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ESTIMATION OF THE DEFLECTION OF VERTICAL COMPONENTS USING PRECISE LEVELLING AND GNSS MEASUREMENTS ON PRECISE GEODETIC VERTICAL CONTROL NETWORK IN SERBIA

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ABSTRACT
This paper presents the combined method of determining the deflection of vertical components ξ and η based on the precise levelling and GNSS measurements. In general, there are several methods, but the combined method simplifies data collecting and subsequent processing of results in comparison with other methods. Components determination was performed on the high precision levelling network in Serbia, by calculating undulation on each point, so as the azimuth and length of each side of the network starting from a reference point. The length of sides is in the range of 20 km to 300 km. A point approximately located in the middle of the network was chosen as a reference point. By applying least-squares method, the estimated values of the components were obtained. In addition to the statistical evaluation of the deflection of vertical components, the paper shows a comparison of the obtained values with the values from the global Geopotential models, which justifies using the combined method for networks of similar design.

Keywords: deflection of vertical, components, geoid, GNSS, levelling

INTRODUCTION
The physical surface of the Earth is a very complex surface, which makes it difficult to define mathematically. For this reason, it is approximated by simpler surfaces such as ellipsoidal surfaces that are geometrically defined and a geoid that is defined in the physical sense. Since different approximations of the physical surfaces of the Earth are different, the ellipsoid and geoid are not overlapping. The difference between these two surfaces is called the undulation N. Based on the undulation, it is possible to define a geoid relative to the reference ellipsoid. In addition to the undulation, the other component involved in the definition of a geoid is called a deflection of the vertical that
represents the spatial angle between the normal and vertical directions at one point as seen in Figure 1. The deflection of vertical is an important parameter of the local gravitational field and is used for various practical purposes, such as the transformation of coordinates and azimuths. The deflection of vertical can be separated into two components, in the north-south direction $\xi$ and east-west $\eta$.

Different global geoid models have been developed to determine the deflection of vertical components, such as EGM96, CG03C, Astro-Geodetic etc. It is important to note that these global models relate to the whole planet. Therefore, it is possible that there are certain regions with less precision in the determination of geoid. For this reason, several techniques for calculating deflection of vertical components have been developed, such as combining the geometric leveling and GNSS measurements. Previous studies carried out by Soler et al. (1989), Vandenberg (1999), Magilevsky and Melzer (1994), Tse and Iz (2006) and Ceylan (2009), showed that this combined method can be also used in calculating the deflection of vertical components [1].

This paper presents the above-mentioned technique for determining the deflection of vertical components. The obtained values $\xi$ and $\eta$ were compared with values from EGM96. The experiment was performed on the precise geodetic vertical control network in Serbia. Each network point has geodetic coordinates $B, L$ and $h$ as well as orthometric height $H$. All calculations refer to the WGS84 ellipsoid.

![Figure 1. Deflection of the vertical and its components (Ceylan, 2009).](image)

On the basis of the given data, the azimuths $\alpha$ and the length of the geodetic lines $\Delta s$ from the central point of the network to the other points, as well as the six points on the perimeter of the network, were calculated. In addition, the undulation differences $\Delta N$ were calculated, after which the estimated values of the deflection of vertical
components were obtained by least-squares adjustment method, which is shown in the paper.

**CALCULATION OF THE DEFLECTION OF VERTICAL**

The relationship between the undulation and the deflection of the vertical in the direction of azimuth is given by the following equation [3]:

\[ \varepsilon = \frac{-dN}{ds}, \]  

which can be seen in the Figure 2.

![Figure 2. Relationship between geoid height and deflection of the vertical (Heiskanen and Moritz, 1984).](image)

On the other hand, the deflection of vertical of a known azimuth on any side can be calculated using the components \( \xi \) and \( \eta \) [3]:

\[ \varepsilon = \xi \cos(\alpha) + \eta \sin(\alpha). \]  

Replacing \( \varepsilon \) from the equation (1) to the equation (2) gives [3]:

\[ -\frac{dN}{ds} = \xi \cos(\alpha) + \eta \sin(\alpha), \]

where \( dN \) and \( ds \) are differentially small values of the undulation difference and the distance between the two close points, respectively.
Equation (3) can be approximated as [1]:

\[
\frac{\Delta N}{\Delta s} \approx \xi \cos(\alpha) + \eta \sin(\alpha),
\]

(4)

where \(\Delta N\) and \(\Delta s\) are obtained based on geodetic measurements.

The undulation difference \(\Delta N\) between two points 1 and 2 is calculated according to:

\[
\Delta N_{12} = N_1 - N_2,
\]

(5)

where \(N_1 = h_1 - H_1\) and \(N_2 = h_2 - H_2\).

By substituting \(N_1\) and \(N_2\) in equation (5), the following can be written:

\[
\Delta N_{12} = N_1 - N_2 = (h_1 - H_1) - (h_2 - H_2) = \Delta h_{12} - \Delta H_{12}.
\]

(6)

Finally, by replacing equation (6) with equation (4), the following is obtained:

\[
- \frac{\Delta h_{12} - \Delta H_{12}}{\Delta s} \approx \xi \cos(\alpha) + \eta \sin(\alpha).
\]

(7)

For the purposes of the experiment, the azimuths are calculated according to the following equation [2]:

\[
\alpha_{12} = \tan^{-1}\left(\frac{-X_{12} \sin(L_1) + Y_{12} \cos(L_1)}{-X_{12} \sin(B_1) \cos(L_1) - Y_{12} \sin(B_1) \sin(L_1) + Z_{12} \cos(B_1)}\right).
\]

(8)

where:

- \(X_{12} = X_2 - X_1\),
- \(Y_{12} = Y_2 - Y_1\) and
- \(Z_{12} = Z_2 - Z_1\).

Geocentric coordinates \(X\), \(Y\) and \(Z\) were obtained by transformation from geodetic coordinates \(B\), \(L\) and \(h\).

Length of geodetic line of one side of the network was obtained on the basis of [2]:

\[
\Delta s = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}.
\]

(9)
PRACTICAL PART

The network (Figure 3) consists of 1073 points, distributed around the Republic of Serbia. Each point of the network is given by its geodetic coordinates B, L and h, as well with the orthometric height H.

![Figure 3. The spatial distribution of the High precision levelling network of Serbia used in calculations](image)

For the center point, a point is chosen which is the geometrically closest to the center of the network, to which the calculated and estimated deflection of vertical components are related. Table 1 and 2 shows the data for the center point.
Table 1: Data of the center point.

<table>
<thead>
<tr>
<th>Point</th>
<th>B [°]</th>
<th>L [°]</th>
<th>h [m]</th>
<th>H [m]</th>
<th>N [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/481</td>
<td>44.183360172</td>
<td>21.111020422</td>
<td>153.034</td>
<td>108.729</td>
<td>44.31</td>
</tr>
</tbody>
</table>

Geodetic coordinates of all points in the network are transformed into geocentric [5], in purpose of calculating $\alpha$ and $\Delta s$ according to formulas (8) and (9). In addition to the aforementioned, the undulation differences $\Delta N_{12}$ from the central point to all points of the network were also calculated. Least-squares adjustment method was applied to the measurements by forming corrections equations based on the equation (4):

$$v = \cos(\alpha)d\xi + \sin(\alpha)d\eta - \frac{\Delta N}{\Delta s}$$  \hfill (10)

Table 2: Geocentric coordinates of the center point.

<table>
<thead>
<tr>
<th>Point</th>
<th>Radius of Curvature in the prime vertical</th>
<th>Geocentric coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nr [m]</td>
<td>X [m]</td>
</tr>
<tr>
<td>4/481</td>
<td>6388532.533</td>
<td>4273925.140</td>
</tr>
</tbody>
</table>

On the basis of (10), a $1068 \times 2$ coefficient matrix $A$ was formed:

$$A = \begin{bmatrix} \cos(\alpha_1) & \sin(\alpha_1) \\ \cos(\alpha_2) & \sin(\alpha_2) \\ \vdots & \vdots \\ \cos(\alpha_{1068}) & \sin(\alpha_{1068}) \end{bmatrix},$$

as well as $1068 \times 1$ vector of absolute terms of correction equations $f$:

$$f = \begin{bmatrix} \Delta N \\ \Delta s_1 \\ \vdots \\ \Delta s_{1068} \end{bmatrix}.$$  \hfill (11)

Normal equations coefficient matrix $N$ was obtained as [4]:

$$N = A^T PA,$$
where \( P = 1/\Delta s_{km} \) represents weight matrix.

Vector of the absolute terms of normal equations \( n \) was obtained as [4]:

\[
    n = A^T Pf. \tag{12}
\]

On the basis of equations (11) and (12), a vector of estimated unknown parameters \( \hat{x} \) is calculated [4]:

\[
    \hat{x} = -N^{-1} n. \tag{13}
\]

The estimated values of the deflection of vertical components after the least-squares adjustment were:

\[
    \hat{\xi} = -0.6552'' \text{ and } \\
    \hat{\eta} = -3.9811''.
\]

Obtained values were compared with values from the global EGM96 model, as shown in the Table 3.

Table 3: Comparison of estimated values with values obtained from EGM2008.

<table>
<thead>
<tr>
<th>Estimated values</th>
<th>EGM2008 values</th>
<th>Deviation of the estimated components</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\xi} [''] )</td>
<td>( \hat{\eta} [''] )</td>
<td>( \xi [''] )</td>
</tr>
<tr>
<td>-0.6552'</td>
<td>-3.9811'</td>
<td>4.95748'</td>
</tr>
</tbody>
</table>

The values of \( \Delta \xi \) and \( \Delta \eta \) in Table 3 represent the deviation of the estimated components values from the values obtained from EGM96.

Although in previous papers and experiments this method has been presented as applicable, based on obtaining deviations from global models, the question arises as to the justification of the application of the combined method in determining the deflection of vertical components. Consequently, the conclusion is drawn that additional tests and research should be carried out to obtain an adequate assessment of the quality of the combined method.

**CONCLUSION**

This paper presents the method for determining the deflection of vertical components by combining the measurement of the geometric leveling and GNSS. For this purpose, the precise geodetic vertical control network was used in Serbia. By geometric leveling the orthometric heights were obtained and by GNSS measurements the ellipsoidal heights
were obtained. The central point was defined on which azimuths and the length of the geodetic lines were calculated. The measurements were adjusted by the least-squares adjustment method, based on which the estimated values of the components $\hat{\xi} = -0.6552$" and $\hat{\eta} = -3.9811$" were obtained.

After comparing with the values of the global geoid model EGM96, it was concluded that there are certain deviations. Therefore, the application of this method is still not definitely justified, but additional testing and research are required.

REFERENCES


