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DEFINING THE PRINCIPAL AXES OF THE QUADRIC CONE - GENERAL CASE WITH ELLIPTIC BASE SECTION CURVE

Aleksandar Čučaković[9]
Magdalena Dimitrijević[10]

RESUME

This paper presents an constructive procedure of determining three mutually orthogonal principal axes (three planes of symmetry) of the quadric cone, the general case with elliptical base section curve.The constructive procedure is based on establishing correlative corespondance between the base curve plane (points and lines) and bundle of lines and planes at the vertex of the cone.At the base curve plane, two pairs of collocal, corelatively associated planes are set. After overlapping two of them, the other two collocal planes become collinear. Three double points in two generally collinear planes are intersection points between three principal axes and base plain of the cone.

***Key words**: The general case of cone; the main axes of cone; corellative transformation; polarity; auto polar tetrahedron*

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1. THE INTRODUCTION

It is well known that each cone has three mutually orthogonal axes, so called – the principal axes of cone. They define three orthogonal planes i.e. three planes of symmetry of cone. The position of the principal axes of an right circular cone are not metter of interest. In case of an quadric cone (general case), with elliptic, parabolic or hyperbolic base curve, with one axis or plain of symmetry specified, a simple geometric construction is available to determine the other two axes, or the other two planes of symmetry. In case of quadric cone (elliptical, parabolic or hyperbolic type of base section curve), with no specified elements, constructive procedure for determining three principal orthogonal axes i.e. three symmetry planes, is complex geometric problem.

1.1 The correlative bundles of lines and planes in space

The cone τ is set with base curve section – ellipse ***k***, center point ***K***, in the horizontal plane ***H***, and vertex ***V*** above. The minimal distance from vertex ***V*** to the base plane is determined by radius of circle k_1. Center point K_1 is orthogonal projection ***V'*** of point ***V*** on the base curve plane ***H***.

The points and lines in base plane ***H***, with vertex *V* of cone ***τ***, form the bundle of lines and planes. They are mutuallly in correlative correspondance.

Each point in the base plane ***H***, related to a base curve - ellipse **k** of cone τ, induces on the correspondent polar line the involutory sequence of points [2;10]. Also, each involutory mapped pair of points on the polar line, with correspondent polar point of plain ***H*** form an auto polar triangle [4;74]. Connection line of vrtex ***V*** and corresponding polar point in the base plane ***H*** is carrier for an involutory pencil of planes respectively to the cone ***τ***.This pencil of planes induces in corresponding polar plane an pencil of lines. Each pair of corresponding lines from involutory pencil of planes (in the polar plane), with corresponding polar line forms an auto polar tetrahedron of cone [2;11]. Each plane, from pencil of planes, intersects cone in two generatricess, and polar plane in one straight line. This three lines are in dual harmonic relation to a polar line. In the involutory pencil of lines there is an orthogonal pair of lines. Regarding this, it is necessary to determine polar line, which is orthogonal to a polar plane.

The observed cone τ defines correlative bundle of lines and planes {□}. The bundle {□} consists of polar axes and associated polar planes, respectively to the cone τ.

A new right circular cone τ_1 is set within the same vertex (***V***). It's axis is perpendicular to a circular base of cone. Diameter of base circle k_1 is equal to perpendicular distance from vertex ***V*** to the plane ***H*** (all the generators make the angle of 45^o to the plane ***H***). This circular right cone determines new correlative bundle of lines and planes { □$_1$}.There are only three mutually orthogonal axes mapped to adequate polar planes in correlative bundle {□}, respectively to cone τ, while in the correlative bundle { □$_1$} respectively to cone τ_1, all polar axes and corresponding planes are mutually orthogonal [2;25].The double lines/double planes of those two corelative bundles are principal axes/planes of summetry for the quadric cone τ.

2. COLLOCAL CORRELATIVE MAPPED PLANES

Plane ***H***, with two base section curves: of cone τ and cone τ_1, intersect two collocal correlative bundles {□} and { □$_1$} in two pairs of collocal corelative corresponding planes (α,β) and (α_1,β_2). After overlaping planes β and β_1, the other two, α and α_1 become collinear.

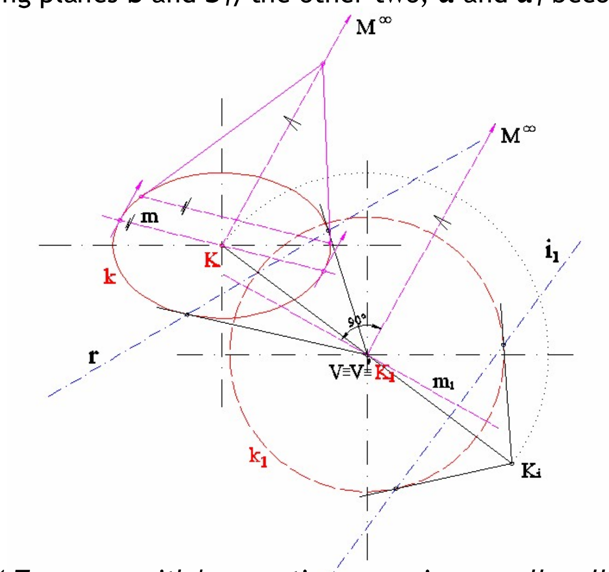


Fig.1 Two cones with base section curves in generally collinear planes

Two collocal, generally collinear planes $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha_1}$ are defined with pair of mapped points $\boldsymbol{K}$ and $\boldsymbol{K_1}$ and vanishing lines $\boldsymbol{r}$ and $\boldsymbol{i_1}$. Vanishing line $\boldsymbol{r}$ is polar line of pole - center point $\boldsymbol{K_1}$, respectively to ellipse $\boldsymbol{k}$.Vanishing line $\boldsymbol{i_1}$ is antipolar line of pole $\boldsymbol{K}$ (or polar line of inversed pole $\boldsymbol{Ki}$), respectively to circle $\boldsymbol{k_1}$. (fig.1) The inversion came out of the fact that cone $\boldsymbol{\tau_1}$ is real representative of the imaginary cone defined with vertex $\boldsymbol{V}$ and absolute conic curve.

In two collinear mapped planes $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha_1}$, centar point $\boldsymbol{K}$ of ellipse $\boldsymbol{k}$, corresponds to center point $\boldsymbol{K_1}$ of circle $\boldsymbol{k_1}$. Both are in correlative correspondence to the infinite line $\boldsymbol{t\infty \equiv t_1\infty}$ in base curve plane $\boldsymbol{\beta \equiv \beta_1}$. Conjugated diameters of elipse $\boldsymbol{k_1}$ form an involutory pencil of lines, while conugated diameters of circle $\boldsymbol{k}$ form circular involutory pencil of lines. Pairs of parallel lines in planes $\boldsymbol{\beta \equiv \beta_1}$ (linking lines between points $\boldsymbol{A^\infty}$, $\boldsymbol{B^\infty}$ and $\boldsymbol{C^\infty}$ on infinite line $\boldsymbol{t\infty \equiv t_1\infty}$ and center points $\boldsymbol{K/K_1}$) have their corresponding lines – conjugated diameters of ellipse $\boldsymbol{k}$ ($\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$) and circle $\boldsymbol{k_1}$ ($\boldsymbol{a}_1$,$\boldsymbol{b}_1$,$\boldsymbol{c}_1$) in planes $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha_1}$.(fig.2)

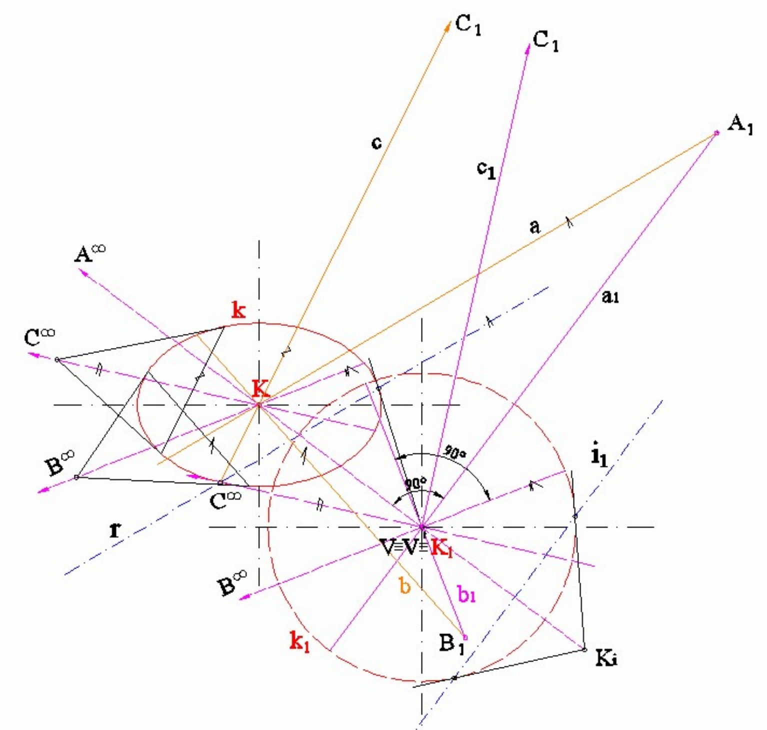


Fig.2 Two involutory pencils of lines respectively to ellipse $\boldsymbol{k}$ *and circle* $\boldsymbol{k_1}$

2.1 Hyperbola of Apollonius as a result of two projective mapped pencils of lines

Conjugated diameters $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{a_1}, \boldsymbol{b_1}, \boldsymbol{c_1}$, in planes $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha_1}$ form two projective mapped pencils of lines in vertexes $\boldsymbol{K}$ and $\boldsymbol{K_1}$ (center points of ellipse $\boldsymbol{k}$ and circle $\boldsymbol{k_1}$**).** They produce an conic curve of 2nd order, in this case, hyperbola $\boldsymbol{h_1}$ with orthogonal asimptotes – hyperbola of Apollonius. (fig.3) Two projective mapped pencils of lines were translated to one overlaping point $\boldsymbol{K \equiv K_1}$ in collocal "position". Using Steiner's construction procedure over sequence of points on 2nd order curve (Steiner's circle), for two perspective mapped pencils of lines (with vertexes $\boldsymbol{S}$ and $\boldsymbol{S_1}$), the axis of perspectivity (double lines $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$) determined directions of asymptotes of hyperbola.

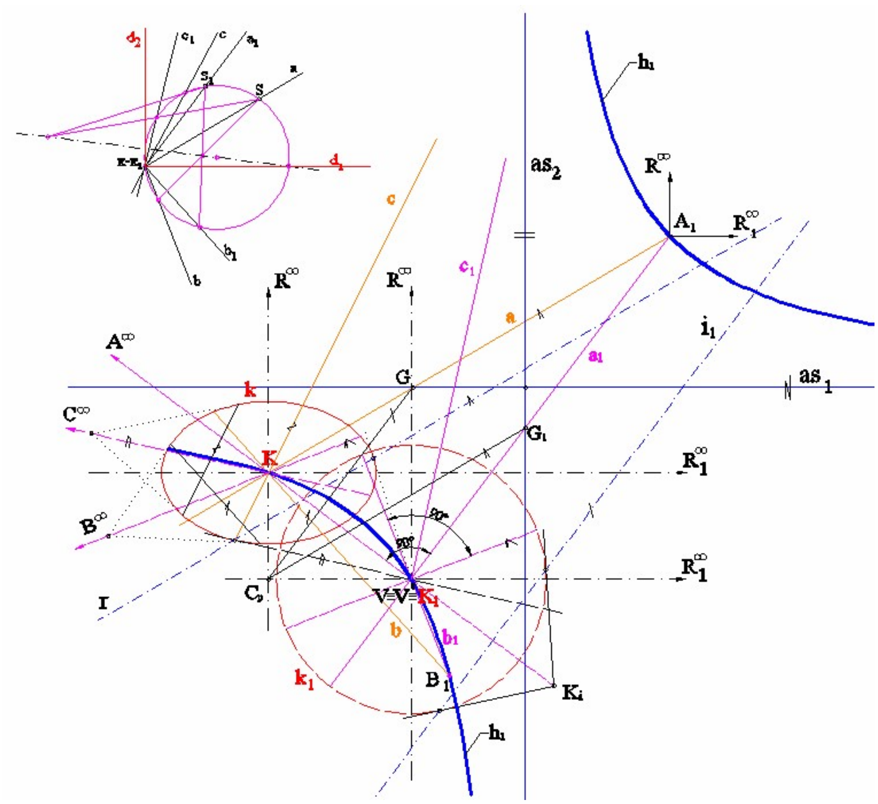


Fig.3 Steiner's construction (left top corner)
Two perspective mapped pencils of lines defining asymptotes of hyperbola

The other two projective mapped pencils of parallel lines ***(R)*** and (***R_1***), with infinite vertexes ***R*** and ***R_1***, determined position of asymptotes ***as_1*** and ***as_2*** and center point of hyperbola. Pencils of lines ***(R)*** and (***R_1***) were intersected with two lines ***a*** and ***a_1***, producing two perspective mapped sequences of points. The vanishing points ***G*** and ***G_1*** on two perspective mapped sequences of points, on lines ***a*** and ***a_1***, are referent points for position of asymptotes ***as_1*** and ***as_2***.(fig.3)

2.2 Projective mapped pencils of lines with vertexes in focal points of two collinear planes

Double points D_1, D_2 and D_3 for two generally collinear planes ***α*** and ***α_1*** are intersecting points between two conic curves (any type) derived from two pairs of projective mapped pencils of lines. One conic curve is hyperbola of Apollonius ***h_1***, with orthogonal asymptotes. The other appropriate choice of conic curve is circle, because of accuracy of intersection (between circle and hyperbola).

In order to get a circle, as a result of mapping, it is necessary to set vertexes of two projective mapped pencils of lines in focal points of planes ***α*** and ***α_1***. In process of constructing focal points, the first step is determination of vanishing points ***P*** and ***O_1*** on vanishing lines r and ***i_1*** in planes ***α*** and ***α_1***. Those are corresponding points to infinite points ***$O\infty$*** and ***$P_1\infty$*** of directions ***$n \perp r$*** and ***$p_1 \perp i_1$***.The main perpendiculars ***g_n*** , ***g_{n1}*** of planes ***α*** and ***α_1*** pass through points ***P*** and ***O_1***.

According to a known rule: „ Some lines in plane ***α*** ,from pencil of parallel lines set through infinite vertexes, have their corresponding lines in plane ***α_1*** passing through corresponding vanishing points and focal point ***L_1***“ (and vice versa), the focal points ***L*** and ***L_1*** were determined [1]. The important fact is that the angle which those corresponding lines make with the adequate vanishing lines has the same value. Following the rule [1], in plane ***α_1*** , the direction ***w_1*** (in vertex ***K_1***) and infinite point ***$W_1\infty$*** was addopted. Corresponding line ***w*** in plane ***α*** (in vertex ***K***) intersects vanishing line ***r*** in vanishing point ***W***. Eventualy, there is a line in plane ***α***, set through point ***W***, which make the angle of 69^{o} with vanishing line ***r*** (the same angle is between direction ***w_1*** and vanishing line ***i_1***, in plane ***α_1***) and intersecting the main perpendicular ***g_n*** in focal point ***L***.The same procedure followed for the focal point ***L_1***, using arbitrary direction ***q*** and point ***$Q\infty$*** in plane ***α***.(fig. 4)

The other pair of focal points M and M_1 is symetrical to L and L_1 respectively to vanishing lines r and i_1.

Two projective mapped pencils of lines with vertexes in focal points L and L_1, will produce a circle l as a conic curve. The circle l is defined with three points: focal points L and L_1 and point of intersection of two main perpendiculars g_n and g_{n1}. There are four points of intersection between circle l and hyperbola h_1.Three of them are double points D_1, D_2 and D_3 of two collocal mapped collinear planes α and α_1.

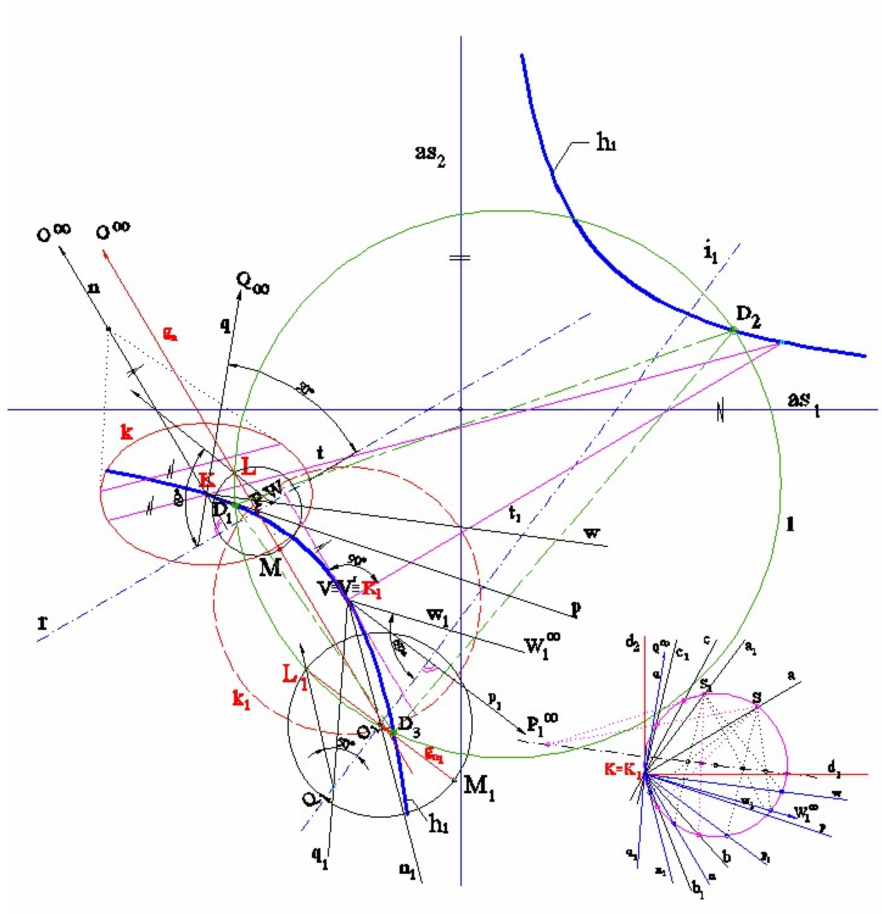


Fig.4 The focal points within planes α *and* α_1
Steiner's supplementary construction (right corner down)

The new circle $\boldsymbol{m}$ was adopted using three points: focal points $\boldsymbol{M}$ and $\boldsymbol{M_1}$ and intersection point between main perpendiculars $\boldsymbol{g_n}$ and $\boldsymbol{g_{n1}}$. Two projective mapped pencils of lines in points $\boldsymbol{M}$ and $\boldsymbol{M_1}$ produce new hyperbola $\boldsymbol{h_2}$, also with orthogonal asymptotes (hyperbola of Apollonius). Constructive procedure: in two projective pencils of lines $\boldsymbol{M}$ ($\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$) and $\boldsymbol{M_1^o}$($\boldsymbol{x_0}$, $\boldsymbol{y_0}$, $\boldsymbol{z_o}$) which produce a circle $\boldsymbol{m}$, one pencil , in vertex $\boldsymbol{M_1^o}$, was symmetrically transformed around axis $\boldsymbol{g_{n1}}$ into pencil $\boldsymbol{M_1}$ ($\boldsymbol{x_1}$, $\boldsymbol{y_1}$, $\boldsymbol{z_1}$), in order to produce hyperbola $\boldsymbol{h_2}$. (fig.5)

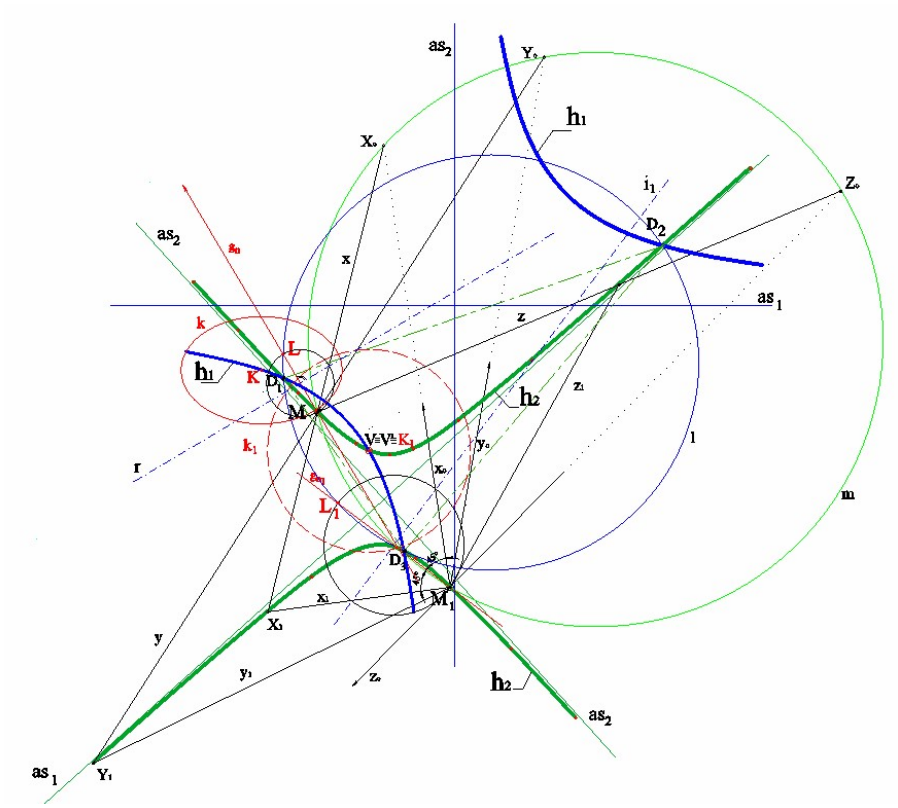
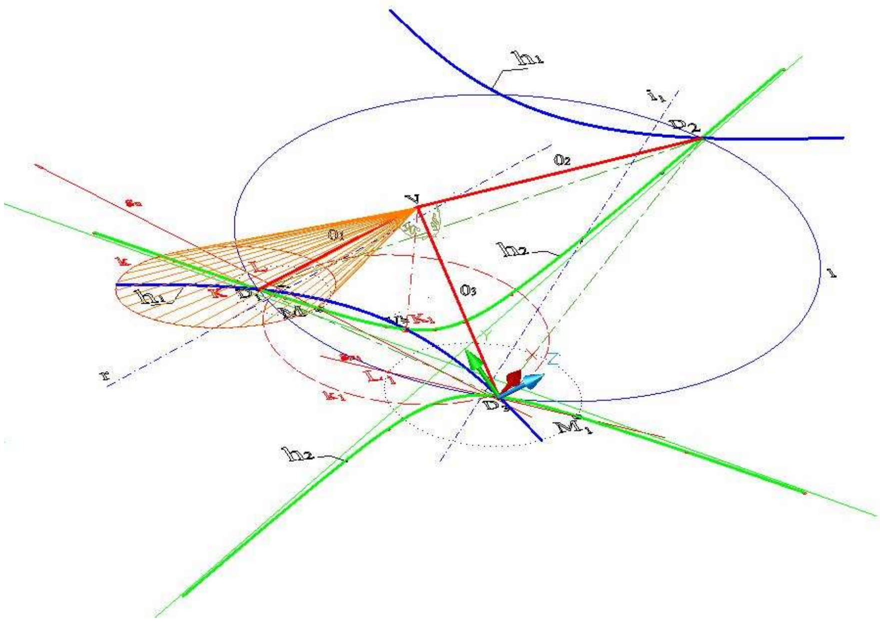


Fig.5 Two projective mapped pencils of lines for hyperbola h_2

As the result of intersection of three curves: hyperbola h_1, circle l, hyperbola h_2, three double points D_1, D_2 and D_3 of an auto polar triangle, in two generally collinear planes α and α_1, appear. Their linking lines o_1, o_2 and o_3 to a vertex V are three principal axes of cone τ, forming an auto polar tetrahedron (in space) with vertexes D_1, D_2, D_3 and V. (fig.6) These axes form 3 mutually orthogonal planes of symmetry of cone τ. (fig.7)



F*ig.6 Three principal axes of quadric cone* τ

3. CONCLUSION

The determination of principal axes of the quadric cone τ, with an elliptic base section curve and vertex V, is based on determination of vertexes (double points) D_1, D_2 and D_3 of an auto polar tetrahedron in generally collinear planes α and α_1. Those are intersection points between two conic curves: hyperbola h_1 and circle l, or circle l and hyperbola h_2, as results of mapping of two pairs of projective pencils of lines in focal points. If determined focal points L, L_1 and M, M_1 for two planes α and α_1, the easiest way to determine an auto polar tetrahedron i.e. axes of symmetry for the quadric cone τ, is to make one more projective transformation in focal points ***M, M$_1$*** deriving hyperbola of Apollonius ($\boldsymbol{h}_2$).

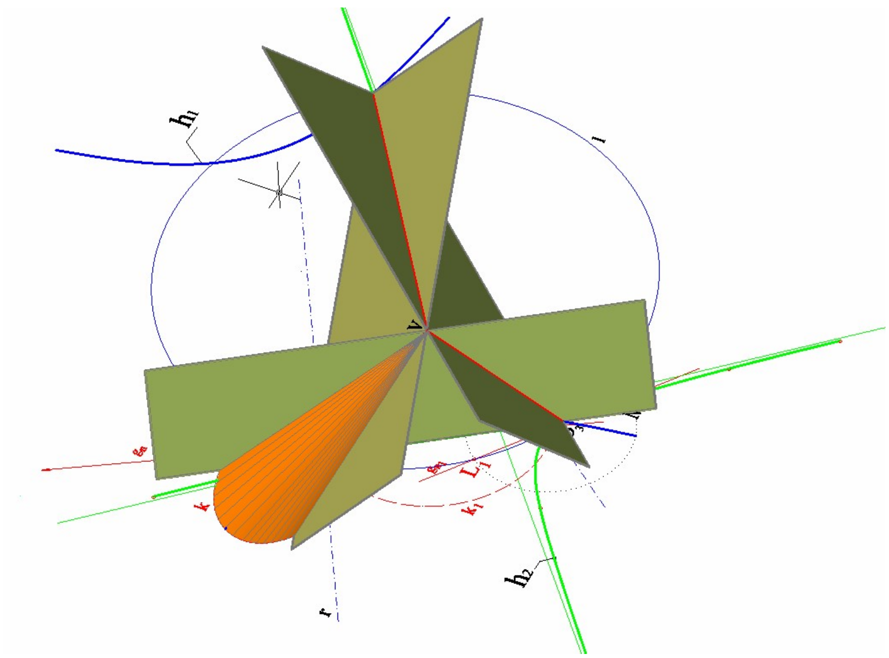


Fig.7 Three planes of symmetry of quadric cone τ

4. LITERATURE

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