CALCULATION OF VERTICAL SEISMIC HYDRODYNAMIC LOADS

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Summary: *This paper analyzes Zangar's method for calculation of seismic hydrodynamic loads acting at the inclined contour of a concrete gravity dam. Different numerical models are compared with the procedure proposed by Zangar, and the regression formula that eliminates the observed discrepancies is proposed.*

Keywords: *seismic hydrodynamic loads, Zangar's hydrodynamic parabolic loads*

1. INTRODUCTION

Zangar [1] determined the hydrodynamic seismic pressure acting on a dam face (sloped or vertical), due to horizontal earthquake, using the electric analogue. He referred to the same assumptions as did Westergard in his study (rigid dam, small displacements, plane strain, infinite reservoir, and negligible compressibility of the water).
Zangar experimented on dams with constant upstream slopes, defined by angles, θ, of 0, 15, 30, 45, 60 and 75 degrees (Figure 1). The results were presented as a family of curves, enabling efficient use of these data.
Zangar found that the pressure distribution is almost parabolic, and for a generic depth Z from the water surface, the pressure can be obtained as:

$$p_z(Z) = C \cdot \alpha \cdot \gamma_w \cdot h \qquad (1)$$

where
- h is the total water depth,
- C is the dimensionless coefficient, giving the distribution and magnitude of pressure:

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$$C = \frac{C_m}{2}\left[\frac{Z}{h}\cdot\left(2-\frac{Z}{h}\right)+\sqrt{\frac{Z}{h}\cdot\left(2-\frac{Z}{h}\right)}\right] \tag{2}$$

where C_m (Table 1) being the maximum value of C,

– γ_W is the unit weight of water,

– α is the seismic coefficient, defined as a ratio between the maximum horizontal acceleration and the gravity acceleration.

Zangar proposed that the horizontal force $P_{Z,Z}$ above arbitrary depth, Z, is calculated as:

$$P_{Z,Z}(Z) = 0.726 \cdot p_Z(Z) \cdot Z \tag{3}$$

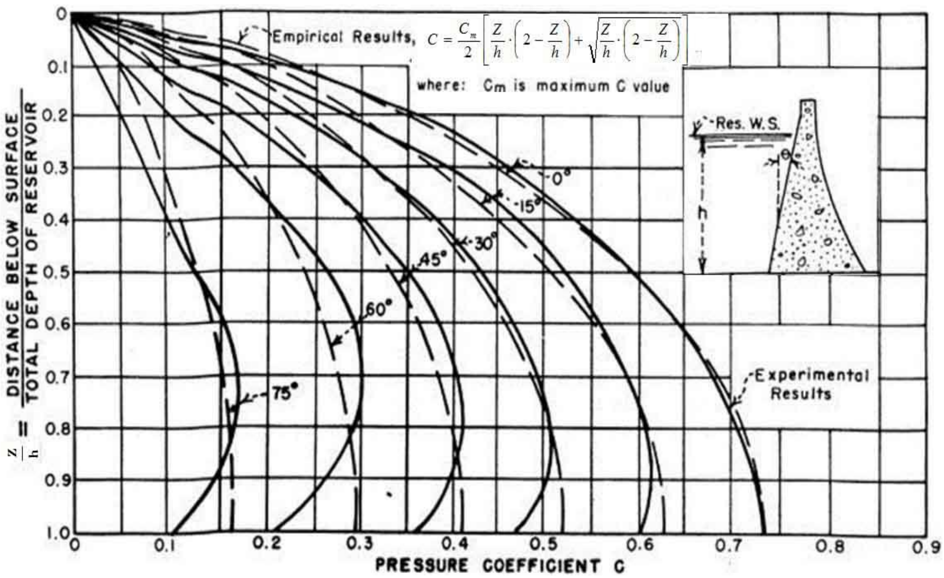

Figure 1: The results of Zangar's experiment

The total moment above Z due to the horizontal pressure is determined by:

$$M_{Z,Z}(Z) = 0.299 \cdot p_Z(Z) \cdot Z^2 \tag{4}$$

In the case of horizontal earthquake, the seismic pressure is perpendicular to the upstream face, hence the vertical and horizontal components of pressure.
The vertical component of pressure at a given depth Z is equal to the horizontal pressure at the same depth:

$$p_{Z,V}(Z) = p_Z(Z) \tag{5}$$

Vertical pressure force is given by:

$$P_{Z,Z}(Z) = 0.726 \cdot p_Z(Z) \cdot Z \cdot \tan(\theta) \qquad (6)$$

Zangar also analyzed the dams with combined vertical and sloping faces. A dam with vertical face longer than half the total height of the dam in contact with water is defined as vertical. However, a dam with sloping face shorter than half of the total height of the face (in contact with water) is classified as a sloping dam.
The angle of the slope, θ, (Figure 1) is obtained when the point of intersection of the dam with the surface of water is joined with the point of intersection of the dam with the terrain.

Table 1 Coefficient Cm as a function of the upstream-face slope

Angle θ (º)	0	15	30	45	60	75	90
C_m	0.735	0.630	0.520	0.410	0.295	0.160	0.000

2. PARADOX WITH ZANGAR'S HYDRODYNAMIC FORCES

In the literature and engineering practice, calculating Zangar's hydrodynamic forces is based on equations (3) and (6), with a constant coefficient (0.726), obtained by integration of the load over the entire depth of the reservoir. Nevertheless, this coefficient should not be a constant, since it depends on the depth, *Z*. While the vertical force (Eq. 6) can be obtained by multiplying the horizontal force (Eq. 3) with *tanθ*, in further analysis, horizontal forces will be considered only. Seismic hydrodynamic pressure curve between points (depths) Z_1 and Z_2 is presented at Figure 2. Parabolic curve is obtained by Zangar's experiment, while the shaded area presents its trapezoidal approximation. It is paradoxical that the force obtained using the expression (3), with a constant coefficient, may produce lower values than the trapezoidal approximation.

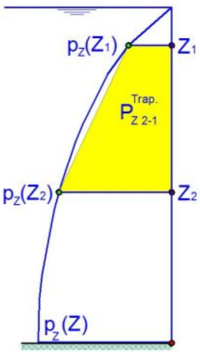

Figure 2: Comparison of the parabolic and trapezoidal loads

Since neither in Zangar's study, nor in the other literature, such a process of the integration was not justified by relevant evidence, there is a need for analysis of the results with appropriate analytical and numerical results.

3. ANALYTICAL SOLUTION

First, a direct integration of Zangar's hydrodynamic pressure along the *Z* axis is performed, giving the function of hydrodynamic force:

$$P_Z(Z) = \int p_Z(Z) \cdot dZ \tag{7}$$

$$P_Z(Z) = \int \frac{C_m}{2}\left[\frac{Z}{h}\cdot\left(2-\frac{Z}{h}\right)+\sqrt{\frac{Z}{h}\cdot\left(2-\frac{Z}{h}\right)}\right]\cdot\alpha\cdot\gamma_W\cdot h\cdot dZ \tag{8}$$

Without affecting the outcome, a transformation is made to a dimensionless coordinate:

$$X = \frac{Z}{h}, \tag{9}$$

whereby, nondimensional depth, *X* is:

$$0 \le X \le 1,$$

Hence the integral (Eq. 8) transforms into:

$$P_Z(Z) = K \cdot \int\left[1-(1-X)^2+\sqrt{1-(1-X)^2}\right]\cdot h\cdot dX \tag{10}$$

whereby:

$$K = \frac{C_m}{2}\cdot\alpha\cdot\gamma_W\cdot h = const \tag{11}$$

From here a replacement is introduced:

$$Y = 1 - X \tag{12}$$

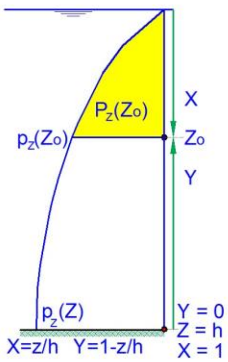

Figure 3: Physical interpretation of the dimensionless coordinates X and Y

Coordinate *Y* physically represents the dimensionless coordinate of the observed section “o” in relation to the bottom (Figure 3). In this manner it is obtained:

$$P_Z(Y) = -K \cdot h \cdot \int \left(1 - Y^2 + \sqrt{1-Y^2}\right) \cdot dY \tag{13}$$

And after integration:

$$P_Z(Y) = -K \cdot h \cdot \left[\frac{Y}{3} \cdot \left(3 - Y^2\right) - \frac{1}{2} \cdot ar\,cos(Y) + \frac{1}{2} \cdot Y \cdot \sqrt{1-Y^2} + C\right] \tag{14}$$

The integration constant is obtained from the condition:

$$P_Z(Y=1) = 0 \Rightarrow C = -\frac{2}{3} \tag{15}$$

The expression for Zangar's hydrodynamic force can be written in the form which was used previously, with dimensionless variable *Y* instead of the constant coefficient 0.726:

$$P_Z(Y) = p_Z(Y) \cdot h \cdot \frac{1}{6} \cdot \frac{3 \cdot ar\,cos(Y) - 2 \cdot Y \cdot \left(3 - Y^2\right) - 3 \cdot Y \cdot \sqrt{1-Y^2} + 4}{1 - Y^2 + \sqrt{1-Y^2}} \tag{16}$$

Equation (16) can be expressed as a function of *X*:

$$P_Z(X) = p_Z(X) \cdot \frac{h}{6} \cdot \frac{3 \cdot ar\,cos(1-X) - 2 \cdot (1-X) \cdot (2 + X \cdot (2-X)) - 3 \cdot (1-X) \cdot \sqrt{X \cdot (2-X)} + 4}{X \cdot (2-X) + \sqrt{X \cdot (2-X)}} \tag{17}$$

4. COMPARISSON BETWEEN ZANGAR’S AND ANALYTICAL SOLUTION

At Figure 4 a nondimensional hydrodynamic force, $\psi = P_Z(X)/P_Z(1)$, as a function of a nondimensional depth, X, is presented.
The analytical solution $\psi_Z(X)$ (Eq. 17) is designated by a solid line, Zangar’s solution $\psi_{Z,Z}(X)$ (Eq. 3) by a dashed line, and numerical approximation of the analytical solution $\psi_{Z,C}(X)$ (Eq. 18) by a dash-dotted line.

Comparing the results obtained using Zangar's expression (3), and the analytical expression (17), one can observe significant disagreement, except for a region close to the water-surface and at the bottom.

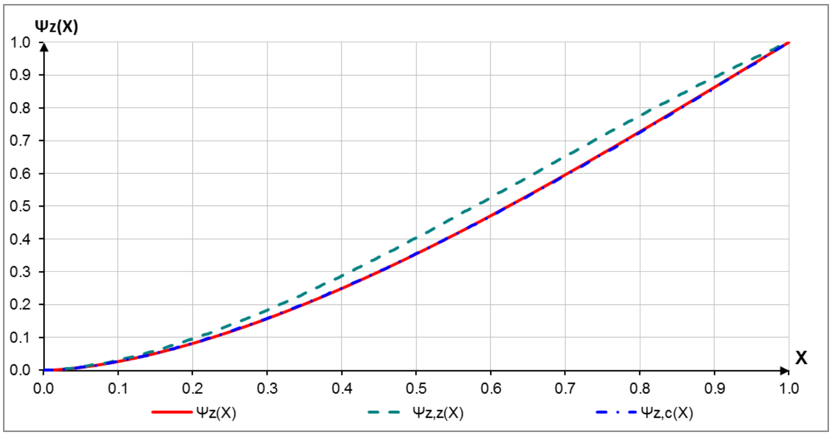

Figure 4: Comparison of hydrodynamic forces obtained along the dimensionless depth X

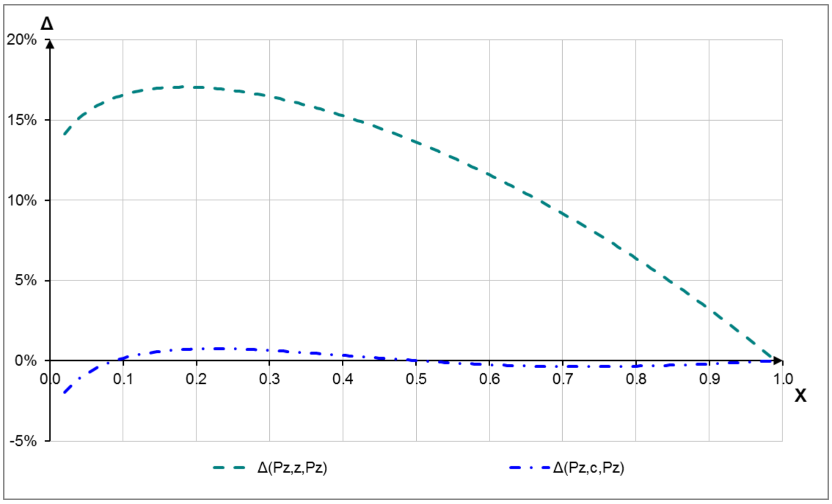

Figure 5: Comparison of relative disagreements, Δ

The largest disagreements occur at about 20% of the depth of the reservoir, reaching a relative error of over Δ = 17% (Figure 5). It should be noticed that the values obtained using Zangar's expression (with a constant coefficient of 0.726) are always larger than those obtained by the analytical solution.

Considering the nature of the observed disagreements, it can be concluded that computing the horizontal hydrodynamic forces by Zangar's expression is on the safe side. However, for certain shapes of the upstream face of the dam (Figure 6) the vertical hydrodynamic forces are not on the safe side:

1. At a horizontal segment of a vertically incised upstream contour, a larger stabilizing vertical force is produced using Zangar's solution than the real one (obtained by the analytical solution).
2. For a partly sloping vertical contour with an overhang, the vertical destabilizing hydrodynamic force computed by Zangar's solution is smaller than the actual one.

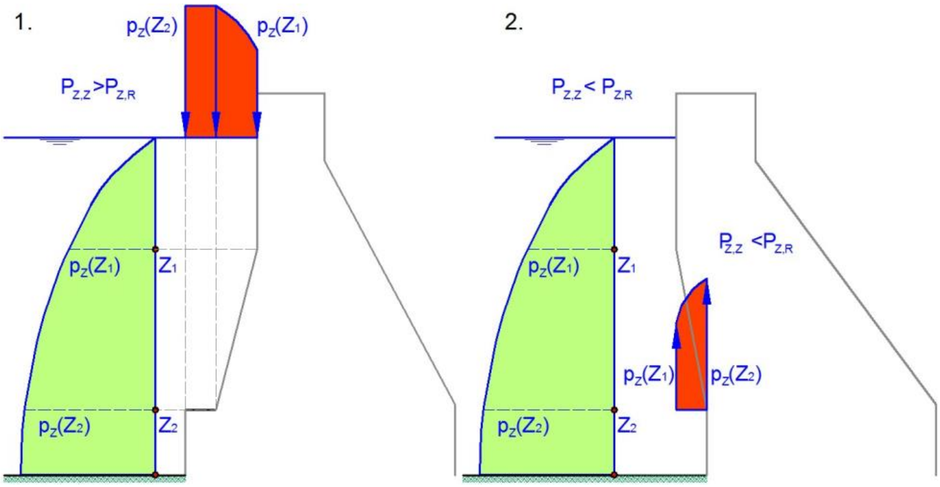

Figure: 6 Shapes of the upstream dam face for which the vertical hydrodynamic forces computed by Zangar's solution are not on the safe side

Since the analytical solution (Eq. 17) is rather complicated for computation, the authors recommend an approximate regression expression which yields insignificant disagreement with the analytical:

$$P_Z(X) = a \cdot X^b \tag{18}$$

where the constants *a* and *b*.
For *a* = 0.859 and *b* = 0.355 maximum disagreements from the values obtained by the analytical solution were less than 2% for the most unfavorable scenarios (Figure 5).

5. CONCLUSIONS

Zangar's method for calculation of seismic hydrodynamic loads acting at the inclined contour of a concrete gravity dam has been reconsidered.
For certain shapes of the upstream face of the dam the vertical hydrodynamic forces computed by Zangar's original solution are not on the safe side. Significant improvement of the Zangar's method has been achieved by analytical integration of the original data curves, enabling much safer computation of the hydrodynamic seismic forces.
An efficient regression expression is developed to approximate rather complicated analytical solution.

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ПРОРАЧУН ВЕРТИКАЛНИХ СЕИЗМИЧКИХ ХИДРОДИНАМИЧКИХ УТИЦАЈА

Резиме: *У раду се анализира Зангаров поступак прорачуна сеизмичких хидродинамичких оптерећења која делују на косу контуру гравитационе бетонске бране. Упоређују се различити нумерички модели са поступком који је предложио Зангар и даје се регресиона формула која елиминише уочена одступања.*

Кључне речи: *сеизмичко хидродинамичко оптерећење, Зангарова парабола*