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Determination of dynamic thermal characteristics of a building wall

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Abstract: In this paper, based on in situ experimental temperature measurements, the following dynamic thermal parameters of a wall were calculated: decrement factor, time shift of thermal transmittance and areal heat capacities. The outdoor and indoor air temperature of a resident building in Belgrade during the three months from spring to summer of the last year were collected by data loggers. The experimental data were processed using the techniques of filtering and Locally Scatter plot Smoothing (LOESS) method, in order to get frequency and amplitude of daily temperature variations. A solution of the Fourier heat equation for this case, in matrix formulation in frequency space, is given. Matrix's elements of the Fourier equation were used for calculation of the dynamic thermal parameters of the wall.

Keywords: Fourier heat equation, temperature data processing, wall's thermal parameters

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1. Introduction

Dynamic thermal parameters describe the thermal behavior of building components subjected to time dependent boundary conditions (BC). These conditions could be variable temperature and heat flow rate on one or both of their boundaries. Generally, the variable condition assume any function of time but often means sinusoidal variation of known frequency. The response of the building component's is twofold: transient, which decrees in time, and steady with the same harmonics as excitation. The amplitudes of the temperature and the heat flux is damped as the excitation traverses the component. The both quantities have time delay compared to surface's BC [1]. This imply that usage of the complex functions could be appropriate for description of the physics involved. In this way, the dynamic parameters of the wall: decrement factor, time shift of thermal transmittance and areal heat capacities have been defined over complex quantities based on the solution of the Fourier heat equation in frequency space [2].

In this paper, a procedure leading to the Fourier heat equation solution in complex matrix form for a multilayered wall, using the Laplace transform, was adopted. Also, the results of simultaneous measurements of the external and internal air temperature in an apartment located on the second floor residential five storey building in Belgrade, were shown. Using techniques of signal processing, temperature variations in form of sine functions on the both surfaces of the wall of the building were extracted [3]. From this, the period of the temperature changes due to daily variation, was determined.

2. Experimental setup and measurement methodology

Figure 1 shows the “data logger” used for measurements of the air temperature inside and outside of the apartment. Inside and outside temperatures were measured at the same moments, every 5 minutes. Response time of the logger to step excitation is 20s. The logger measures and stores up to 16382 temperature readings over range from -35^{0}C to 80^{0}C. The logger is protected against water and dust when the plastic cap and seal are fitted. Figure 2 represents the measurement accuracy of the air temperature according to manufacturer specifications. Within a measuring range of 0^0C to 40^0C the maximum absolute error of measurement is less than $\pm 1^0$C.

Facade of the living room wall faces south. Outdoor temperatures are measured in vicinity of the building close to the same wall in a place sheltered from direct sunlight. We used two measuring devices of the same manufacturer and type.

Figure 1. Data logger used for measuring air temperature and relative humidity

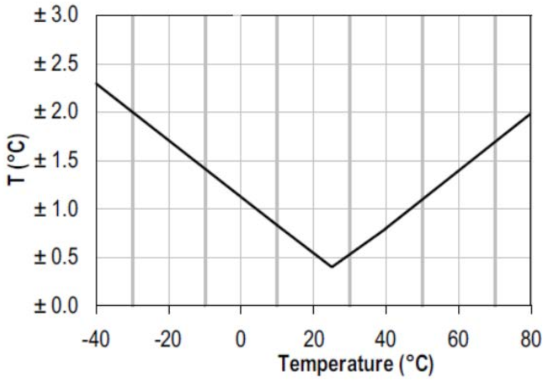


Figure 2. The data logger accuracy of air temperature measurement, according to the manufacturer.

Data logger were placed at the living room of the apartment, on the dresser (Figure 3) and in the window of ventilated garage on the ground floor of the building (Figure 4). Windows of the apartment were circled with rectangle on the figure 4. During entire course of measurements there were no people present and no power consumption in the apartment.

Figure 3. The living room of the apartment

Figure 4. The building and marked windows of the apartment at the second floor.

3. Solution of the Fourier equation in matrix form

The Fourier equation for the heat conduction in a homogeneous wall of thickness d, without heat sources and sinks is:

$$\frac{\partial T(x,t)}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 T(x,t)}{\partial x^2}, \tag{1}$$

where λ is coefficient of the thermal conductivity, ρ is density and $T(x,t)$ is temperature field inside the wall. BCs subjected to the surfaces at both sides are of the mixed type:

$$\begin{aligned} q_u &= -\lambda \frac{\partial T(x,t)}{\partial x}\bigg|_{x=0} = \alpha_u (T_u(t) - T_{zu}(t)) \\ q_s &= -\lambda \frac{\partial T(x,t)}{\partial x}\bigg|_{x=d} = \alpha_s (T_{zs}(t) - T_s(t)) \end{aligned} \tag{2}$$

Subscripts u and s stand for indoor and outdoor. α is a convection coefficient. Eqn (1) is the partial differential equation and could be transformed into the ordinary differential equation using the Laplace-transform formalism, $T(j\omega, x) = \int\limits_0^\infty T(x,t) e^{-j\omega t} dt$, (j-imaginary unit):

$$j\omega T(j\omega, x) - T_0(x) = \frac{\lambda}{\rho c} \frac{\partial^2 T(j\omega, x)}{\partial x^2}, \tag{3}$$

where $T_0(x)$- initial condition for the temperature field in the wall at $t = 0$. We adopt $T_0(x) = 0$, because we are interested in steady-state solution. With a short mark $k = \rho c / \lambda$, eqn (3) reads:

$$\frac{\partial^2 T(j\omega, x)}{\partial x^2} - j\omega \cdot k T(j\omega, x) = 0 \tag{4}$$

Solution of eqn (5) has well known form $T(j\omega, x) = e^{\eta(j\omega)x}$, where $\eta(j\omega)$ is solution of characteristic equation $\eta^2(j\omega) - j\omega \cdot k = 0$, $\eta_{1,2}(j\omega) = \pm\sqrt{j\omega \cdot k}$. Now, general solution of eqn (4) is:

$$T(j\omega, x) = C_1 e^{\sqrt{j\omega \cdot k}x} + C_2 e^{-\sqrt{j\omega \cdot k}x} \tag{6}$$

Applying BC (2) in $x = 0$ we arrive to:

$$C_1 + C_2 = T(j\omega, 0) = T_{zu}(j\omega)$$

and

$$q_u = q_u(j\omega, x = 0) = -\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = -\lambda\sqrt{j\omega \cdot k}\left[C_1 + C_2\right]. \tag{7}$$

Eqns (7) give unknown constants C_1 and C_2:

$$\begin{aligned} C_1 &= \frac{1}{2} T_{zu}(j\omega) - \frac{q_u(j\omega)}{2\lambda\sqrt{kj\omega}} \\ C_2 &= \frac{1}{2} T_{zu}(j\omega) + \frac{q_u(j\omega)}{2\lambda\sqrt{kj\omega}} \end{aligned} \tag{8}$$

The temperature field in the wall is:

$$T(j\omega, x) = T_{zu}(j\omega) ch\left(\sqrt{j\omega \cdot k}x\right) + \frac{q_u(j\omega)}{\lambda\sqrt{kj\omega}} sh\left(\sqrt{j\omega \cdot k}x\right) \tag{9}$$

Our aim is to relate the temperatures and the heat fluxes at both surfaces. In order to achieve this the Laplace-transform of the heat flux on outdoor surface of the wall is used:

$$q_s(j\omega) = -\lambda \frac{\partial T(j\omega, x)}{\partial x}\bigg|_{x=d} = -\lambda\sqrt{j\omega \cdot k} T_{zu} sh\left(\sqrt{j\omega \cdot k}d\right) + q_u(j\omega) ch\left(\sqrt{j\omega \cdot k}d\right) \tag{10}$$

Also, the outdoor surface temperature is:

$$T_{zs}(j\omega)=T_{zu}(j\omega)ch\left(\sqrt{j\omega\cdot k}d\right)-\frac{q_u(j\omega)}{\lambda\sqrt{kj\omega}}sh\left(\sqrt{j\omega\cdot k}d\right) \tag{11}$$

In matrix form eqns (10) and (11) reads:

$$\begin{bmatrix} T_{zs}(j\omega) \\ q_s(j\omega) \end{bmatrix}=\begin{bmatrix} ch\left(\sqrt{j\omega\cdot k}d\right) & \dfrac{-sh\left(\sqrt{j\omega\cdot k}d\right)}{\lambda\sqrt{j\omega\cdot k}} \\ -\lambda\sqrt{j\omega\cdot k}sh\left(\sqrt{j\omega\cdot k}d\right) & ch\left(\sqrt{j\omega\cdot k}d\right) \end{bmatrix}\begin{bmatrix} T_{zu}(j\omega) \\ q_u(j\omega) \end{bmatrix} \tag{12}$$

In eqn (12) the temperatures of the wall surfaces could be expressed over the air temperatures $T_s(j\omega)$ and $T_u(j\omega)$. Using BC (2) in matrix form we get:

$$q_u(j\omega)=\begin{bmatrix} 1 & -\dfrac{1}{\alpha_u} \end{bmatrix}\begin{bmatrix} T_u(j\omega) \\ q_u(j\omega) \end{bmatrix},\ q_s(j\omega)=\begin{bmatrix} 1 & -\dfrac{1}{\alpha_s} \end{bmatrix}\begin{bmatrix} T_{zs}(j\omega) \\ q_s(j\omega) \end{bmatrix} \tag{13}$$

Substituting into eqn (12) we get an equation relating the temperatures and the heat flow rates at the both side of the wall:

$$\begin{bmatrix} T_s(j\omega) \\ q_s(j\omega) \end{bmatrix}=Z_s\cdot Z_1\cdot Z_u\begin{bmatrix} T_u(j\omega) \\ q_u(j\omega) \end{bmatrix}, \tag{14}$$

$$Z_s=\begin{bmatrix} 1 & -\dfrac{1}{\alpha_s} \\ 0 & 1 \end{bmatrix},\ Z_u=\begin{bmatrix} 1 & -\dfrac{1}{\alpha_u} \\ 0 & 1 \end{bmatrix},Z_1=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}=\begin{bmatrix} ch\left(\sqrt{j\omega\cdot k}d\right) & \dfrac{-sh\left(\sqrt{j\omega\cdot k}d\right)}{\lambda\sqrt{j\omega\cdot k}} \\ -\lambda\sqrt{j\omega\cdot k}sh\left(\sqrt{j\omega\cdot k}d\right) & ch\left(\sqrt{j\omega\cdot k}d\right) \end{bmatrix}.$$

If the wall is multilayered with n-layers eqn (14) is:

$$\begin{bmatrix} T_s(j\omega) \\ q_s(j\omega) \end{bmatrix}=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\cdot\begin{bmatrix} T_u(j\omega) \\ q_u(j\omega) \end{bmatrix}=Z\cdot\begin{bmatrix} T_u(j\omega) \\ q_u(j\omega) \end{bmatrix},\quad Z=Z_s\cdot Z_1\cdot Z_2 \ldots\cdot Z_n\cdot Z_u, \tag{15}$$

where $Z_1, Z_2, \ldots Z_n$ stands for matrix of each layer.

4. Dynamic thermal parameters of the wall

As stated in the first chapter the following parameters describe the most important physics of processes: decrement factor, time shift of thermal transmittance and areal heat capacities. The decrement factor f relates periodic and steady-state thermal transmittance, more precisely, the modulus ratio of modulus of the periodic transmittance and steady-state transmittance U_0 [2] is:

$$f=\frac{1}{U_0}\left|-\frac{1}{Z_{12}}\right|. \tag{16}$$

f illustrates the heat flux damping capability of a wall when the flux changes in the time as a sinusoidal function. The decrement factor has value between zero and one and if it is close to zero it means that the wall has strong damping capabilities. This is the characteristic of the massive constructions. The thermal transmittance does not include the thermal bridges.

The time shift Δt_f represents the period which takes the amplitude of the heat flux to traverses construction element [2]:

$$\Delta t_f = \frac{T}{2\pi} \arg(Z_{12}) \, [h] \tag{17}$$

As in the case of the decrement factor, for the lighter constructions the time shift is shorter than for massive one.

The areal heat capacity is the heat capacity per unit area of construction element. The two definitions exist [2]:

$$\kappa_1 = \frac{T}{2\pi} \left| \frac{Z_{11} - 1}{Z_{12}} \right|, \quad \kappa_2 = \frac{T}{2\pi} \left| \frac{Z_{22} - 1}{Z_{12}} \right| \tag{18}$$

Distinction between these definitions is that the first one relates to external period of the temperature variations and the later to internal one.

5. Numerical results and discussion

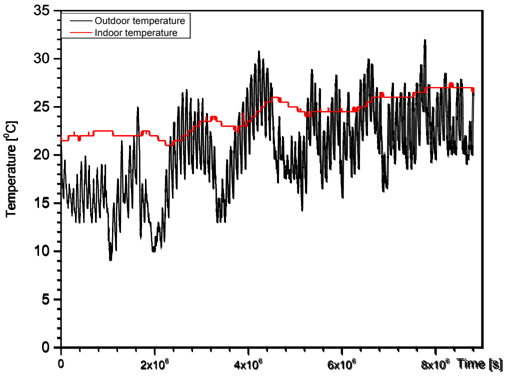


Figure 5. Outdoor (black) and indoor (red) air temperature.

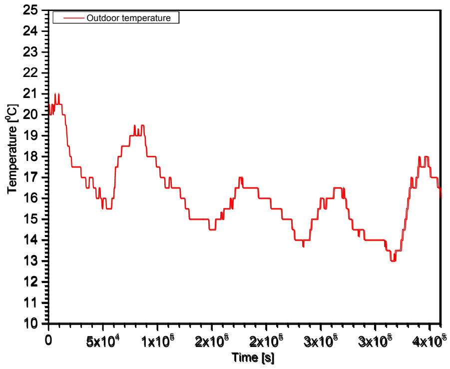
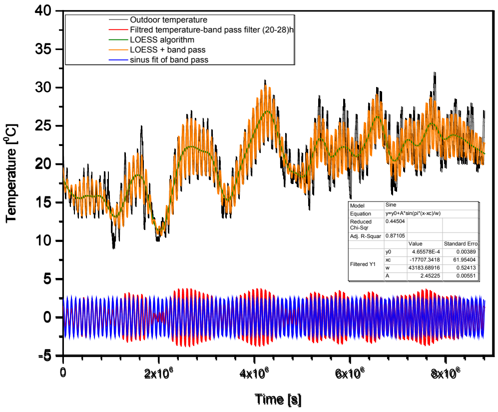
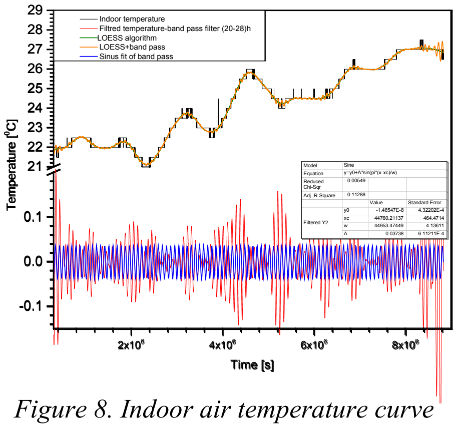


Figure 6. Outdoor temperature variations for the first 5 days.



Model	Sine		
Equation	y=y0+A*sin(pi*(x-xc)/w)		
Reduced Chi-Sqr	0.44504		
Adj. R-Squar	0.87105		
		Value	Standard Erro
Filtered Y1	y0	4.65578E-4	0.00389
	xc	-17707.3418	61.95404
	w	43183.68916	0.52413
	A	2.45225	0.00551

Figure 7. Outdoor air temperature curve (black), band pass filter curve (red), sinus fit of band pass curve (blue), LOESS algorithm curve (green)



Model	Sine		
Equation	y=y0+A*sin(pi*(x-xc)/w)		
Reduced Chi-Sqr	0.00549		
Adj. R-Square	0.11288		
		Value	Standard Error
Filtered Y2	y0	-1.46547E-8	4.32202E-4
	xc	44760.21137	464.4714
	w	44953.47449	4.13611
	A	0.03738	6.11211E-4

Figure 8. Indoor air temperature curve (black), band pass filter curve (red), sine fit of band pass curve (blue), LOESS algorithm (green)

In Figure 5 the measured outdoor and indoor air temperature were shown. The measurements in the period from 22.04.2014. to 02.08.2014 were made. In this period each logger recorded 29326 readings. Red curve in Figure 5 corresponds to the indoor and black to the outdoor temperatures. As expected, indoor curve has much lower amplitude than the outdoor one. To calculate the matrix elements Z_{11}, Z_{12}, Z_{21} and Z_{22}, it is necessary, beside material parameters, to know the angular frequency of the sinusoidal input temperatures. The FFT spectra of the outdoor temperatures for Belgrade, for period of last fifty years, have several picks at frequencies corresponding to daily, seasonal, half year and year temperature variations [4]. The most influential frequency on the numerical values of the matrix elements are the daily temperature variations because in this case the corresponding angular frequency has maximal value. In Figure 6 a typical outlook of the daily air temperature variations, recorded in our outdoor measurements is shown. To extract the frequency of interest from the data, filtering was used. All temperature changes lasting mach longer and shorter than a day has been removed by a band-pass filter. The bandwidth filter, with corresponding time interval from 20h to 28h was chosen, because the characteristic temperature variations do not follow exactly day time. To justify this bandwidth interval, the LOESS smoothing algorithm on temperature curves was applied. In Figures 7 and 8 smoothed, original, sinus fitted and band pass curves for outdoor and indoor air temperature, were shown respectively. Orange curve represents sum of two curves: smoothed with the LOESS algorithm and band pass. Excellent agreement of original (measured) data and processed one was obtained. Filtered curves are fitted at sine function of the form $y_0 + A\sin(\pi(x - x_c)/\omega)$, where y_0, A, x_c and ω are variable parameters. In Figures 7 and 8 with blue color filtered curves were shown.

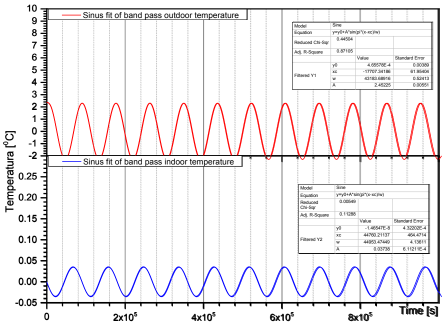


Figure 9. Sinusoidal outdoor (red) and indoor (blue) air temperature changes curves

In Figure 9 extracted sinusoidal outdoor (red) and indoor (blue) air temperature were drawn. These curves has been used for determination of the period T of input temperature functions from both sides of the wall. Corresponding periods are 88162s and 88189s for outdoor and indoor temperature respectively. A mean values of 88170s was chosen as a common period (1day has 86400s). In Figure 7 and 8 all fitting parameters including the angular frequency were given.

Dynamic thermal parameters of the wall: the decrement factor, time shift of thermal transmittance and areal heat capacities were calculated as explained in chapters 3 and 4. The structure and numerical values of dimensions and physical characteristics of the wall materials are shown in Table 1. In the Table 2 numerical values of these parameters were given. The wall is massive because it has decrement factor close to zero, meaning that it has strong damping capability. The massive constructions also have greater value of the time shift and in the Table 2 these values were shown.

Table 1. Physical characteristics of wall materials

Material	*d[cm]*	*λ[W/mK]*	*ρ [kg/m³]*	*c [J/kgK]*
mortar	2	1,4	2100	1050
ferroconcrete	20	2,33	2500	960
extruded polystyrene foam	7	0,035	33	1500
rabic grout	3	0,58	1200	920
plaster	2	0,7	1850	1050

Table 2. Numerical values of dynamic parameters

f	Δt_f [h]	$\kappa_1\ [J/(m^2K)]$	$\kappa_2\ [J/(m^2K)]$
0,26	-2,57	56389	171854

6. Conclusion

In this paper, the experimental data of indoor and outdoor temperature of a residential building in Belgrade, out of heating season, during three months, were processed and used to extract the period of characteristic daily sine temperature variations necessary for calculation of the dynamical parameters. The band pass filter has been used for data extraction for the daily temperature variations. The fitting to sine function and LOESS smoothing algorithm are used to check the results obtained by band pass filter. The physics of the heat transfer in the wall has been modeled by the Laplace transform approach which relates the outdoor and indoor temperatures and corresponding the heat fluxes in the matrix form. The matrix elements for the calculation of the parameters were used. The thermal properties of the construction in dynamical regime could be estimated from the dynamical parameters.

7. References

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