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AFFINE CONFORM PLANES AS THE SPECIAL CASES OF GENERAL COLLINEAR PLANES

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ABSTRACT: General collinear planes which double straight line is in infinity are affine collinear planes. Affine collinear planes which double points of the collocation on the infinitely distant double straight line are the absolute points or their real representatives, are conform planes. The paper considers conform affine transformations: homothety, translation, rotation and axial symmetry, as well as their combinations, by defining their types of collocation as: elliptic, hyperbolic, parabolic or perspectively collinear.

INTRODUCTION

General collinear planes are two perspectively associated planes that belong to the same plane, whereby they contain three double points (the points correspondent to themselves), one of which will be certainly real. These three double points determine three double straight lines, one of which will be real, too. Depending on whether two double points or double straight lines are real or conjunctively imaginary, general collinear planes are distinguished as: hyperbolic, elliptic and marginal, parabolic types. The planes have only one pair of infinitely distant correspondent points. If there will exist just one more pair of such points, they would determine, with the first pair of points - infinitely distant straight line, which would be the double straight line of these collinear planes. In this case, general collinear planes become AFFINE.

1. CONFORM PLANES

The special case of general affine planes is the one in which each picture of one plane is mapped into a self-similar, conform picture of another plane. Some of elementary conform transformations in plane are well known, such as: homothety, translation, rotation and axial symmetry. Except of homothety, the others are also equiform transformations. Each one of them can be observed as the special case of general collinear, or more specified - general affine planes, in function to determine the type of collinear planes obtained by their combining. The purpose of determining the types of these planes are not only theoretical, but helps the applying the adequate constructive methods for the direct mapping of points from one plane to another. Let's reconsider, therefore, these four transformations, firstly individually, and afterwards, their possible combinations.

1.1 HOMOTHETY

Homothety is a perspective transformation, for having a center of perspectivity and an axis of perspectivity - the infinitely distant straight line of the plane. Therefore, two homothetic pictures can be considered as pictures in two homothetic planes, the special cases of affine planes; their double point is just the center of homothety (perspectivity), and the double straight line is the infinitely distant axis of perspectivity, on which there is a collocal set of identical, incidental sets, each point of which represent the double point.

The special case of homothety is a central symmetry, whereby the correspondent pair of points are at the opposite sides, and equally distant from the double point, respectively - the center of homothety.

1.2 TRANSLATION

Translation is also a special case of homothety, whereat the center of homothety is in infinity, so is the axis. In that manner, it is a case of affine elation. Collocal sets on the infinitely distant straight line, are identical and incidental, again.

1.3 ROTATION

In case of rotation, assigned canter of rotation represents the double point of these planes, while the infinite double straight line is the "carrier" of such a collocal set, obtained by translator shifting one of these two identical sets along the line, so the interval between two correspondent points on the set will be constant. The interval will distinguish the angle of the rotation in the definite double point. However, where are the remaining two double points of these general affine planes?

If the collocal set, which is now in infinity, would be in definitiveness, its double points would be infinitely distant, incidental points on the course determined by the double straight line, the carrier of the set. That can be easily proofed by using the Schteiner's circle method. According to the fact, it would be the matter of parabolic set. But, in the actual case, all the correspondent points are already in infinity, so the existence of incidence of any two correspondent points is not possible, because interval of translation on the set is infinitely long, independently on the rotation angle size. It is happening because infinity looses the system of measurement; the measurable dimensions indefinitiveness become neglectedly small in infinity, so the appearing of the parabolic set on a translatorly shifted collocal set is possible in definitiveness. Applying this on the equivalent set on the infinite straight line, the case becomes elliptic, more specifically - circular, because now, as the double points of the collocation on the line, appear just the absolute points. This means that the rotation is actually general affine transformation of circular type, which is a special case of elliptic general affine planes.

1.4 AXIAL SYMMETRY

Axial symmetry is interesting because in this case, on the infinitely distant straight line, which represent the double straight line of these planes, a hyperbolic collocal set appears. The double points of the set are determined by the course of the symmetry axis and the course of the rays of symmetry perpendicular to the axis, which itself is the double straight line, the carrier of identical incidental sets, for being also an affinity axis of these planes. Axial symmetry is a perspectively collinear transformation, as well as homothety, only now the axis is in definitiveness, and the center is in infinity. Each ray of symmetry can be considered as the double straight line that carries the hyperbolic collocal set. Specially interesting is, however, the infinite double straight line on which there are double points, the real representatives of the absolute points. This will show as significant in various combinations of axial symmetry and the other conform transformations. Thereby, the collocal set on the infinitely distant double straight line is symmetrical, related to the point of intersection of symmetry axis and the infinite line, so it is obvious that it is the matter of involutary set of hyperbolic type.

The results of combining each of the described transformations with themselves, are already well known: homothety and homothety will give another homothety. That means, if two homothetic pictures are confirmed as the pictures of general affine homothetic planes, and then one of them is transformed again, to a new picture homothetic to the former, for the arbitrary taken center of homothety - the first and the third picture will be also homothetic. Their new center will lie on the connection straight line of two former centers; thus, there are considered only original and the final plane, while the between plane is in the known, obtained relations to the observed planes.

Translation and translation will result with translation, too. The course of it will depend on vector of the translations.

Two rotation will give a new rotation with the center in the new double point of these general affine planes and the angle equal to the summary of previous rotations' angles.

The pair of axial symmetries will result with rotation again, the angle of which would be equal to the double value of the angle formed by symmetry axises.

2. COMBINING THE TRANSFORMATIONS

Reconsidering, furthermore, the mutual combinations of these transformations, there can be distinguished the six different cases:

2.1 HOMOTHETY AND TRANSLATION

Homothety and translation as the relative perspectively collinear transformations, will give as the result the general affine planes of homothetic type, because more general case predominates the special. The center of the new homothety will be placed on the straight line that includes the former center of homothety and is parallel to the course of translation (adequately to the combination of two homotheties). The obtained planes are, therefore, the perspectively collinear - AFFINE planes.

2.2 HOMOTHETY AND ROTATION

Homothety and rotation will form general affine planes of circular type. The planes will have one real, definite double point and a real infinitely distant double straight line, on which there would be the circular collocal set. The other two of the double points would be the absolute points, the double points of the collocation on the infinite straight line. The position of the definite double point it is possible to determine by one of the following methods:

- by intersection of the affinity axises, which are determined by the consecutive points of pair of parallel associated straight lines of the planes, belonging to the perspective straight line sets, which centers of perspectivity are infinite, correspondent points, defined by the courses of the adjusted parallel lines. Two affinity axises are enough to determine, by their intersection - the double point of the planes;

- by the intersection of two circles obtained as products of associated projective pencils established in pairs of correlative points. In this manner, collocal, incidental sets on the infinite double straight line of homothety, become translatorly shifted by the function of the rotation. The double points of the newly obtained collocal set, will be the absolute points, which is proofed by the appearance of the circles as the basis second order curves of the double points, knowing that each definite circle intersect the infinite double straight line in the absolute points.

2.3 HOMOTHETY AND AXIAL SYMMETRY

Homothety and axial symmetry create general affine planes of hyperbolic type with one of the double points in definitiveness, and the reaming two in infinity, which courses are perpendicular, and parallel to the courses of the axis and the rays of symmetry. Whenever axial symmetry takes part, the planes' orientation changes, which means that planes are converted into their equivalent planes of hyperbolic type, with the real representatives of the absolute points.

It is possible to determine the definite double point, according to the previous case, by the intersection of the affinity axises of two associated sets of parallel straight lines. However, the second order curves obtained by association of projective pencils of straight lines from the definite points, in this case will be the Ptolemy's hyperbolas which asymptotes are determined by the course of the axis and the rays of symmetry, respectively, they passes through the pair of infinite double points. All the hyperbolas obtained in such a manner will be homothetic, rectangle, and will contain the three double points, the permanent basic points of four intersection points. One of the double points will be definite, and two definite perpendicular double straight lines (also determined by the course of the other two infinite double points) will be concurrent to it.

The fastest way for the direct mapping the points from so obtained planes one to another, is by using the affinity axis as in all the other cases, although the Schteiner's circle method can be used, in dependence of the particular case, on the definite double straight lines of these planes.

2.4 TRANSLATION AND ROTATION

Translation and rotation identify the case to the one with the combination of homothety and rotation, described in number 2.2. The obtained planes are of circular type, in this case, also - equiform.

2.5 TRANSLATION AND AXIAL SYMMETRY

Translation and axial symmetry produce general affine planes of a special type, which can be named (specified) as parabolicly-hyperbolic. In this case, the axis of associated perspective sets of straight lines will be all mutually parallel, respectively, will intersect in the double point determined by the course of symmetry axis. This suggests that there will be no definite double points. Second order curves obtained as products of associated projective pencils in correspondent definite points, would be Ptolemy's hyperbolas, again. Their asymptotes would be determined by double, infinitely distant points, of symmetry axis and rays of symmetry. However, now will appear an asymptote that will be common to the whole set of the hyperbolas. It will represent the definite double straight line of so obtained planes. The sets of curves have the common tangent (the same, mentioned asymptote). All the curves osculate in the infinite point of the tangent, which means that they intersect in two infinitely close points. In this manner, collocal set on the definite double straight line will be obtained by translatorly shifted, identical sets, mentioned in the number 1.3. The double points of the sets are incidental i.e. overlapped in infinity, so the set is parabolic. On the infinite double straight line there is a hyperbolic set with the double points: the osculation point (the double point of the parabolic set on the definite straight line) and another double point, of perpendicular course to the previous, determined by the course of the rays of symmetry.

Such planes can be distinguished as shifted symmetrical planes, because it is possible to get a clear, axial symmetry, by coincidence of the identical sets on the double straight line.

2.6 ROTATION AND AXIAL SYMMETRY

Rotation and axial symmetry will result with the planes of the same type as the previous, only now the asymptotes of these homothetic Ptolemy's hyperbolas will not be parallel to the axis nor the rays of symmetry, because the rotation caused the shifting the one of the symmetrical sets along the infinite double straight line. Now, the different pair of double points will appear, comparing to the symmetrical sets on the line; the set on the definite double straight line - the common asymptote of the hyperbolas, will be parabolic, as in the case of translation and axial symmetry.

For the fast and the efficient mapping the points of so obtained planes from one to another, it is enough to find the course of the affinity axis, automatically the course perpendicular to it. Through the half a distance between two of the correspondent definite points will pass the double straight line of the planes, and the distances from it to each pair of other correspondent points will be equal.

The triple and multiple combinations of these cases can be deduced on some of the observed cases.

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