c_{2n} \quad (7)$$

$$M(w_n) = m_1 \times w_n + m_2 \quad (8)$$

Pavement deterioration model was formulated with the following equation (9):

$$s_1 e^{f_1 t_n} = s_2 \quad (9)$$

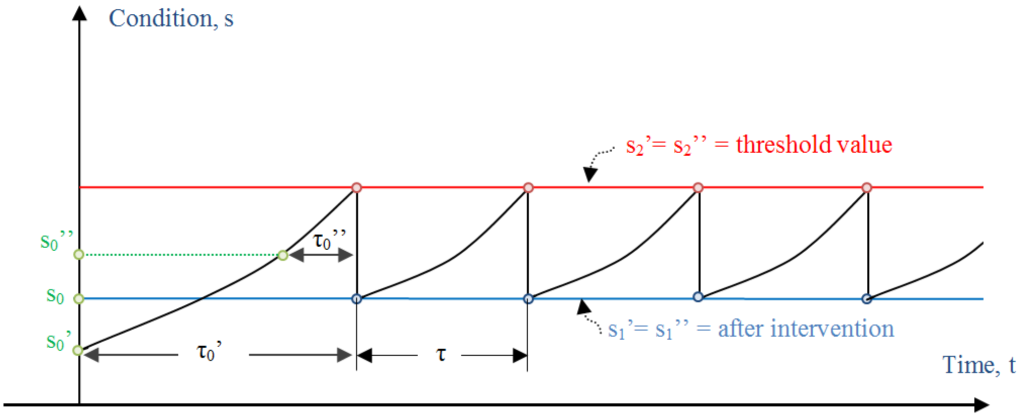

Fig. 2. Models formulation-closed form solution

Based on the equation (6) the required overlay thickness can be expressed as:

$$w_n = \left[\frac{s_{2n}(1-g_2-e^{-f_1\times t_n})-g_3}{g_1}\right]^2 \quad (10)$$

Road user costs are discounted with a discount rate r, to present value, and calculated based on the pavement condition in every year of period between two interventions, τ, according to the following equation:

$$C = \int_0^{\tau}(c_1 \times s_1 \times e^{f_1\tau} + c_2) \times e^{-r\tau}\,\delta t \quad (11)$$

If the analysis period is unlimited (time goes from 0 to +infinity), and the pavement is in "steady state" condition, equation (1) can be solved as a closed form problem, as shown by equation (12),

$$J(s_1,\tau) = \frac{\left\{m_1\left[\frac{s_1\times\left[(1-g_2)\times e^{f_1\times\tau}-1\right]-g_3}{g_1}\right]^2 + m_2\right\}\times e^{-r\times\tau} + \frac{c_1\times s_1}{f_1-r}\left(e^{(f_1-r)\times\tau}-1\right)}{1-e^{-r\times\tau}} \quad (12)$$

or expressed based on the thickness of asphalt overlay (w), and time between the two interventions, as presented by equation (13):

$$J(w,\tau) = \frac{(m_1\times w+m_2)\times e^{-r\times\tau} + \frac{c_1}{f_1-r}\left(e^{(f_1-r)\times\tau}-1\right)\times\frac{g_1\times\sqrt{w}+g_3}{e^{f_1\times\tau}\times(1-g_2)-1}}{1-e^{-r\times\tau}} \quad (13)$$

In the GA procedure, the same object function (Equation 1) is used and the same constrains (Equations 2, 3 and 4) are applied as in the close form solution procedure. The difference is that the time between the treatments is not constant (τ) but it changes through the analysis period, meaning that it may be different between every two subsequent interventions (t_n). Similarly, the thickness of asphalt overlay (w_n) can vary from treatment to treatment, which can lead to the more cost efficient solution based on variable intensity and timing of maintenance treatments through the analysis period. In GA solution it is possible to set constrains for minimal and maximal roughness after the treatment intervention is applied, i.e. from IRI = 1.0 m/km to IRI = 2.5 m/km; analysis period is set to be 30 years, with constraint for second and third treatment that cannot occur in interval shorter than 6 years.

GAs procedure is based on the laws of natural selection and gives approximate minimum of the objective function. Generations of solution are improving through iterations with the use of mutation, crossover and selection functions until reaching the point of local minimum. However, although the solution is approximate, the procedure allows great flexibility in the choice of maintenance strategies (different combinations of timing and intensity of the maintenance treatments). Therefore it is expected to obtain improved solution for local minimum compared to the previous methodology.

4. Case study

The case study demonstrates the application of methodology on five existing road sections with substantially different traffic from the Serbian main road network. Table 1 provides the most important characteristics of road sections.

Table 1. Data about the real sections from Serbian Road Database

Section ID	Section name	L [m]	AADT [veh./day/year]	SN [in]	CBR [%]	IRI [m/km]	Age [years]	$ESAL_{80kN}$ [mill.per year]
1	Klokočevac-Plavna, M-24	11,107	606	4.2	7.44	4.52	10	0.098
2	Granica BiH/SR-Kremna 1, M-5	17,380	1,031	4.1	19.43	3.10	5	0.109
3	Niška Banja – Crvena reka, M-1.12	21,490	5,050	4.8	10.03	2.01	1	1.351
4	M.Požarevac 2 - Vlaško Polje, M-23	7,688	9,909	3.9	6.19	2.50	1	0.902
5	Airport „N. Tesla“ – Beograd1 (Zmaj), M-1	6,770	49,230	5.4	6.70	2.99	5	5.421

The traffic ranges between 606 vehicles/day/year and 49,230 vehicles/day/year, while the current condition of these roads varies from IRI 2.05 m/km for Section 3 to IRI 4.52 m/km for Section 1.

4.1. Calibration procedure

The calibration of the road deterioration model (equation 9) was performed using the available data for each section: climate, terrain, traffic volumes, axle loading, structural number and the pavement age.

Program RONET (Archondo-Callao, 2009) uses simplified incremental deterioration model from HDM-4, which is to the great extent in accordance to the original HDM-4 pavement deterioration model, having R^2=0.999. That model was applied herein as well, because of its simplicity. RONET model calculates increment in roughness for each year of the analysis period according to the following equation:

$$dIRI = K_{gp}\left(a_0 \times e^{(K_{gm}\times m\times AGE)}(1+SNC\times a_1)^{-5}ESSO + a_2\times AGE\right) + K_{gm}\times m\times RI_a \qquad (14)$$

After implementing the values of HDM-4 calibration coefficients, equation (14) becomes:

$$dIRI = 134\times e^{0.06\times AGE}\times(1+SNC\times 0.7947)^{-5}\times ESSO + 0.0054\times AGE + 0.06\times RI_a \qquad (15)$$

The modified structural number depends on the value of the structural number of the pavement and of the California Bearing Ratio (CBR %) of the subsoil, as shown by equation (16):

$$SNC = SN + 3.51\times log(CBR) - 0.85\times(log(CBR))^2 - 1.43 \qquad (16)$$

Roughness at the end of each year (Equation 17) is then calculated as sum of roughness at the end of the previous year and yearly increment in roughness, dIRI. That increment, as shown by Equation 14, depends on the climate characteristics, traffic levels, structural number of the existing pavement, bearing capacity of the subsoil and the current pavement surface condition.

$$IRI(t)=IRI_{st}+dIRI \qquad (17)$$

Figure 3 shows calibrated pavement deterioration models for five sections over the 20-year analysis period.

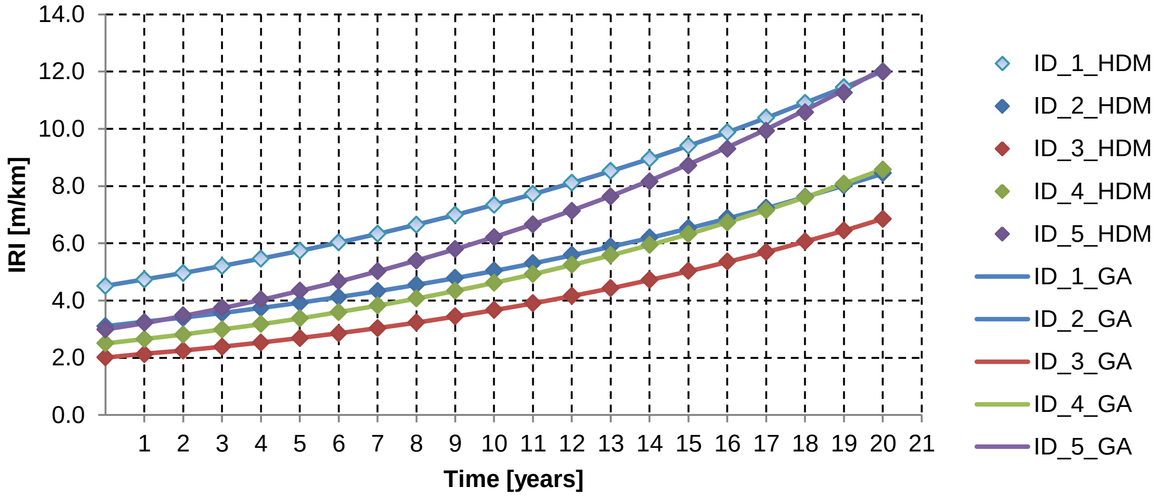

Fig. 3. Deterioration curves

Calibration procedure was performed using nonlinear regression, based on determination of minimum of sum of squared residuals. Table 2 shows values of calibration coefficients for all five sections, together with coefficients of determination which are close to 1.0.

Table 2. Calibrated coefficients for deterioration curves

Coefficient	Section ID				
	1	2	3	4	5
f_0	4.51	3.10	2.01	2.50	2.99
f_1	0.048981	0.049766	0.061144	0.061723	0.07053
R^2	1.000	0.999	1.000	1.000	0.998

Coefficient f_o actually represents the current condition of the road section that is s_0. Coefficient f_1 shows the rate of deterioration which is also in line with the calculated equivalent standard axle loading.

Road user costs depend on the pavement roughness, as shown by Equation (18).

$$\text{Unit Road User Costs } (\$/\textit{vehicle-km}) = b_0 + b_1\, IRI + b_2\, IRI^2 + b_3\, IRI^3 \qquad (18)$$

Table 3. Calibrated coefficients for deterioration curves

Coefficient	Section ID				
	1	2	3	4	5
AADT	606	1,031	5,050	9,909	49,230
b_0	0.32043	0.2671	0.41899	0.25754	0.3303
b_1	-0.00255	-0.00293	-0.00157	-0.00357	-0.00289
b_2	0.00132	0.0011	0.00185	0.00111	0.00116
b_3	-2.60E-05	-2.20E-05	-3.80E-05	-2.20E-05	-2.90E-05
c_1	0.016	0.010	0.017	0.009	0.010
c_2	0.247	0.228	0.376	0.222	0.289
R^2	0.989	0.975	0.973	0.964	0.987

Coefficients b_0 to b_3 are calculated using RUCKS software (World Bank, 2011). They depend on the road and vehicle characteristics, such as tire abrasion, consumption of motor fuel, consumption of fuel (diesel or gasoline), and yearly amortization cost for new vehicle, as well as of traffic composition on the particular road sections which was obtained by traffic counts. The coefficients are calculated separately for each of five sections (as shown in Table 3). The calibration of models was performed using nonlinear regression. The coefficients. c1 and c2 of Equation 7 are shown in Table 3. It can be noted here that the newly obtained User Cost model is in accordance with RUCKS model, having very satisfactory values of coefficients od determination, that is R2 is close to 1.0.

Several equations which describe the improvement function can be found in the literature. In this papers models for improvement function according to (Equation 19: Ouyang 2007; Equation 20: Ouyang and Madanat, 2006; Equation 21: Tsunokawa and Schofer, 1994) are calibrated based on Paterson's bilinear model, and later compared in order to find the best fit (as shown in Table 4). Method used was once again nonlinear regression, based on the minimum of sum of squared residuals. In all explored cases, the improvement function, G, depends on the thickness of the overlay (w_n), and on the current condition of the pavement (s_n), prior to intervention.

$$G(w_n, s_n) = \frac{g_1 * w_n}{g_2 * s_n + g_3} \quad (19)$$

$$G(w_n, s_n) = \frac{g_1 * w_n}{g_2 + \frac{g_3}{s_n}} \quad (20)$$

$$G(w_n, s_n) = g_1 * \sqrt{w_n} + g_2 * s_n + g_3 \quad (21)$$

Table 4. The calibration coefficients for the improvement function

Eq.	Coefficient			R^2
	g_1	g_2	g_3	
(19)	0.001	-0.002	0.028	0.696
(20)	0.012	-0.050	2.101	0.801
(21)	0.413	0.769	-4.617	0.911

Based on the results of fitted models (as presented in Table 4) Equation 21 was chosen as the closest to the Paterson model.

Road agency costs are expressed through equation (8). Cost of asphalt overlay depends on the thickness of the overlay (w_n), and was calibrated using two coefficients, m_1 that expresses the relationship between the cost per km and unit cost of asphalt, and m_2 that expresses the cost of equipment and personnel. Based on the local prices in Serbia, those coefficients are:

- m_1=2400 US$/km (7-m wide road equivalent), and
- m_2= 10000 US$.

4.2. Discussion of the results

Table 5 shows results obtained by solving optimization problem as a closed form solution, accepting all the above mentioned assumptions (i.e. that pavement is in steady-state condition, and that current condition of the pavement is actually condition after the intervention, i.e. s_1.).

Table. 5 The closed form solution

Parameter	Unit	Section ID				
		1	2	3	4	5
s_1	m/km	1.5	1.5	1.5	1.5	1.5
t	years	42	35	17	21	11
s_2	m/km	19.7	14.3	4.5	6.2	2.7
w	mm	-	-	128	150	106
j	US$	103,156,330	164,557,261	693,688,164	472,982,772	1,583,074,399

The results show logical relationships between the equivalent loading and proposed maintenance treatments and threshold values. This approach in general proposes more intensive resurfacing treatments with thickness from

11cm on a heavy loaded motorway section (Section 5) to “No interventions” on low volume road sections (Section 1 and Section 2).

Table. 6. The solution obtained through GA procedure

Treatment	Parameter	Unit	Section ID				
			1	2	3	4	5
1	s_{11}	m/km	3.3	3.4	1.5	1.5	1.5
	t_{11}	years	7	16	5	1	2
	s_{21}	m/km	6.4	6.9	2.7	2.7	3.4
	w_1	mm	41	41	82	81	89
2	s_{12}	m/km	3	2.8	1.5	1.5	1.5
	t_{12}	years	8	4	8	10	9.5
	s_{22}	m/km	4.9	4.1	2.4	2.8	2.9
	w_2	mm	40	41	79	82	84
3	s_{13}	m/km	2.9	2.7	1.5	1.5	1.5
	t_{13}	years	8	5	9	10	9.5
	s_{23}	m/km	4.4	3.6	2.6	2.8	2.9
	w_3	mm	40	40	80	82	84
4	s_{14}	m/km					
	t_{14}	years	7	5	8	9	9
	s_{24}	m/km	4.1	3.5	2.4	2.6	2.8
	w_4	mm					
	j		9,934,330	20,893,334	190,300,957	78,376,165	431,911,406

The genetic algorithms solution (Table 6) is consistent with the solution shown in table 5 for Sections 1 and 5, which are border cases in this study, Section 1 having very low traffic volume which doesn’t justify any intervention on the pavement, and Section 5 having relatively high level of traffic demanding regular interventions on the pavement. For Sections 3, 4 and 5 the proposed solution implies lower threshold values and thinner corresponding thicknesses of resurfacing. Consequently, the total discounted costs for a 30-year period are lower for the solution obtained with the use of GAs by approximately 30%.

5. Conclusions

This paper presented an application of two approached to develop optimal maintenance strategy for rad sections: the use of genetic algorithms and closed form solution. Important aspect of both methodologies was calibration of pavement deterioration and user cost model to be in accordance with simplified HDM-4 and RUCKS models, respectively. The pavement improvement function was calibrated based on Paterson bilinear model.

The closed-form solution showed logical relationship between frequency of the maintenance treatments and equivalent traffic loading. However, it implies generally more intensive pavement resurfacing interventions. The corresponding threshold values are relatively high, especially for sections with low or moderate traffic levels. GAs are more flexible in all the parameters. They recommend the frequency between the treatments between 4 and 16 years, and the intensity of the resurfacing between 4 and 9 cm, but generally advising more frequent resurfacing with thinner treatments, which leads to an overall better condition of the road network and lower total discounted costs. The limitation of the GA procedure is that it is not resistant to local minimum as well as the “optimal” solution may not be always practical and should be checked in terms of engineering judgment.

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