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# THREE-DIMENSIONAL ROSETTES BASED ON THE GEOMETRY OF CONCAVE DELTAHEDRAL SURFACES

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**Abstract:** In this paper concave deltahedral surfaces are applied to link the two concepts of geometric rosette design – the polar distribution of the unit element (circular arc) around the center of the contour circle and the rosettes obtained by means of regular polygons. Forming composite polyhedral structures based on the geometry of concave deltahedral surfaces over a  $n$ -sided polygonal base, we have demonstrated one possible method of geometrical generation of three-dimensional rosettes. The concave polyhedral surfaces are lateral surfaces of the concave polyhedrons of the second, fourth and higher sorts, consisting of series of equilateral triangles, grouped into spatial pentahedrons and hexahedrons. Positioned polarly around the central axis of the regular polygon in the polyhedron's basis and linked by triangles, the spatial pentahedrons and hexahedrons form the deltahedral surface. The sort of the concave polyhedron is determined by the number of equilateral triangle rows in thus obtained polyhedron's net. In this study, composite polyhedral structures whose surface areas form the three-dimensional rosette are obtained through the combination of concave cupolae of the second sort (CC-II), concave cupolae of the fourth sort (CC-IV), concave antiprisms of the second sort (CA-II) and concave pyramids (CP). By means of elongation, gyro-elongation and augmentation of the listed concave polyhedrons it was possible to generate complex polyhedral structures, which can be used to create three-dimensional rosettes. The parameters of the solids were determined constructively by geometric methods and analytical methods which use iterative numerical procedures.

**Keywords:** rosette, polyhedron, architecture, triangle, geometry

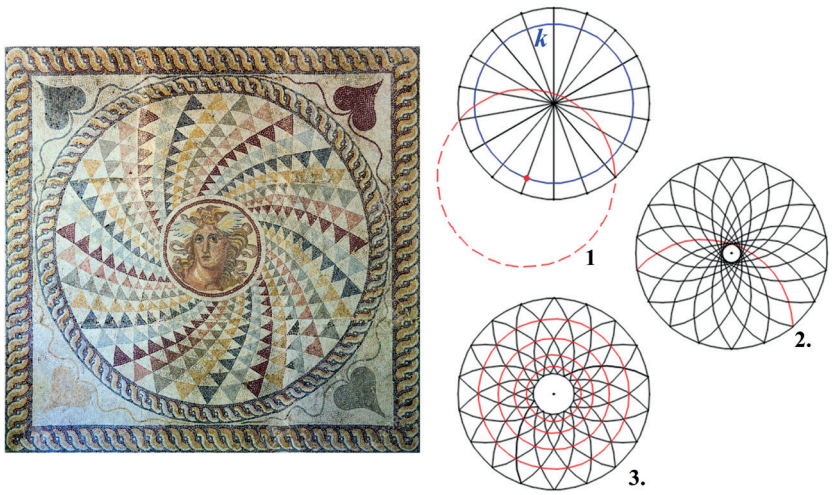


Fig. 1

**INTRODUCTION**

Mosaic, as a sort of a decorative finish of wall and floor surfaces, has been developing the link between geometry and design since the earliest times. The subset of thus finished decorative surfaces is formed within a circle of larger or smaller diameter. Figure 1 shows the mosaic floor with Medusa head found in Zea, Piraeus, 2<sup>nd</sup> century AD. The design is based on the geometrical construction of circular arcs whose curve centres are located on the referent circle, marked as *k*. The petal motif shown below is obtained by rotating the given circular arc around the centre of the contour circle. The petal number is equal to the chosen number of sections of the contour circle. To facilitate the rest of the procedure, the petals of thus deformed geometrical figure are further divided by concentric circles whose centers coincide with

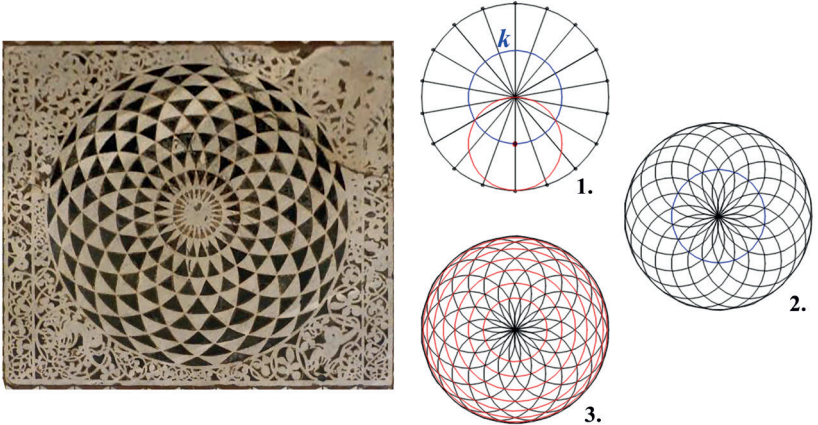


Fig. 2

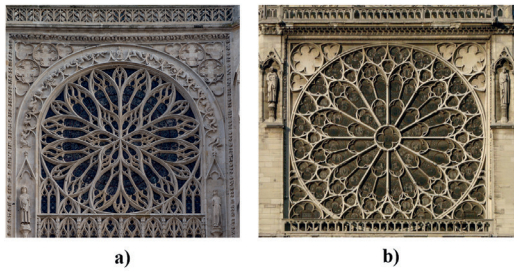


Fig. 3

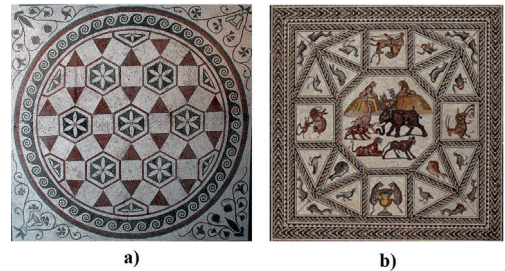


Fig. 4

the contour circle center, and whose radius is determined by the intersection points of two neighboring petals. The radii of concentric notch circles grow progressively, thus eventually forming the concentric bands. The number and width of thus obtained bands depends on the radius of the referent circle  $k$ .

Geometrical reconstruction of the decorative circular motif shown in Figure 2 is based on the polar distribution of circles whose centres lie on the referent circle  $k$ . The radius of the referent circle  $k$  is half that of the contour circle. Similarly to the illustration above, the petals of thus defined geometric figure are further divided by concentric circles whose centre coincides with that of the contour circle, and whose radius is determined by the intersection points of each two neighboring petals. The width of the concentric bands obtained by this division decreases as the distance from the centre of the contour circle centre, so that the obtained geometrical figure creates an optical illusion of a spherical surface.

We have shown only two characteristic illustrations of the geometrical construction of circular decorative surfaces which Albrecht Dürer termed “rosette”<sup>1</sup>. The rosette becomes a dominant architectural detail in the first half of the 12<sup>th</sup> century in France, when the builders of religious architecture, under the influence of Neoplatonic ideas, became fascinated by light as a means to connect with God.<sup>2</sup> Conceptually, the Christian church became the temple of light – a temple bathed in the light of God. With an altered construction system and the use of new constructive elements, it became possible to install spectacular stained glass windows, through which the filtered sunlight entered the cathedral. In this system, the unique rosette-shaped window opening stands out. Its circular shape and complex geometry have made this architectural element the most representative product of Gothic applied art.

We can now identify several basic characteristics of the shown rosettes. Their primary feature is the geometrical construction based on polar distribution, i.e. rotation of the unit figure around the contour circle centre. The second is the application of modular elements, which facilitates implementation. The final characteristic is the presence of color in the final rosette design.

The rosettes from Roman mosaics in Figure 4 indicate that the tendency to form rosettes by applying polygonal shapes is rather old. More precisely, these examples contain a triangle, a square, a hexagon, an octagon and a dodecagon. We can observe the geometric pattern of 2D tiling, where polygonal shapes are arranged in a circular motif. The geometric construction is characterized by rotational symmetry, which is the fundamental link with Gothic rosettes, in themselves a three-dimensional stone law-reliefs.

1 K. Williams, “Spirals and Rosettes in Architectural Ornament”, *Nexus Network Journal*, Volume I, Issue 1–2, 1999, 129–138.

2 A. Samper and B. Herrera, “A Study of the Roughness of Gothic Rose Windows”, *Nexus Network Journal*, July 2016, Volume 18, Issue 2, pp 397–417.

Islamic geometrical patterns fully exploit regular polygons as a matrix to shape decorative architectural elements. Their application is a full integration of geometry with architecture. The base of these patterns consists of regular constructible polygons (such as hexagons and octagons) and star-shaped polygons that are formed from them. These shapes represent the rosette leaves. Depending on the number of vertices ( $n$ ) of regular polygons generated from them, a new term is introduced in the classification – the ‘geometrical  $n$ -point pattern’<sup>3</sup>, where the type of the rosette depends on the polygon from which it is derived. The evolution of Islamic geometrical patterns can be followed through the type of use of  $n$ -sided polygons, from the hexagon to more complex polygon types and through the rosettes formed from them.

Geometric patterns with 6 and 8 points, obtained by applying the hexagon and the octagon, begin to dominate the Islamic architecture as early as the end of the 9<sup>th</sup> century. There was also an art movement (11<sup>th</sup>–13<sup>th</sup> century) which brought about a radical change in how the conventional geometrical patterns are applied when it introduced the patterns with 7, 9, 11 and 13 points. Today, we notice the use of non-constructive polygons in the applied art, which bears importance for this study.

Biggest value lies in how they are mutually combined 6-, 8-, 10-, 12-, and 16-point pattern geometrical patterns. Such combination was the guideline followed by the new art movement in the history of Islamic geometrical patterns. During the 15<sup>th</sup> and the early 16<sup>th</sup> century geometrical 16-point patterns and combined geometrical patterns were highly popular<sup>4</sup>.

## CONCAVE PYRAMIDS OF THE SECOND SORT

In this study we applied concave deltahedral surfaces to link the two concepts of geometric rosette design presented above: the one where a rosette is obtained by polar distribution of the given element (circular

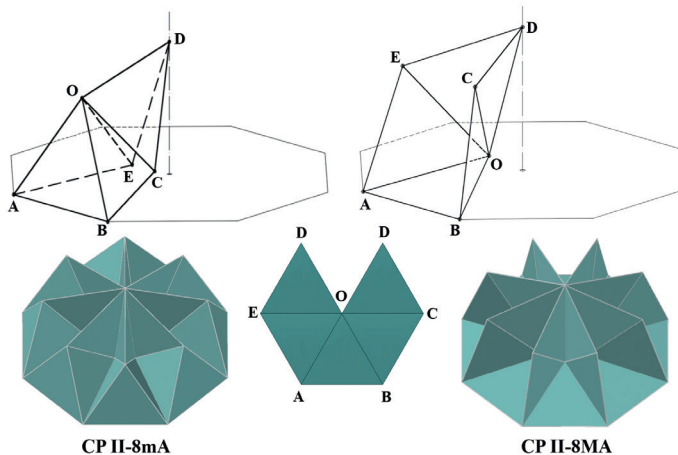


Fig. 5

3 Y. Abdullahi and M. Rashid Bin Embi, “Evolution of Islamic geometric patterns”, *Frontiers of Architectural Research*, Volume 2, Issue 2, 2013, 243–251.

4 Dabbour Loai M.: Geometric proportions: The underlying structure of desing process for Islamic geometric patterns”, *Frontiers of Architectural Research*, Volume 1, Issue 4, December 2012, pp 380–391

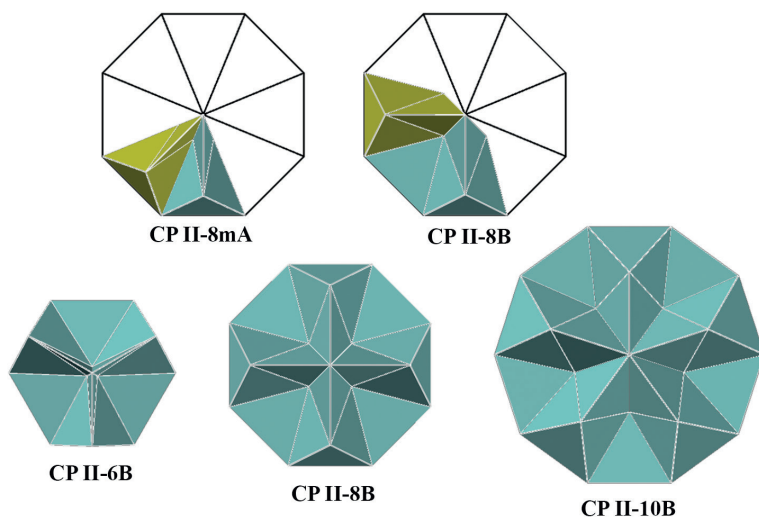


Fig. 6

arc) around the center of the contour circle and the other in which the rosette is produced through application of regular polygons.

We start from the concave pyramids of the second sort (CP-II), which are formed by polar distribution of the unit cell around the axis vertical to the plane of the regular polygonal base. In Figure 5, the base is the regular octagon. The unit cell is a set of five equilateral triangles in a plane which, grouped around the common vertex  $O$ , form a spatial pentahedron. Thus formed deltahedral lateral surface is the convolution surface. The sort of the pyramid is determined by the number of equilateral triangle rows in the planar net of deltahedral lateral surface. In concave pyramids of the second sort there are two rows of equilateral triangles.

Geometrical generation of concave pyramids of the second sort is based on finding the position of the vertex of the spatial pentahedron which meets the condition that the vertices  $A$  and  $B$  are located on the sides, with vertex  $D$  on the axis of the polygonal base. In our previous studies<sup>5</sup>, we showed that there are two types of concave pyramids of the second sort above the same polygonal base. If the common vertex  $O$  is indented, we obtain the concave pyramid of greater height (CP II-nM). Conversely, if the common vertex  $O$  is protruding, the placement of others vertex of the spatial pentahedron generates the concave pyramid of smaller height (CP II-nm).

Apart from the height-based division of the concave pyramids of the second sort, which inevitably results in a differently shaped deltahedral net, our research also showed that there are two ways to generate concave pyramids<sup>6</sup>. In the first manner, termed type A, the lateral surface contains the number of unit cells which is equal to the number of sides of the polygonal base (Figure 6). In other words, the

5 M. Obradović, S. Mišić and B. Popkonstantinović, "Concave Pyramids of Second Sort – The Occurrence, Types, Variations", in: *Proceedings of the 4<sup>th</sup> International Scientific Conference on Geometry and Graphics, moNGeometrija 2014*, ed. S. Krsaić, Vol 2. Vlasina, 2014, 157–168.

6 M. Obradović, S. Mišić and B. Popkonstantinović, "Variations of Concave Pyramids of Second Sort with an Even Number of Base Sides", *Journal of Industrial Design and Engineering Graphics (JIDEG) – The SORGING Journal, Volume 10, Special Issue, Fascicle 1*, ed. D. Marin, Brasov, 2015, 45–51.

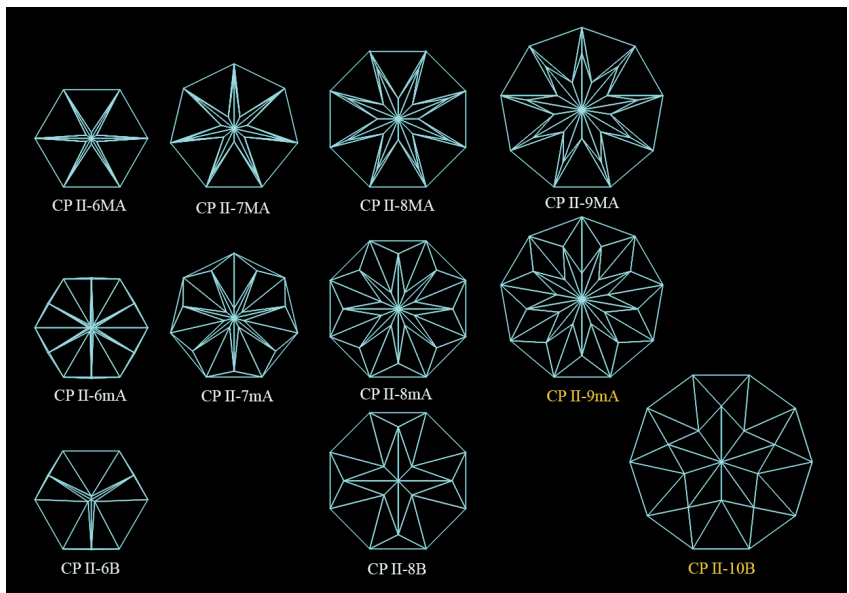


Fig. 7

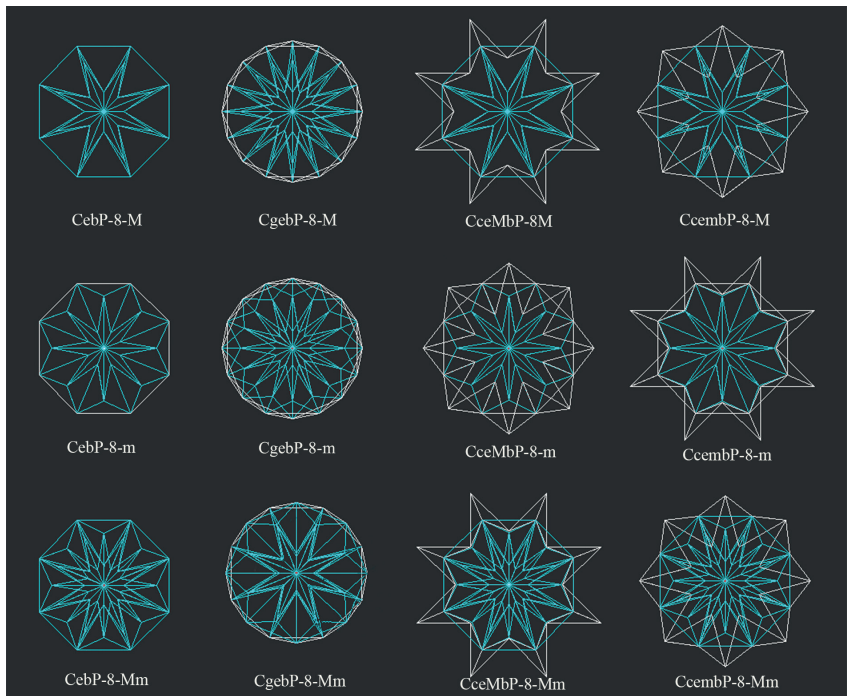


Fig. 8

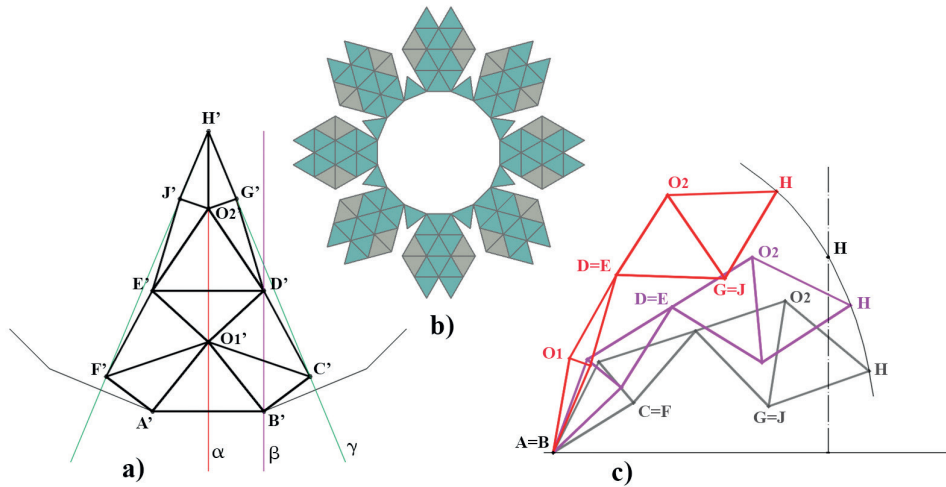


Fig. 9

unit cell is developed above every side of the polygonal base. In the other generation method, the unit cells are developed above every other side, and are mutually linked by isosceles triangles. This manner, termed type B, can be applied only to concave pyramids of the second sort above a polygonal base with even number of sides.

In terms of 'geometrical n-point patterns', the process of generation of concave pyramids of the second sort reveals 6-, 7-, 8-, 9- and 10-point patterns. In other words, they can be developed over polygonal bases with respective number of sides. Concave pyramids CP II-9mA and CP II-10B differ from the others, as their deltahedral net penetrates the plane of the polygonal base. Figure 7 shows orthogonal projections on the base plane of all concave pyramids of the second sort, treated as spatial grids consisting exclusively of bars, so that these concave deltahedral surfaces can also be viewed as rosettes. Pyramid unit cells, spatial pentahedrons, consist only of equilateral triangle sides.

When concave pyramids of the second sort with the same polygonal base are combined and joined, concave bipyramids of the second sort are obtained, and these can be further elongated, gyroelongated, conca-elongated, creating a distinctive family of diverse three-dimensional rosettes. In elongation, we use prisms, antiprisms and concave antiprisms (CA II-nM and CA II-nm), whose generation was discussed in our previous studies<sup>7</sup>. The characteristic star-like shape of the three-dimensional rosettes based on the geometry of concave pyramids of the second sort is obtained when these are elongated with concave antiprisms of the second sort (Fig. 8).

## CONCAVE PYRAMIDS OF THE FOURTH SORT

In this paper we show how Concave pyramids of the fourth sort, type B, can be generated above the regular hexadecagonal polygonal base (CP IV-16B). The unit cell is a spatial hexahedron and a spatial pentahedron with a mutual side (marked as side ED in Figure 9). The figure shows the orthogonal projection and

7 M. Obradović, B. Popkonstantinović and S. Mišić, "On the Properties of the Concave Antiprisms of Second Sort", *FME Transactions*. Vol. 41 No 3, (Belgrade), 2013, 256–263.



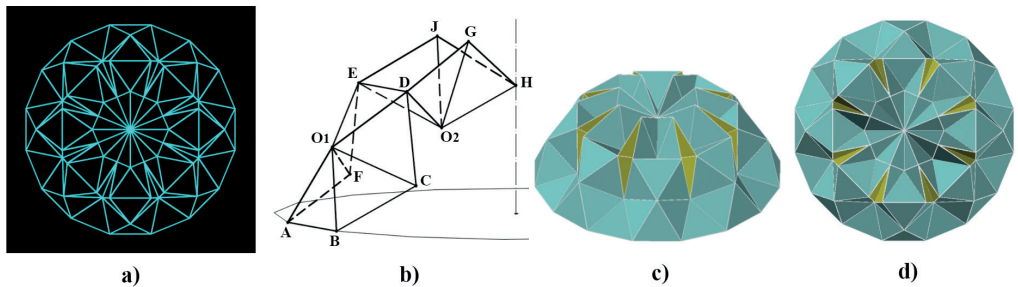


Fig. 10

the spatial model of thus formed unit cell, whose polar distribution around the axis vertical on the base plane forms CP IV-16B. Deltahedral lateral net is a convolution surface shown in Figure 9b. Geometrical generation is based on finding such a position of the unit cell vertices that the vertexes A and B lie on the sides, while vertex H is located on the axis of the polygonal base.

The construction is based on the constructive procedure to generate concave cupolae of the fourth sort<sup>8</sup>. More precisely, it relies on the fact that the distance between the neighboring vertexes of the unit cell is always equal to the side of the used equilateral triangles<sup>9</sup>. Auxiliary spheres whose centers lie in neighboring vertices of the spatial hexahedron (vertices  $O_1$  and C in Figure 9) are cut by the vertical plane containing vertices B and D. The intersection of thus obtained intersecting circles determines the position of vertices B and D. By repeating the constructive procedure above and by determining the position of all the vertices of the unit cell for multiple initial positions of vertex  $O_1$  we generate the trajectory of the vertex H. By cutting thus obtained trajectory of the vertex H with the vertical plane of the axis of the polygonal base, we obtain the sought position of the vertex H, and consequently, the final position of all the other vertices of the unit cell.

We will demonstrate two possible shapes of the concave pyramid of the fourth sort above a regular hexadecagonal polygonal base (CP IV-16B). In the first illustration (Figure 10) the spatial hexahedron

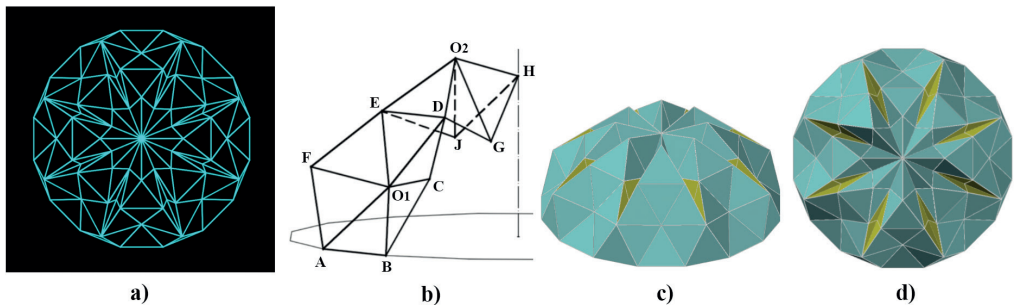


Fig. 11

8 S. Mišić and M. Obradović, "Forming the cupolae with concave polyhedral surfaces by corrugating a fourfold strip of equilateral triangles", in: *Proceedings of the 2<sup>th</sup> International Scientific Conference on Geometry and Graphics, moNGeometrija 2010*, ed. M. Nestorović, Beograd, 2010, 363–374.

9 S. Mišić, M. Obradović and G. Đukanović, "Composite Concave Cupolae as Geometric and Architectural Forms", *Journal for Geometry and Graphics*, Volume 19, No. 1, 2015, 79–91

has a protruding middle vertex  $O_1$  and an indented middle vertex  $O_2$  of the spatial pentahedron. In the second illustration (Fig. 11), the situation is reverse. Observation of the spatial model of the unit cell for both shapes of CP IV-16B reveals expansion and compression of the very structure in the two different examples of formation of the concave pyramid of the fourth sort above the same polygonal base. We also show the orthogonal projections and the spatial shape of the concave pyramid of the fourth sort with the interventions in the colour of the deltahedral lateral surface.

## COMPOSITE POLYHEDRONS WITH CONCAVE DELTAHEDRAL LATERAL SURFACE

Three-dimensional rosettes with the geometry of concave deltahedral surfaces can also be obtained by means of combining – joining of concave cupolas of the second sort (CC II-nM and CC II-nm)<sup>10</sup>, concave cupolas of the fourth sort (CC IV), concave antiprisms of the second sort (CA II) and concave pyramids (CP).

We select several examples of thus obtained three-dimensional rosettes with the geometry of concave deltahedral surfaces. Figure 12 presents examples of three-dimensional rosettes, each accompanied by the description of the type of polyhedrons from which it is produced.

## CONCLUSION

The presented three-dimensional rosettes with the geometry of concave deltahedral surfaces are, similarly to the rosettes in Gothic and Islamic architecture, characterized by multiple symmetry, proportion and order. Their geometrical construction is based on the rotation of the unit cell around the central axis accompanied by modularity which facilitates implementation. When three-dimensional rosettes are observed as a composite structure consisting of concave polyhedral surfaces, their modularity lies in the fact that the complex patterns are obtain through the application of a single element – equilateral triangle. On the other hand, the same structure can be viewed as a three-dimensional grid, whose modularity lies in the application of same-length bars.

Further research should focus on exploring the application of three-dimensional rosettes as architectural elements, as contemporary architecture recognizes the quality of modular solutions which reduce the construction time and cost. Building blocks of three-dimensional rosettes (equilateral triangles) can be treated as glass surfaces of different colours, where refraction of light and the movement of shadows can be explored as an asset to architectural design. In that manner, analogously to the Gothic architecture, the filtered light can penetrate and enrich the modern interior. Three-dimensional rosettes based on the geometry of concave polyhedral surfaces can be applied as a façade architectural element, as cupolas, roof constructions or as independent spatial constructions (entire objects). Three-dimensional rosettes, unlike two-dimensional, allow the overlapping of the shadows cast through their (triangular) sides depending on the angle of the light shining through them. This shadows, in turn, shape dynamic pictures (patterns and compositions), which change with hours and seasons. Moreover, these will neither be mere projections of flat images nor familiar pictures, but complex compositions obtained through an interplay of their positions and spatial relationships. Future research can focus on the engineering of these composition – on how to obtain a certain distribution of light and shadows at a given time of day or on a given day in a year, as an additional architectural accomplishment.

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10 M. Obradović and S. Mišić, "Concave Regular Faced Cupolae of Second Sort", in: *Proceedings of 13th ICGG*, ed. G. Weiss, Dresden, 2008, El. Book: 1–10

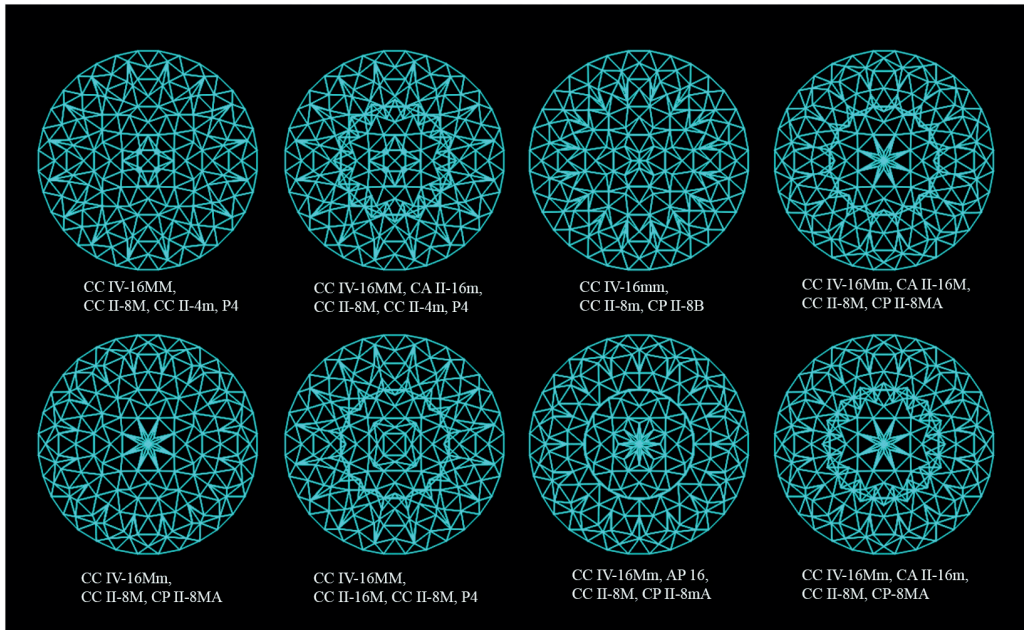
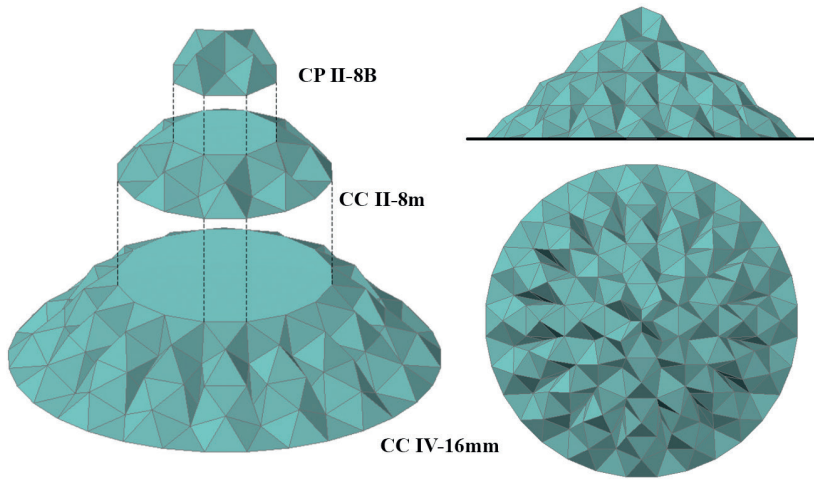


Fig. 12

### Acknowledgement

This paper is supported by the Ministry of Education, Science and Technology Development of Serbia, grant No. III44006.

## ILLUSTRATIONS

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[https://commons.wikimedia.org/wiki/File:Rose\\_du\\_transept\\_Sud\\_Notre-Dame\\_de\\_Paris\\_170208\\_02.jpg](https://commons.wikimedia.org/wiki/File:Rose_du_transept_Sud_Notre-Dame_de_Paris_170208_02.jpg)
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CP IV-16B (испучено теме O2 и удубљено теме O1), а) ортогонална пројекција, б) јединична ћелија, ц) просторни модел, д) ортогонална пројекција

12: Three-dimensional rosettes of the geometry of composite polyhedrons with concave deltahedral lateral surface  
Тродимензионалне розете геометрије композитних полиедара са конкавним делтаедарским омотачем

## REFERENCES

- AbdullahiYahya, Rashid Bin Embi Mohamed: "Evolution of Islamic geometric patterns", *Frontiers of Architectural Research*, Volume 2, Issue 2, June 2013, pp 243–251.
- DabbourLoai M. "Geometric proportions: The underlying structure of desing process for Islamic geometric patterns", *Frontiers of Architectural Research*, Volume 1, Issue 4, December 2012, pp 380–391.
- Mišić, Slobodan, Obradović, Marija. „Forming the cupolae with concave polyhedral surfaces by corrugating a fourfold strip of equilateral triangles“, in: *Proceedings of the 2<sup>th</sup> International Scientific Conference on Geometry and Graphics, moNGeometrija 2010*, ed. M. Nestorović, Beograd, 2010, 363–374.
- Mišić, Slobodan, Obradović, Marija and Đukanović, Gordana. "Composite Concave Cupolae as Geometric and Architectural Forms", *Journal for Geometry and Graphics*, Volume 19, No. 1, 2015, 79–91.
- Obradović, Marija, Mišić, Slobodan and Popkonstantinović, Branislav. "Concave Pyramids of Second Sort – The Occurrence, Types, Variations", in: *Proceedings of the 4<sup>th</sup> International Scientific Conference on Geometry and Graphics, moNGeometrija 2014*, ed. S. Krasić, Vol 2. Vlasina, 2014, 157–168.
- Obradović, Marija, Mišić, Slobodan and Popkonstantinović, Branislav. "Variations of Concave Pyramids of Second Sort with an Even Number of Base Sides", *Journal of Industrial Design and Engineering Graphics (JIDEG) – The SORGING Journal*, Volume 10, Special Issue, Fascicle 1, ed. D. Marin, University 'Transilvania' of Brasov / Romanian Society of Engineering Graphics SORGING, Brasov, 2015, 45–51.
- Obradović, Marija, Popkonstantinović, Branislav and Mišić, Slobodan. "On the Properties of the Concave Antiprisms of Second Sort", *FME Transactions*. Vol. 41 No 3, (Belgrade), 2013, 256–263.
- Obradović, Marija and Slobodan Mišić. "Concave Regular Faced Cupolae of Second Sort", in: *Proceedings of 13th ICGG*, ed. G. Weiss, Dresden, 2008, El. Book: 1–10.
- Samper Albert, Herrera Blas: "A Study of the Roughness of Gothic Rose Windows", *Nexus Network Journal*, July 2016, Volume 18, Issue 2, pp 397–417.
- Williams Kim: "Spirals and Rosettes in Architectural Ornament", *Nexus Network Journal*, June 1999, Volume 1, Issue 1–2, pp 129–138.

## ABBREVIATIONS:

- CP II-nM – concave pyramid of the second sort, major type  
CP II-nm – concave pyramid of the second sort, minor type  
CP II-nB – concave pyramid of the second sort, type B  
CP IV-nB – concave pyramid of the fourth sort, type B  
CA II-nM – concave antiprism of the second sort, major type  
CA II-nm – concave antiprism of the second sort, minor type  
CC II – concave cupola of the second sort  
CC IV – concave cupola of the fourth sort

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## ТРОДИМЕНЗИОНАЛНЕ РОЗЕТЕ ЗАСНОВАНЕ НА ГЕОМЕТРИЈИ КОНКАВНИХ ДЕЛТАЕДАРСКИХ ПОВРШИ

**Резиме:** Розета постаје доминантан архитектонски детаљ у првој половини XII века у Француској, када су градитељи религиозне архитектуре, под утицајем неоплатонских идеја, постали очарани светлошћу као средством за повезивање са Богом. Концептуално је хришћанска црква постала храм светлости – храм окупан у Божјој светлости. Измењеним системом конструкције и употребом нових конструктивних елемената постало је могуће уградити спектакуларне витраже кроз које је филтрирано сунчево светло ушло у катедралу. У овом систему се истиче јединствени отвор прозора у облику розете. Кружни облик и сложена геометрија учинили су овај архитектонски елемент најрепрезентативнијим производом примене готске уметности. Њихова основна геометријска карактеристика је конструкција заснована на поларној дистрибуцији – ротацији јединичне ћелије око средишта контурне кружнице.

У истом периоду исламски геометријски узорци се користе као матрица за обликовање украсних архитектонских елемената. Њихова примена је потпуна интеграција геометрије са архитектуром. Основа ових образаца састоји се у примени правилних полигона, и полигона у облику звезде који се формирају од њих. Ови облици представљају лишће розете. У зависности од броја ( $n$ ) темена правилних полигона генерисаних из њих, у класификацију се уводи нови појам – геометријски узорак  $n$ -тачака, при чему врста розете зависи од полигона из којег је изведена. Еволуција исламских геометријских узорака може се пратити кроз употребу  $n$ -тостраних полигона, од правилног шестоугла до сложенијих врста полигона и кроз розете формиране од њих.

У овом раду, веза између ова два начина формирања розета остварена је применом конкавних полиедарских површи. Формирајући композитне полиедарске структуре засноване на геометрији конкавних делтаедарских површи преко  $n$ -тостраних полигоналних основа, показали смо једну могућу методу геометријског генерисања тродимензионалних розета. Конкавне полиедарске површи су омотачи конкавних полиедара друге, четврте и виших врста, који се састоје од низа једнакостраничних троуглова, груписаних у јединичне ћелије. Позициониране поларно око средишње осе правилног полигона у основи полиедра, просторне јединичне ћелије формирају површину делтаедра. Врста конкавног полиедра одређена је бројем редова једнакостраничних троуглова у развијеној мрежи полиедра. У овом истраживању композитне полиедарске структуре, чије површине формирају тродимензионалну розету, добијају се комбинацијом конкавних купола друге врсте (CC II), конкавних купола четврте врсте (CC IV), конкавних антипризми друге врсте (CA II) и конкавних пирамида (CP). Параметри површи су конструктивно одређени геометријским и аналитичким методама. Ортогоналне пројекције на раван основе тако формираних композитних полиедарских структура су розете у равни јединствене геометрије. Оне су, баш као и готске и исламске розете, карактеристичне по вишеструкој симетрији, пропорцији и реду. Сложеност њихових конфигурација резултат је поступка генерисања полиедарске површи, а њихова модуларност лежи у чињеници да се сложени обрасци добијају применом само једног елемента – једнакостраничног троугла.

**Кључне речи:** розета, полиедар, архитектура, троугао, геометрија.

## ЗБОРНИК РАДОВА:

ПРВА МЕЂУНАРОДНА КОНФЕРЕНЦИЈА  
SMARTART – УМЕТНОСТ И НАУКА У ПРИМЕНИ  
„Од инспирације до интеракције”

### Превод и лектура (српски и енглески)

Агенција за преводилачке услуге „Идиома”  
Београд, Србија

### Дизајн и техничко уређење

Оливера Стојадиновић  
Оливера Батајић Сретеновић

### Штампа

### Тираж

120

ISBN 978-86-80245-40-9

Реализацију Зборника радова Конференције SmartArt 2019 – Уметност и наука у примени „Од инспирације до интеракције” финансијски је подржало Министарство просвете, науке и технолошког развоја Републике Србије.



МИНИСТАРСТВО ПРОСВЕТЕ,  
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА



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НАУКА И УМЕТНОСТИ



ФАКУЛТЕТ  
ПРИМЉЕНИХ  
УМЕТНОСТИ

## PROCEEDINGS:

FIRST INTERNATIONAL CONFERENCE SMARTART –  
ART AND SCIENCE APPLIED  
“From Inspiration to Interaction”

### Translation and proofreading (Serbian and English)

Translation Services Agency “Idioma”  
Belgrade, Serbia

### Design and layout

Olivera Stojadinović  
Olivera Batajić Sretenović

### Printed by

### No. of copies

120

ISBN 978-86-80245-40-9

The realization of the Proceedings of the Conference SmartArt – Art and Science Applied “From Inspiration to Interaction” was financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia.

