

PRIOLOG PRORAČUNU SPREGNUTIH STUBOVA OD ČELIKA I BETONA

CONTRIBUTION TO ANALYSIS OF COMPOSITE STEEL AND CONCRETE COLUMNS

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PRETHODNO SAOPŠTENJE

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1 UVOD

Proračun spregnutih stubova prema Evrokodu 4 [2],[10],[11] se zasniva na konceptu graničnih stanja kao što je to detaljno objašnjeno u prethodnom radu istih autora [1]. U ovom radu je, na konkretnom brojnom primeru, ilustrovana primena uprošćene metode proračuna spegnutog stuba izloženog pritisku i dvoosnom savijanju momentima.

Prema Evrokodu 4 [2], analiza nosivosti, a time i stabilnosti pri aksijalnom pritisku izolovanog stuba bazira se na primeni Evropskih krivih izvijanja. Provera nosivosti pri kombinaciji aksijalnog pritiska i savijanja momentima zasniva se na interakcionoj krivoj koja se određuje pri analizi nosivosti poprečnog preseka posmatranog stuba [2], [5], [6], [8]. Pri tome, treba uzeti u obzir i uticaje drugog reda kao i uticaje usled skupljanja i tečenja betona [7], [9], [12],[13].

1 INTRODUCTION

The analysis of composite columns according to Eurocode 4 [2],[10],[11] is based on the concept of limit states as it is presented in detail in the previous paper of the same authors [1]. In this paper, the simplified design method is illustrated on the numerical example of composite column subjected to compression and biaxial bending.

According to the Eurocode 4 [2], the resistance analysis and, therefore, the stability analysis of an individual column subjected to compression are based on the use of European buckling curves. The resistance analysis of the column subjected to compression and bending is based on the interaction curve determined from the section capacity analysis [2], [5], [6], [8]. Also, second order effects and creep and shrinkage effects should be taken into account [7], [9], [12],[13].

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2 BROJNI PRIMER

Prikazaće se proračun nosivosti spregnutog stuba izloženog aksijalnom pritisku i savijanju oko jedne (y ili z ose) ili oko obe ose, prema Evrokodu 4 [2].

Dati su sledeći podaci:

- Dužina stuba $L = 5\text{m}$
- Čelični profil HEB 260 S355
- Armatura $8\text{Ø}16$ S500
- Beton C40/50
- Globalnom analizom, za merodavnu kombinaciju opterećenja, određeni su sledeći uticaji: $N_{Ed}=4200\text{kN}$, pri čemu je $N_{G,Ed}=3000\text{kN}$, $M_{y,Ed,vrh}=150\text{kNm}$, $M_{y,Ed,dno}=0$, $M_{z,Ed,vrh}=20\text{kNm}$, $M_{y,Ed,dno}=0$. Nema poprečnog opterećenja duž ose stuba.

2.1 Geometrijske karakteristike preseka

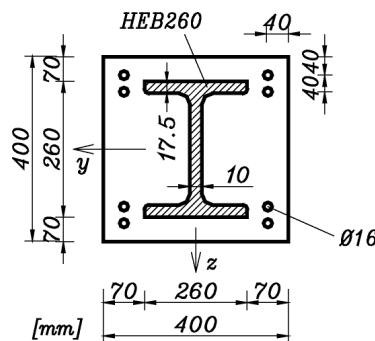
2 NUMERICAL EXAMPLE

The design of a composite column subjected to axial compression and bending about one axis (y or z axis) or both axes, according to Eurocode 4 [2], is presented.

The following data are given:

- Column length $L = 5\text{m}$
- Steel section HEB 260 S355
- Reinforcement $8\text{Ø}16$ S500
- Concrete C40/50
- From global analysis, for the most unfavourable load arrangement, the following forces are obtained: $N_{Ed}=4200\text{kN}$, with $N_{G,Ed}=3000\text{kN}$, $M_{y,Ed,top}=150\text{kNm}$, $M_{y,Ed,bottom}=0$, $M_{z,Ed,top}=20\text{kNm}$, $M_{y,Ed,bottom}=0$. There was no lateral loading.

2.1 Geometrical properties of the cross section



Slika 1. Dimenzije poprečnog preseka spregnutog stuba
Figure 1 Dimensions of the composite cross section

Dimenzije poprečnog preseka spregnutog stuba date su na slici 1, pa je $b_c=h_c=400\text{mm}$, $b=h=260\text{mm}$ i $c_y=c_z=200-130=70\text{mm}$.

Zaštitni sloj betona ispunjava uslove iz Evrokoda 4 [2]:

Dimensions of the column cross section are given in the Figure 1: $b_c=h_c=400\text{mm}$, $b=h=260\text{mm}$ and $c_y=c_z=200-130=70\text{mm}$.

Dimensions of concrete cover satisfy conditions of Eurocode 4 [2]:

$$40\text{ mm} < c_y < 0.4 \cdot 260 = 104\text{mm}$$

$$40\text{ mm} < c_z < 0.3 \cdot 260 = 78\text{mm}$$

$$c_z > \frac{260}{6} = 43.3\text{mm}$$

Površina armature iznosi: $A_s = 8 \cdot 64p = 1608\text{mm}^2$

Površina čeličnog profila iznosi: $A_a = 11800\text{mm}^2$

Površina betona iznosi: $A_c = 400^2 - 11800 - 1608 = 146592\text{mm}^2$

Procenat armiranosti betona iznosi: $\frac{A_s}{A_c} = \frac{1608}{146592} = 1.1\% < 6\%$ $1.1\% > 0.3\%$

Momenti inercije pojedinih delova spregnutog preseka, oko ose y, su:

Area of reinforcement: $A_s = 8 \cdot 64p = 1608\text{mm}^2$

Area of steel section: $A_a = 11800\text{mm}^2$

Area of concrete: $A_c = 400^2 - 11800 - 1608 = 146592\text{mm}^2$

Percent of reinforcement in the concrete: $\frac{A_s}{A_c} = \frac{1608}{146592} = 1.1\% < 6\%$ $1.1\% > 0.3\%$

Second moments of area for parts of composite section, about y axis:

$$I_{a,y} = 149.2 \cdot 10^6\text{mm}^4,$$

$$I_{s,y} = 4 \cdot 201 \cdot (160^2 + 120^2) = 32.2 \cdot 10^6\text{mm}^4,$$

$$I_{c,y} = \frac{400^4}{12} - 149.2 \cdot 10^6 - 32.2 \cdot 10^6 = 1952 \cdot 10^6\text{mm}^4.$$

Momenti inercije pojedinih delova spregnutog preseka, oko ose z, su:

Second moments of area for parts of composite section, about z axis:

$$I_{a,z} = 51.3 \cdot 10^6 \text{ mm}^4,$$
$$I_{s,z} = 8 \cdot 201 \cdot 160^2 = 41.2 \cdot 10^6 \text{ mm}^4,$$
$$I_{c,z} = \frac{400^4}{12} - 51.3 \cdot 10^6 - 41.2 \cdot 10^6 = 2040.8 \cdot 10^6 \text{ mm}^4.$$

Ostale karakteristike čeličnog profila su:
 $W_{pa,y} = 1282 \cdot 10^3 \text{ mm}^3$ i $W_{pa,z} = 602 \cdot 10^3 \text{ mm}^3$.

Additional properties of steel section:
 $W_{pa,y} = 1282 \cdot 10^3 \text{ mm}^3$ i $W_{pa,z} = 602 \cdot 10^3 \text{ mm}^3$.

2.2 Karakteristike materijala

2.2 Properties of the materials

Čelični nosač S355:

Steel section S355:

$$f_y = 355 \text{ N/mm}^2, f_{yd} = \frac{355}{1.0} = 355 \text{ N/mm}^2, E_a = 210 \text{ kN/mm}^2.$$

Armatura S500:

Reinforcement S500:

$$f_{sk} = 500 \text{ N/mm}^2, f_{sd} = \frac{500}{1.15} = 435 \text{ N/mm}^2, E_s = 210 \text{ kN/mm}^2.$$

Beton C40/50:

Concrete C40/50:

$$f_{ck} = 40 \text{ N/mm}^2, f_{cd} = \frac{40}{1.5} = 26.7 \text{ N/mm}^2, 0.85 f_{cd} = 22.7 \text{ N/mm}^2, E_{cm} = 35 \text{ kN/mm}^2.$$

2.3 Nosivost poprečnog preseka spregnutog stuba

2.3 Composite cross-section resistance

Nosivost potpuno plastifikovanog poprečnog preseka pri aksijalnom pritisku je:

Resistance of fully plastified cross section to axial compression is:

$$N_{pl,Rd} = 11.8 \cdot 355 + 146.592 \cdot 22.7 + 1.608 \cdot 435 = 4189 + 3327.6 + 699.5 = 8216.1 \text{ kN}$$

Koeficijent doprinosa čeličnog nosača iznosi:

The steel contribution ratio:

$$d = \frac{4189}{8216.1} = 0.51$$

i nalazi se unutar propisanih granica za spregnute stubove.

is within the prescribed limits for composite columns.

Provera nosivosti poprečnog preseka pri aksijalnom pritisku i savijanju i određivanje odgovarajućih interakcionih krivih će se prikazati u okviru proračuna nosivosti stuba.

The verification of cross-section resistance to axial compression and bending and construction of interaction curves is given in the composite column resistance analysis section.

2.4 Proračun nosivosti spregnutog stuba

2.4 Composite column resistance analysis

2.4.1 Nosivost stuba pri aksijalnom pritisku i savijanju oko y ose

2.4.1 Resistance to compression and bending about y axis

Karakteristična vrednost nosivosti poprečnog preseka pri pritisku je:

The characteristic value of the resistance to compression is:

$$N_{pl,Rk} = 4189 + 1.5 \cdot 3327.6 + 1.15 \cdot 699.5 = 9984.8 \text{ kN}$$

Koeficijent tečenja pri dugotrajnom opterećenju φ_t koji odgovara uslovima unutrašnje sredine (vlažnost 50%) i starosti betona pri opterećenju od $t_o=30$ dana se može odrediti prema Evrokodu 2 [2], na sledeći način.

Obim dela preseka koji je u dodiru sa vazduhom je

The creep coefficient φ_t for long time loading, inside conditions (relative humidity 50%) and age of concrete at loading equal to $t_o=30$ days can be found from Eurocode 2 [2].

The perimeter of the part of the concrete which is

$u = 2(b_c + h_c) = 1600 \text{ mm}$, pa je nominalna dimenzija poprečnog preseka betonskog elementa $h_o = 2A_c/u = 2 \cdot 146592/1600 = 183 \text{ mm}$. Prema Evrokodu 2 [4], koeficijent tečenja, za beton klase C40/50, iznosi $j_t = 1.9$.

Efektivni modul elastičnosti betona je:

$$E_{c,eff} = 35 \frac{1}{1 + (3000/4200) \cdot 1.9} = 14.8 \text{ kN/mm}^2$$

Efektivna krutost preseka iznosi:

$$(EI_y)_{eff} = 210 \cdot 149.2 \cdot 10^6 + 210 \cdot 32.2 \cdot 10^6 + 0.6 \cdot 14.8 \cdot 1952 \cdot 10^6 = 55428 \cdot 10^6 \text{ kNmm}^2$$

Elastična kritična sila izvijanja oko y ose je (pretpostavljeni su uslovi oslanjanja na krajevima stuba koji sprečavaju pomeranje, ali ne i rotaciju krajnjih preseka, pa je dužina izvijanja stuba jednaka dužini stuba):

$$N_{cr,y} = \frac{p^2 \cdot 55428}{5.0^2} = 21882 \text{ kN}$$

Relativna vitkost je jednaka:

$$\bar{I}_y = \sqrt{\frac{9984.8}{21882}} = 0.676 < 2.0$$

Redukcioni faktor C : kriva izvijanja b $\Rightarrow a = 0.34$

$$f = 0.5 \left[1 + 0.34(0.676 - 0.2) + (0.676)^2 \right] = 0.809$$

$$c = \frac{1}{0.809 + \sqrt{0.809^2 - 0.676^2}} = 0.798 \leq 1$$

Kontrola nosivosti pri aksijalnom pritisku:

$$N_{Ed} \leq c \cdot N_{pl,Rd} \quad 4200 \leq 0.798 \cdot 8216.1 = 6556 \text{ kN}$$

Nosivost stuba pri aksijalnom pritisku je zadovoljena.

2.4.2 Uticaj poprečne smičuće sile $V_{z,Ed}$

$$V_{z,pl,Rd} = V_{z,pl,a,Rd} = 3715 \cdot \frac{0.355}{\sqrt{3}} = 761.4 \text{ kN}$$

gde je, prema Evrokodu 3 [3], odgovarajuća površina smicanja:

$$A_{v,z} = A - 2 \cdot b \cdot t_f + (t_w + 2r) \cdot t_f = 11800 - 2 \cdot 260 \cdot 17.5 + (10 + 2 \cdot 24) \cdot 17.5 = 3715 \text{ mm}^2$$

$$V_{z,Ed} = 150/5 = 30 \text{ kN} < 0.5V_{z,pl,Rd}$$

Dakle, ne mora se uzeti u obzir uticaj poprečne smičuće sile $V_{z,Ed}$.

2.4.3 Kriva interakcije za savijanje oko y ose

Određiće se koordinate karakterističnih tačaka krive interakcije (slika 2).

Tačka A:

exposed to drying is $u = 2(b_c + h_c) = 1600 \text{ mm}$, so the notational size is $h_o = 2A_c/u = 2 \cdot 146592/1600 = 183 \text{ mm}$. According to Eurocode 2 [4], the creep coefficient for concrete class C40/50 is $j_t = 1.9$.

Effective modulus of elasticity is:

Effective flexural stiffness is:

The elastic critical buckling force about y axis is (the assumed end conditions are such that end displacements are restrained, but not end rotations, and thus the effective length is equal to the column length):

Relative slenderness:

Reduction coefficient C : buckling curve b $\Rightarrow a = 0.34$

Resistance to axial compression:

The resistance to axial compression is satisfied.

2.4.2 Influence of transverse shear force $V_{z,Ed}$

Where the relevant shear area according to Eurocode 3 [3] is:

Therefore, the influence of shear force $V_{z,Ed}$ can be neglected.

2.4.3 Interaction curve for bending about y axis

The coordinates of the characteristic points on the interaction curve are determined (Figure 2).

Point A:

$$N_A = N_{pl,Rd} = 8216.1 \text{ kN}$$

$$M_A = 0$$

Tačke B, C, D:

Points B, C, D:

$$N_C = N_{pm,Rd} = 146592 \cdot 22.7 = 3327.6 \text{ kN}$$

$$N_D = N_{pm,Rd} / 2 = 1663.8 \text{ kN}$$

$$W_{ps} = 4 \cdot 201 \cdot (160 + 120) = 225.120 \cdot 10^3 \text{ mm}^3$$

$$W_{pc} = \frac{400 \cdot 400^2}{4} - 225.120 \cdot 10^3 - 1282 \cdot 10^3 = 14.49 \cdot 10^6 \text{ mm}^3$$

$$M_D = M_{max,Rd} = \frac{1}{2} 14.49 \cdot 22.7 + 1.282 \cdot 355 + 0.225 \cdot 435 = 717.4 \text{ kNm}$$

Određivanje položaja neutralne ose za tačke B i C:

Calculation of the position the neutral axis for points B and C:

$$h_n = \frac{3327.6 \cdot 10^3}{2 \cdot 400 \cdot 22.7 + 2 \cdot 10(2 \cdot 355 - 22.7)} = 104 \text{ mm}$$

Dakle, neutralna osa jeste u rebru.

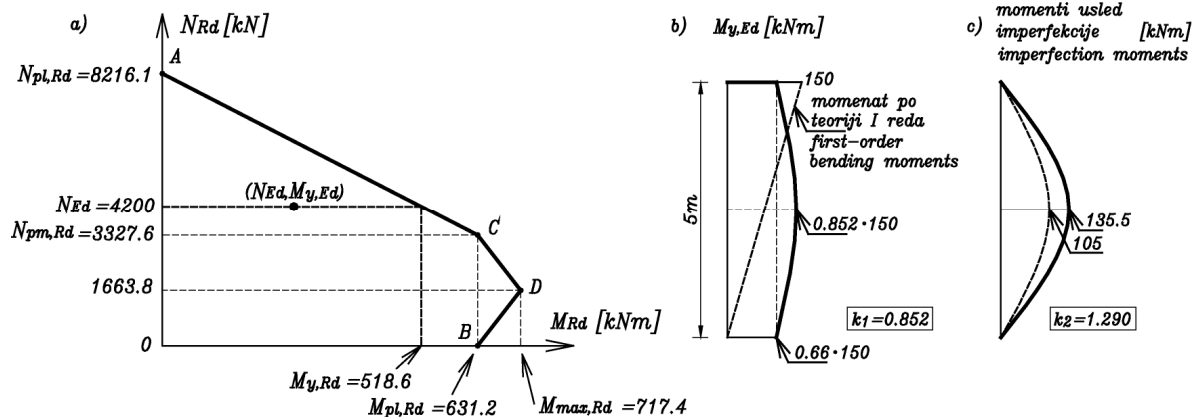
So, the neutral axis passes through the web (the assumption is correct).

$$W_{pan} = 10 \cdot 0.104^2 \cdot 10^6 = 0.108 \cdot 10^6 \text{ mm}^3$$

$$W_{pcn} = 400 \cdot 0.104^2 \cdot 10^6 - 0.108 \cdot 10^6 = 4.218 \cdot 10^6 \text{ mm}^3$$

$$M_{n,Rd} = 0.108 \cdot 355 + \frac{1}{2} 4.218 \cdot 22.7 = 86.2 \text{ kNm}$$

$$M_{pl,Rd} = 717.4 - 86.2 = 631.2 \text{ kNm}$$



Slika 2. a) Kriva interakcije N-My; b) povećanje momenata na krajevima; c) povećanje imperfekcijskog momenta
 Figure 2. a) Interaction curve N-My; b) increase of end moments; c) increase of imperfection moment

2.4.4 Momenti po teoriji II reda – savijanje oko y ose

2.4.4 Second-order bending moment – bending about y axis

Da bi se proverilo da li uticaji drugog reda mogu da se zanemare, potrebno je odrediti proračunsku vrednost efektivne krutosti na savijanje $(EI)_{eff,II}$.

In order to check whether the second-order effects can or cannot be neglected, the effective flexural stiffness $(EI)_{eff,II}$ is determined:

$$(EI_y)_{eff,II} = 0.9 \cdot 10^6 (210 \cdot 149.2 + 210 \cdot 32.2 + 0.5 \cdot 14.8 \cdot 1952) = 47.3 \cdot 10^9 \text{ kNmm}^2$$

$$N_{cr,y,eff} = \frac{p^2 \cdot 47300}{5.0^2} = 18673 \text{ kN} < 10N_{Ed}$$

Dakle, uticaji drugog reda se ne mogu zanemariti.

Therefore, second order effects cannot be

Raspodela momenata sračunata prema teoriji I reda je prikazana na slici 2.b:

neglected.

The first-order bending moment distribution is shown in Figure 2.b:

$$M_{y,Ed,vrh} = 150 \text{ kNm}, \quad M_{y,Ed,dno} = 0.$$

Ovakvoj raspodeli momenata savijanja odgovara, prema tabeli 3 [1] za $r=0$, koeficijent $b = 0.66$, pa je:

For this bending moment distribution, according to Table 3 [1] for $r=0$, $b = 0.66$, so:

$$k_1 = \frac{0.66}{1 - 4200/18673} = 0.852$$

Moment savijanja na sredini stuba usled imperfekcije iznosi:

Moment at mid-span due to imperfection is:

$$e_{0,z} = \frac{L}{200} = 25 \text{ mm}$$

$$e_{0,z} = \frac{L}{200} = 25 \text{ mm}$$

$N_{Ed}e_{0,z} = 4200 \cdot 0.025 = 105 \text{ kNm}$ (slika 2.c). Za ovaj moment savijanja, $b = 1.0$, pa je

$N_{Ed}e_{0,z} = 4200 \cdot 0.025 = 105 \text{ kNm}$ (Figure 2.c). For this bending moment, $b = 1.0$, so

$$k_2 = \frac{1.0}{1 - 4200/18673} = 1.290.$$

$$k_2 = \frac{1.0}{1 - 4200/18673} = 1.290.$$

Uvećani moment usled imperfekcije iznosi $1.290 \cdot 105 = 135.5 \text{ kNm}$.

The increased bending moment due to imperfection is $1.290 \cdot 105 = 135.5 \text{ kNm}$.

Ukupan moment na sredini stuba je: $0.852 \cdot 150 + 1.290 \cdot 105 = 127.8 + 135.5 = 263.3 \text{ kNm}$ i on je veći od momenta na kraju stuba $M_{y,Ed,vrh} = 150 \text{ kNm}$, pa je $M_{y,Ed,max} = 263.3 \text{ kNm}$ moment prema kome treba vršiti proveru nosivosti stuba.

The total bending moment at mid-span is: $0.852 \cdot 150 + 1.290 \cdot 105 = 127.8 + 135.5 = 263.3 \text{ kNm}$

which is greater than maximal end moment $M_{y,Ed,top} = 150 \text{ kNm}$, so $M_{y,Ed,max} = 263.3 \text{ kNm}$ is the moment that governs the design.

Tačka $(N_{Ed}; M_{z,Ed,max}) = (4200; 263.3)$ leži unutar oblasti ograničene krivom interakcije, slika 2.a. Na osnovu vrednosti datih na slici 2.a, može se sračunati $M_{y,Rd}$:

Point $(N_{Ed}; M_{z,Ed,max}) = (4200; 263.3)$ is inside the area limited with the interaction curve, Figure 2.a. From values given in Figure 2.a, the $M_{y,Rd}$ can be found:

$$M_{y,Rd} = (631.2 \cdot (8216.1 - 4200)) / (8216.1 - 3327.6) = 518.6 \text{ kNm},$$

$$\alpha_M M_{y,Rd} = 0.9 \cdot 518.6 = 466.74 \text{ kNm} > M_{y,Ed,max}$$

Nosivost stuba pri aksijalnom pritisku i savijanju oko y ose je zadovoljena.

Resistance to compression and bending about y axis is satisfied.

2.4.5 Nosivost stuba pri aksijalnom pritisku i savijanju oko z ose

2.4.5 Resistance to compression and bending about z axis

Efektivna krutost preseka je:

Effective flexural stiffness is:

$$(EI_z)_{eff} = 210 \cdot 51.3 \cdot 10^6 + 210 \cdot 41.2 \cdot 10^6 + 0.6 \cdot 14.8 \cdot 2040.8 \cdot 10^6 = 37547 \cdot 10^6 \text{ kNmm}^2$$

$$N_{cr,z} = \frac{p^2 \cdot 37547}{5.0^2} = 14823 \text{ kN}$$

Relativna vitkost je jednaka:

Relative slenderness:

$$\bar{\lambda}_z = \sqrt{\frac{9984.8}{14823}} = 0.821 < 2.0$$

Redukcioni faktor C : kriva izvijanja $c \Rightarrow a = 0.49$

Reduction coefficient C : buckling curve $c \Rightarrow a = 0.49$

$$f = 0.5 [1 + 0.49(0.821 - 0.2) + (0.821)^2] = 0.989$$

$$c = \frac{1}{0.989 + \sqrt{0.989^2 - 0.821^2}} = 0.649 \leq 1$$

Kontrola nosivosti pri aksijalnom pritisku:

Resistance to axial compression:

$$N_{Ed} \leq c \cdot N_{pl,Rd} \quad 4200 \leq 0.649 \cdot 8216.1 = 5332 \text{ kN}$$

Nosivost stuba pri aksijalnom pritisku je

The resistance to axial compression is satisfied.

zadovoljena.

2.4.6 Influence of transverse shear force $V_{y,Ed}$

2.4.6 Uticaj poprečne smičuće sile $V_{y,Ed}$

$$V_{y,pl,Rd} = V_{y,pl,a,Rd} = 9100 \cdot \frac{0.355}{\sqrt{3}} = 1865.1 \text{ kN}$$

gde je, prema Evrokodu 3 [3], odgovarajuća površina smicanja:

Where the relevant shear area according to Eurocode 3 [3] is:

$$A_{v,y} = 2bt_f = 2 \cdot 260 \cdot 17.5 = 9100 \text{ mm}^2$$

$$V_{y,Ed} = 20/5 = 4 \text{ kN} < 0.5V_{y,pl,Rd}$$

Dakle, ne mora se uzeti u obzir uticaj poprečne smičuće sile $V_{y,Ed}$.

Therefore, the influence of shear force $V_{z,Ed}$ can be neglected.

2.4.7 Kriva interakcije za savijanje oko z ose

2.4.7 Interaction curve for bending about z axis

Određiće se koordinate karakterističnih tačaka krive interakcije (slika 3).

The coordinates of the characteristic points on the interaction curve are determined (Figure 3).

Tačka A:

Points A:

$$N_A = N_{pl,Rd} = 8216.1 \text{ kN}$$

$$M_A = 0$$

Tačke B, C, D:

Points B, C, D:

$$N_C = N_{pm,Rd} = 146592 \cdot 22.7 = 3327.6 \text{ kN}$$

$$N_D = N_{pm,Rd} / 2 = 1663.8 \text{ kN}$$

$$W_{ps} = 2 \cdot 4 \cdot 201 \cdot 160 = 257.280 \cdot 10^3 \text{ mm}^3$$

$$W_{pc} = \frac{400 \cdot 400^2}{4} - 257.280 \cdot 10^3 - 602 \cdot 10^3 = 15.141 \cdot 10^6 \text{ mm}^3$$

$$M_D = M_{max,Rd} = \frac{1}{2} 15.141 \cdot 22.7 + 0.602 \cdot 355 + 0.257 \cdot 435 = 497.4 \text{ kNm}$$

Određivanje položaja neutralne ose za tačke B i C:

Calculation of the position the neutral axis for points B and C:

$$h_n = \frac{3327.6 \cdot 10^3 + 10(2 \cdot 17.5 - 260)(2 \cdot 355 - 22.7)}{2 \cdot 400 \cdot 22.7 + 4 \cdot 17.5(2 \cdot 355 - 22.7)} = 27 \text{ mm}$$

Dakle, neutralna osa jeste u rebru.

So, the neutral axis passes through the web (the assumption is correct).

$$W_{pnn} = 2 \cdot 17.5 \cdot 0.027^2 \cdot 10^6 + \frac{(260 - 2 \cdot 17.5) \cdot 0.01^2}{4} \cdot 10^6 = 0.031 \cdot 10^6 \text{ mm}^3$$

$$W_{pcn} = 400 \cdot 0.027^2 \cdot 10^6 - 0.031 \cdot 10^6 = 0.261 \cdot 10^6 \text{ mm}^3$$

$$M_{n,Rd} = 0.031 \cdot 355 + \frac{1}{2} 0.261 \cdot 22.7 = 13.97 \text{ kNm}$$

$$M_{pl,Rd} = 497.4 - 13.97 = 483.43 \text{ kNm}$$

2.4.8 Momenti po teoriji II reda – savijanje oko z ose

2.4.8 Second-order bending moment – bending about z axis

$$(EI_z)_{eff,II} = 0.9 \cdot 10^6 (210 \cdot 51.3 + 210 \cdot 41.2 + 0.5 \cdot 14.8 \cdot 2040.8) = 31.07 \cdot 10^9 \text{ kNmm}^2$$

$$N_{cr,z,eff} = \frac{p^2 \cdot 31070}{5.0^2} = 12266 \text{ kN} < 10N_{Ed}$$

Uticaji drugog reda se ne mogu zanemariti.

Vrednosti momenata savijanja prema teoriji I reda

su:

$$M_{z,Ed,vrh} = 20 \text{ kNm}, \quad M_{z,Ed,dno} = 0.$$

$$k_1 = \frac{0.66}{1 - 4200/12266} = 1.004$$

$$e_{0,y} = \frac{L}{150} = 33.3 \text{ mm}$$

Momenat usled imperfekcije iznosi:

Imperfection is: $e_{0,y} = \frac{L}{150} = 33.3 \text{ mm}$ and imperfection moment is:

$$N_{Ed}e_{0,y} = 4200 \cdot 0.0333 = 140 \text{ kNm}$$

$$k_2 = \frac{1.0}{1 - 4200/12266} = 1.521.$$

Uvećani moment usled imperfekcije iznosi $1.521 \cdot 140 = 212.94 \text{ kNm}$.

Ukupan moment na sredini stuba je: $1.004 \cdot 20 + 1.521 \cdot 140 = 233 \text{ kNm}$ i on je veći od $M_{z,Ed,vrh} = 20 \text{ kNm}$, pa je $M_{z,Ed,max} = 233 \text{ kNm}$.

Tačka $(N_{Ed}; M_{z,Ed,max}) = (4200; 233)$ leži unutar oblasti ograničene krivom interakcije, slika 3. Na osnovu vrednosti datih na slici 3, može se sračunati $M_{z,Rd}$:

$$M_{z,Rd} = (483.4 \cdot (8216.1 - 4200)) / (8216.1 - 3327.6) = 397.1 \text{ kNm},$$

$$a_M M_{z,Rd} = 0.9 \cdot 397.1 = 357.4 \text{ kNm} > M_{z,Ed,max}$$

Therefore, second order effects cannot be neglected.

The first-order bending moments are:

tion moment is:

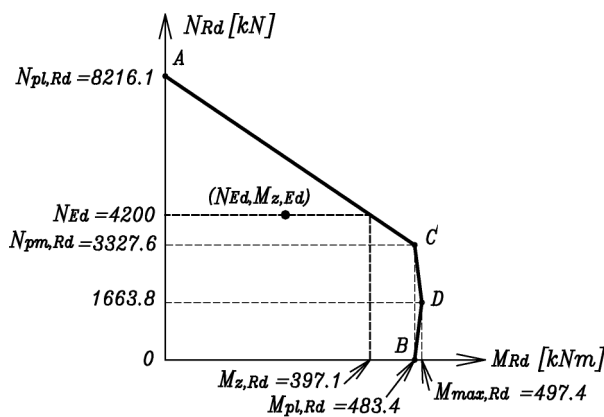
The increased bending moment due to imperfection is $1.521 \cdot 140 = 212.94 \text{ kNm}$.

The total bending moment at mid-span is: $1.004 \cdot 20 + 1.521 \cdot 140 = 233 \text{ kNm}$, and is greater than $M_{z,Ed,top} = 20 \text{ kNm}$, so $M_{z,Ed,max} = 233 \text{ kNm}$.

Point $(N_{Ed}; M_{z,Ed,max}) = (4200; 233)$ is inside the area limited with the interaction curve, Figure 3. From values given in Figure 3, the $M_{z,Rd}$ can be found:

Nosivost stuba pri aksijalnom pritisku i savijanju oko z ose je zadovoljena.

Resistance to compression and bending about z axis is satisfied.



Slika 3. Kriva interakcije N-Mz
Figure 3. Interaction curve N-Mz

2.4.9 Nosivost stuba pri aksijalnom pritisku i dvoosnom savijanju

Kada je stub izložen aksijalnom pritisku i dvoosnom savijanju, vrši se provera nosivosti za svaku ravan savijanja pojedinačno, kao što je prethodno prikazano, ali se imperfekcija uzima u obzir samo u ravni u kojoj se očekuje pojava loma, a to je u ovom slučaju pri savijanju oko ose z ($N_{Ed}e_0$ je veće za savijanje oko ose z), pa je $M_{y,Ed}=127.8$ kNm (slika 2.b)

Konačno,

$$\frac{127.8}{518.6} + \frac{233}{397.1} = 0.833 \leq 1,0$$

Kako je ovaj uslov ispunjen, a prethodno je dokazana i nosivost pri aksijalnom pritisku, to sledi da je: **nosivost stuba pri pritisku i dvoosnom savijanju zadovoljena.**

3 KRIVE INTERAKCIJE

Na osnovu datog brojnog primera, može se zaključiti da znatan deo proračuna spregnutih stubova izloženih kombinaciji aksijalnog pritiska i savijanja momentima (oko jedne ili oko obe glavne ose inercije) predstavlja određivanje N-M interakcionih krivih. U ovom delu rada je, za stub iz datog brojnog primera, prikazana analiza zavisnosti interakcionih krivih od količine podužne armature i od marke betona. Takođe su konstruisane i kontinualne krive interakcije za uobičajene dimenzije spregnutih stubova sa ubetoniranim I čeličnim profilom i za najčešće klase betona. Date krive, sa svojim referentnim vrednostima, olakšavaju dimenzionisanje ove grupe stubova pojednostavljujući izbor dimenzija kako betonskog dela preseka, tako i čeličnog profila, kao i izbor klase betona.

3.1 Uticaj klase betona

Najpre je analiziran uticaj klase betona na graničnu nosivost spregnutog preseka. Ova zavisnost će biti ilustrovana na krivama interakcije za poprečni presek stuba iz brojnog primera, za betone sledećih klasa: C20/25, C30/37, C40/50 i C50/60. Za ostale klase betona, odgovarajuće krive je moguće dobiti linearnom interpolacijom. Odgovarajuće krive su prikazane na slici 4.

Može se zaključiti da se krive interakcije približno izotropno šire sa povećanjem klase betona.

3.2 Uticaj podužne armature preseka

Na istom poprečnom preseku spregnutog stuba, sa betonom klase C40/50, biće prikazan i uticaj količine podužne armature stuba na graničnu nosivost preseka. Posmatran je presek bez podužne armature i preseci armirani sa 4Φ12, 4Φ16 i 8Φ16, što odgovara, respektivno, sledećim procentima armiranja betona: 0%, 0.3%, 0.55% i 1.1%. Odgovarajuće krive interakcije su prikazane na slici 5.

Može se, ponovo, uočiti širenje površi interakcije sa povećanjem procenta armiranja. Međutim, ono sada nije izotropno. Budući da je doprinos armature veći kod

2.4.9 Resistance to axial compression and biaxial bending

For composite column subjected to compression and biaxial bending, the resistance for each bending plane, separately, as shown in previous section, needs to be satisfied. Imperfections are considered only in the plane in which failure is expected to occur, which is in this case bending about z axis ($N_{Ed}e_0$ is greater for bending about z axis), so $M_{y,Ed}=127.8$ kNm (Figure 2.b).

Finally,

Since this condition is satisfied, and previously is shown that resistance to axial compression is also satisfied, we conclude: **resistance to axial compression and biaxial bending is satisfied.**

3 INTERACTION CURVES

As shown in the previous numerical example, construction of N-M interaction curves represents a large part of the calculation of composite columns subjected to combined axial compression and bending (uniaxial or biaxial). Therefore, in the paper, the influence of the percent of longitudinal reinforcement and concrete classes on interaction curves is studied on the cross section from the numerical example. Also, the continuous interaction curves for common dimensions of square cross sections with encased steel I section and for common concrete classes are constructed. These curves, with their referent values, simplify the design of composite columns and choice of dimensions of concrete section, steel section and concrete class.

3.1 Change of concrete class

The influence of change of concrete class on the composite cross section ultimate capacity is analysed firstly. This dependency is illustrated on interaction curves for the composite cross-section from the previous numerical example, for concrete classes: C20/25, C30/37, C40/50 and C50/60. For other concrete classes, interaction curves can be obtained by linear interpolation. These curves are given in Figure 4.

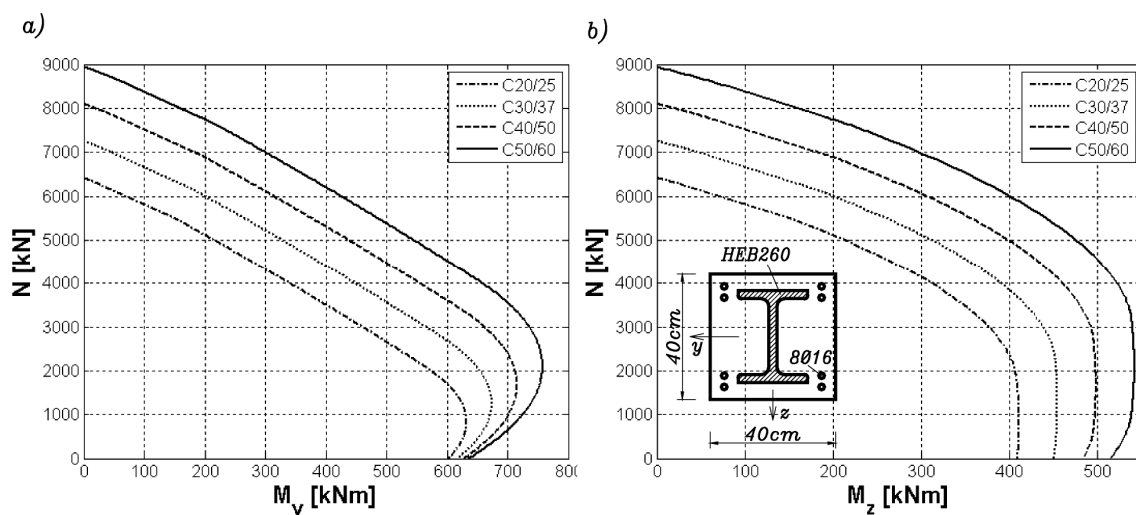
3.2 Change of longitudinal reinforcement

On the same composite cross section, with concrete class C40/50, the change of interaction curves with change in percent of reinforcement in the concrete is shown. The following reinforcement arrangements are studied: no reinforcement, 4Φ12, 4Φ16 and 8Φ16, which is equal to the following percent of reinforcement in the concrete, respectively: 0%, 0.3%, 0.55% and 1.1%. The corresponding interaction curves are given in Figure 5.

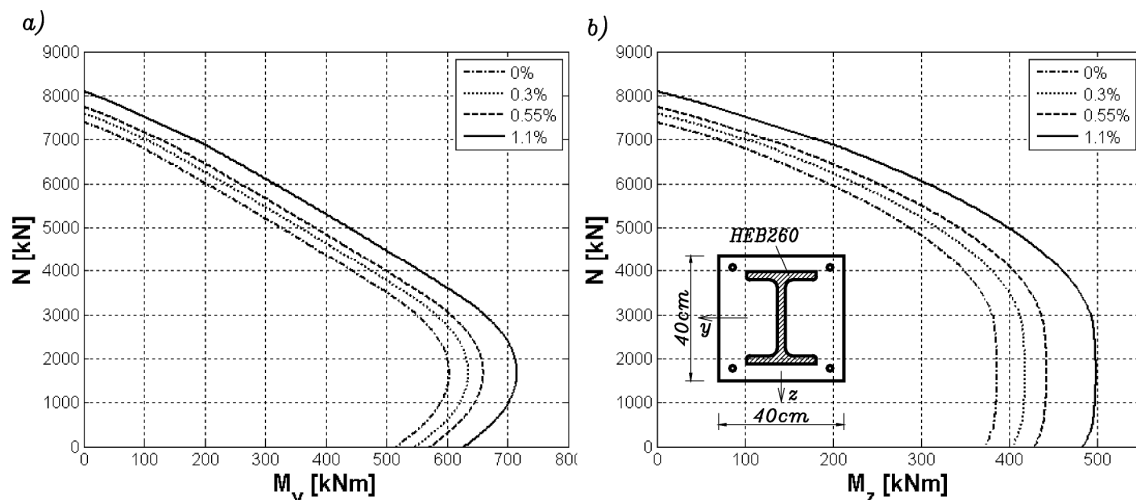
As before, the interaction curves expand as the percent of reinforcement in the concrete increases. However, this expansion is not uniform. The reinforcement increases moment capacity more than

momenata savijanja, procentualno veći prirast odgovara momentima savijanja (npr. $M_{\max,Rd}$) nego normalnoj sili (npr. $N_{pl,Rd}$).

axial compression capacity, so higher increment, in percent, corresponds to bending moments (i.e. $M_{\max,Rd}$) than to axial force (i.e. $N_{pl,Rd}$).



Slika¹ 4. Promena krive interakcije sa promenom klase betona
Figure 4. Interaction curves for different concrete classes



Slika 5. Promena krive interakcije sa promenom procenta armiranosti betona
Figure 5. Interaction curves for different percent of reinforcement in the concrete

3.3 Krive interakcije za neke čelične profile

Kako bi se olakšalo dimenzionisanje spregnutih stubova, konstruisane su kontinualne krive interakcije za spregnute stubove kvadratnog poprečnog preseka, sa ubetoniranim sledećim čeličnim profilima: HEA220, HEA240, HEA260, HEA280, HEA300 i HEA320 (slike 6-11). Posmatrani stubovi imaju stranice poprečnog preseka od 30 cm do 60 cm. Za svaki od pomenutih čeličnih profila, varirane su dimenzije betonskog stuba

3.3 Interaction curves for some steel sections

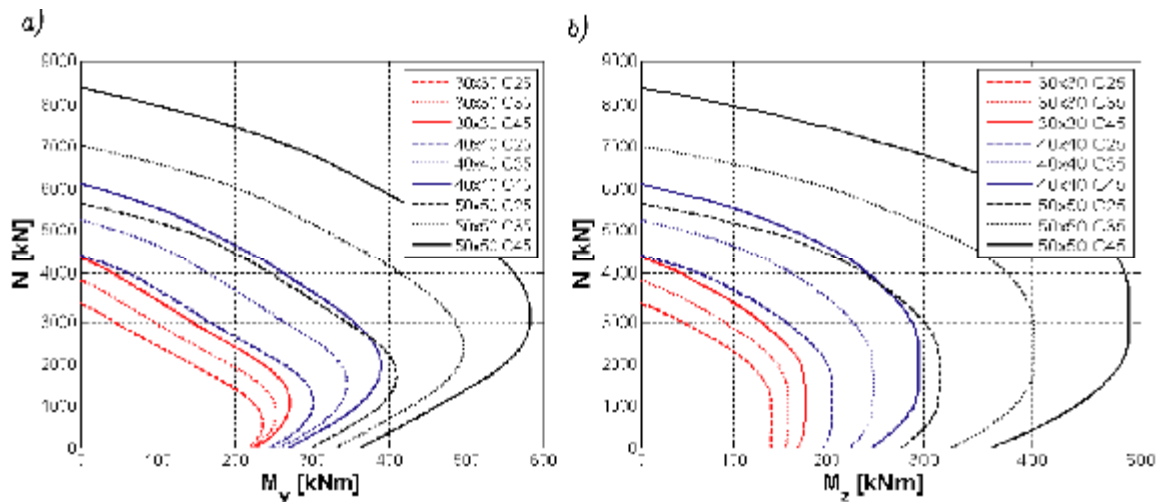
In order to simplify the design of composite columns, continuous interaction curves for composite square cross sections with encased steel sections HEA220, HEA240, HEA260, HEA280, HEA300 i HEA320 are constructed (Figures 6-11). Dimension of side of concrete column varies between 30 and 60cm. For each of studied steel sections, dimensions of concrete are determined to satisfy the Eurocode 4 requirements about

¹ Interakcione krive većeg formata se mogu preuzeti sa sledeće internet adrese
<http://www.grf.bg.ac.rs/~svetlanakostic/KriviInterakcije.pdf>

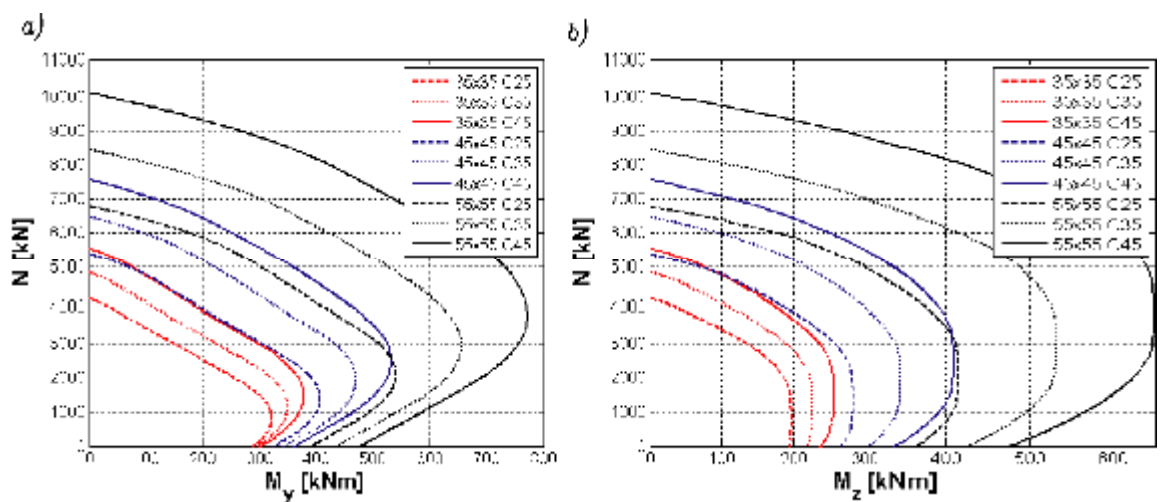
* Larger figures of interaction curves can be downloaded from the following web address
<http://www.grf.bg.ac.rs/~svetlanakostic/KriviInterakcije.pdf>

vodeći računa o ograničenjima koja postoje u Evrokodu 4 za veličinu zaštitnog sloja betona, kao i uslov da koeficijent doprinosa čelika δ mora biti u granicama od 0.2 do 0.9. Takođe, za svaki od preseka je varirana i klasa betona, pa su prikazane krive za klase C25/30, C35/45 i C45/55. Pri proračunu je zanemaren uticaj podužne armature stubova, pa su dobijene krive interakcije na strani sigurnosti. Međutim, na osnovu prethodno prikazane analize uticaja armature (deo 3.2), može se pretpostaviti položaj krive za armirani presek.

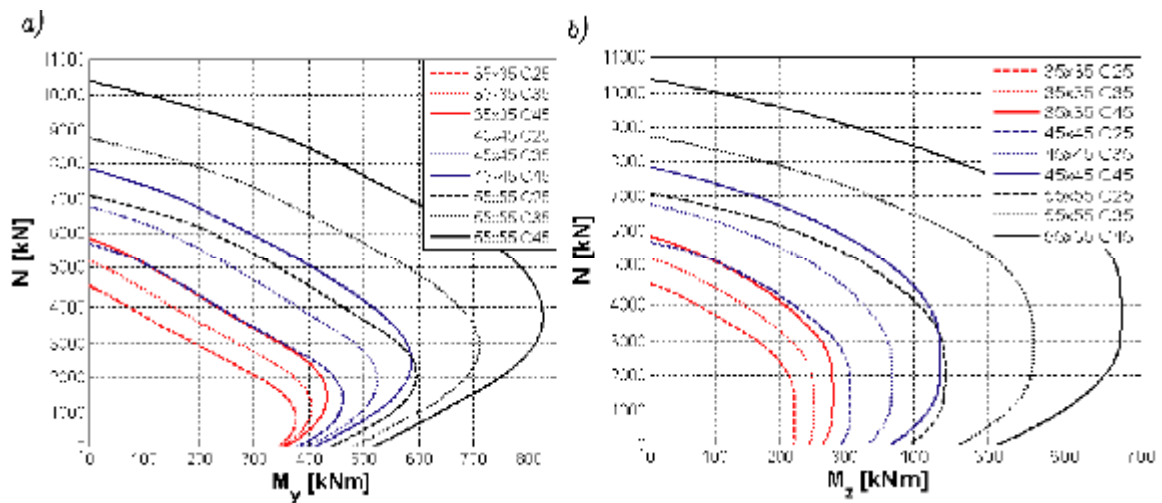
dimensions of concrete cover, and to have steel contribution ratio between 0.2 and 0.9. Also, for each studied composite cross section, the concrete class is varied and the curves are given for the following classes C25/30, C35/45 and C45/55. Reinforcement is neglected, and therefore, the shown interaction curves are on the safe side. However, based on the conclusions from the previous section 3.2, the position of the interaction curve for reinforced section can be predicted.



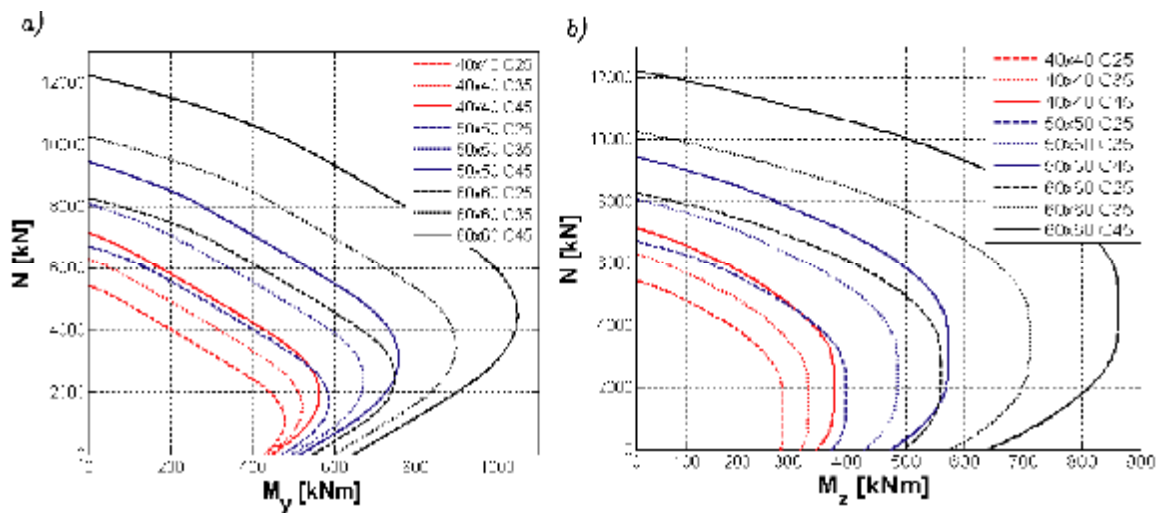
Slika 6. HEA 220 profil: krive interakcije za stubove dimenzija 30x30, 40x40 i 50x50cm
Figure 6. HEA 220 section: interaction curves for columns 30x30, 40x40 and 50x50cm



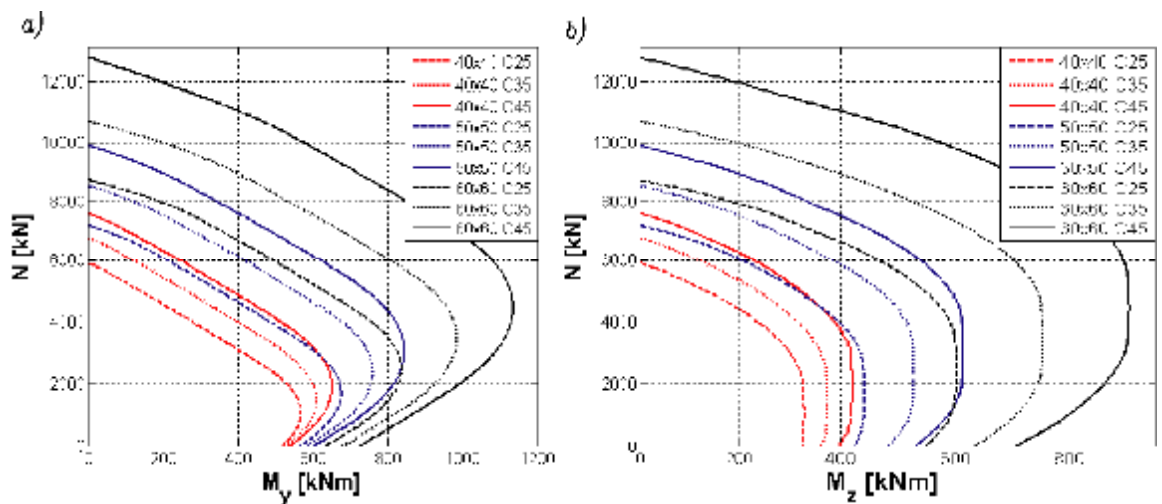
Slika 7. HEA 240 profil: krive interakcije za stubove dimenzija 35x35, 45x45 i 55x55cm
Figure 7. HEA 240 section: interaction curves for columns 35x35, 45x45 and 55x55cm



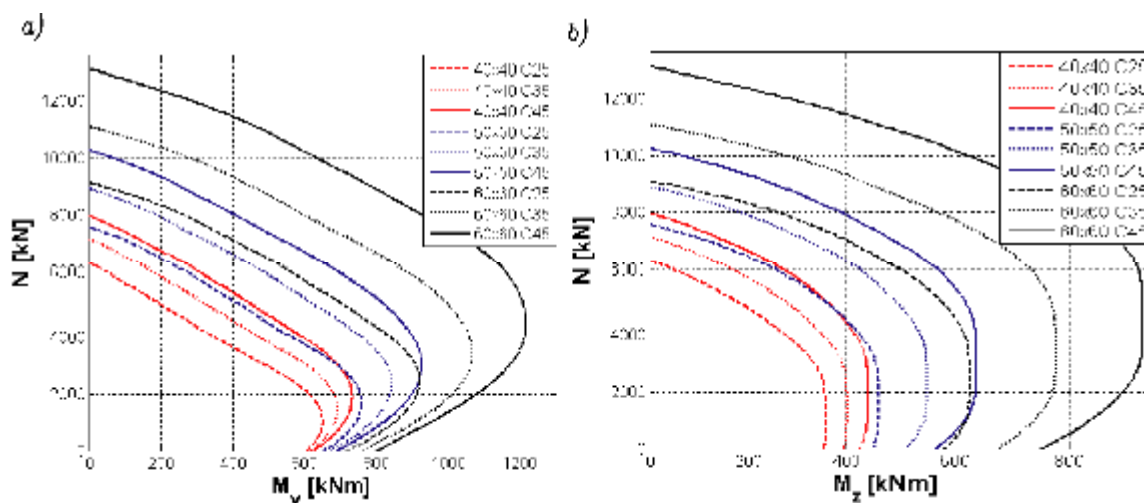
Slika 8. HEA 260 profil: krive interakcije za stubove dimenzija 35x35, 45x45 i 55x55cm
 Figure 8. HEA 260 section: interaction curves for columns 35x35, 45x45 and 55x55cm



Slika 9. HEA 280 profil: krive interakcije za stubove dimenzija 40x40, 50x50 i 60x60cm
 Figure 9. HEA 280 section: interaction curves for columns 40x40, 50x50 and 60x60cm



Slika 10. HEA 300 profil: krive interakcije za stubove dimenzija 40x40, 50x50 i 60x60cm
 Figure 10. HEA 300 section: interaction curves for columns 40x40, 50x50 and 60x60cm



Slika 11. HEA 320 profil: krive interakcije za stubove dimenzija 40x40, 50x50 i 60x60
 Figure 11. HEA 320 section: interaction curves for columns 40x40, 50x50 and 60x60cm

Dati dijagrami olakšavaju dimenzionisanje spregnutih stubova sa potpuno ubetoniranim čeličnim I profilima, budući da pojednostavljaju izbor dimenzija betonskog dela preseka, čeličnog profila i klase betona, a time i smanjuju broj iteracija pri dimenzionisanju.

4 ZAKLJUČAK

U radu je, na detaljnom brojnom primeru, ilustrovan proračun nosivosti spregnutih stubova prema važećem evropskom standardu za proračun spregnutih konstrukcija od čelika i betona - Evrokodu 4. Proračuna nosivosti je urađen za spregnuti stub kod koga je čelični I profil potpuno obložen betonom. Stub je izložen istovremenom uticaju aksijalnog pritiska i savijanja momentima. Uzeti su u obzir uticaji drugog reda kao i uticaji skupljanja i tečenja betona.

Za poprečni presek spregnutog stuba kod koga je čelični I profil potpuno obložen betonom, za proveru nosivosti pri kombinaciji aksijalnog pritiska i savijanja, analiziran je i uticaj promene marke betona i procenta armiranja na graničnu nosivost poprečnog preseka. Za posmatrani tip poprečnog preseka i grupu od 6 valjanih HEA čeličnih profila, konstruisane su kontinualne krive interakcije za uobičajene klase betona. Ove krive su vrlo pogodne za praktičnu primenu i olakšavaju dimenzionisanje ove grupe nosača, budući da pojednostavljaju izbor dimenzija betonskog dela preseka, čeličnog profila i klase betona i time smanjuju broj iteracija pri dimenzionisanju.

NAPOMENA:

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The given curves simplify design of composite columns with encased I steel section by simplifying choice of dimensions of concrete section, steel section and concrete class. Consequently, number of iterations during design reduces.

4 CONCLUSION

In the paper, on the detailed numerical example, analysis of composite columns according to Eurocode 4 is illustrated. The analysis is done for composite column with encased I steel section. The column is subjected to compression and bending. The second order effects and creep and shrinkage effects are taken into account.

For the composite column with encased I steel section under compression and bending, the influence of change of concrete class and reinforcement percent on the ultimate capacity is studied. For this type of sections and 6 HEA steel sections, the continuous interaction curves are constructed for common classes of concrete. These curves are very practical since simplify the design of composite columns and choice of dimensions of concrete section, steel section and concrete class.

NOTE:

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REZIME

PRILOG PRORAČUNU SPREGNUTIH STUBOVA OD ČELIKA I BETONA

Svetlana KOSTIĆ
Sasa STOŠIĆ
Biljana DERETIĆ-STOJANOVIĆ

U radu se, na detaljnom brojnom primeru spregnutog stuba kod koga je čelični I profil potpuno obložen betonom, ilustruje proračun nosivosti spregnutih stubova prema važećem evropskom standardu za proračun spregnutih konstrukcija od čelika i betona - Evrokodu 4 koji je detaljno objašnjen u prethodnom radu iste grupe autora.

Određena je nosivost spregnutog stuba izloženog samo aksijalnom pritisku i izloženog istovremenom uticaju aksijalnog pritiska i savijanja momentima. Analiza nosivosti, a time i stabilnosti pri aksijalnom pritisku izolovanog stuba bazira se na primeni Evropskih krivih izvijanja. Provera nosivosti pri kombinaciji aksijalnog pritiska i savijanja momentima zasniva se na interakcionoj krivoj koja se određuje pri analizi nosivosti poprečnog preseka posmatranog stuba. Uzeti su u obzir uticaji drugog reda kao i uticaji skupljanja i tečenja betona.

Za poprečni presek spregnutog stuba kod koga je čelični I profil potpuno obložen betonom, prikazan je uticaj marke betona i procenta armiranja na graničnu nosivost poprečnog preseka. Za dati tip spregnutog poprečnog preseka i nekoliko valjanih čeličnih profila, konstruisane su kontinualne krive interakcije za uobičajene klase betona. Ove krive su vrlo pogodne za praktičnu primenu i olakšavaju dimenzionisanje spregnutih stubova, budući da pojednostavljaju izbor dimenzija betonskog dela preseka, čeličnog profila i klase betona.

Ključne reči: spregnuti stubovi, uticaji drugog reda, klasa betona, krive interakcije

SUMMARY

CONTRIBUTION TO ANALYSIS OF COMPOSITE STEEL AND CONCRETE COLUMNS

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Sasa STOŠIĆ
Biljana DERETIĆ-STOJANOVIĆ

In the paper, design of composite steel-concrete column according to Eurocode 4 is illustrated on detailed numerical example. The theoretical foundations of the Eurocode 4 design procedure is explained before, in the paper of the same group of authors.

The resistance of a composite column subjected to axial compression and biaxial bending is determined. Capacity and stability analysis of the individual composite column under axial compression is based on the use of European buckling curves. The verification of column bearing capacity is based on use of interaction diagram determined from capacity analysis of composite cross-section. Second order effects and effects of creep and shrinkage of concrete are taken into account.

For the cross section of fully encased column with I steel section, the dependency of concrete class and reinforcement ratio on ultimate cross section capacity is studied. For this type of composite cross section and 6 different steel sections, the continuous interaction curves are constructed, for common concrete classes. These curves are very practical since simplify the design of composite columns and choice of dimensions of concrete section, steel section and concrete class.

Key words: composite columns, second-order effects, concrete class, interaction curves