METODE ANALIZE FLATERA U FREKVENTNOM I VREMENSKOM DOMENU

FREQUENCY- AND TIME-DOMAIN METHODS RELATED TO FLUTTER INSTABILITY PROBLEM

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1 UVOD

Kompleksno neustaljeno strujanje vetra oko tela obično je praćeno odvajanjem struje vetra i eventualnim ponovnim pripajanjem, što dovodi do fluktuirajućih površinskih pritisaka koji rezultuju dinamičkim silama vetra. Ova vrsta opterećenja naziva se aerodinamičko opterećenje.

U principu, modeli sila (modeli opterećenja) koriste se za opisivanje efekata opterećenja vetrom. Jedan jednostavan model opterećenja odnosi se na razmatranje nedeformabilne, fiksne konstrukcije i naziva se ustaljeni model opterećenja. Ukoliko se zanemare fluktuacije izazvane turbulencijom, nastali pritisci rezultuju osrednjenim silama: silom otpora duž pravca delovanja vetra *D*, silom uzgona upravnom na pravac vetra *L* i momentom *M*. Na osnovu ovih sila, ustaljeni koeficijenti (ili koeficijenti sile) za otpor C_D, uzgon C_L i momenat C_M dobijeni su kao:

1 INTRODUCTION

Complex unsteady wind flow around the body which is usually followed by flow separation and eventual reattachments gives rise to fluctuating surface pressures resulting in dynamic wind forces. This kind of load is referred as aerodynamic load.

In general, force models (load models) are used to describe the loading effects from the wind. One simple load model is related to the consideration of non-deformable, fixed structure and it is called the steady load model. If the fluctuations due to the turbulence are neglected, the created pressures result in mean forces such as: along-wind drag force *D*, an across-wind lift force *L* and a pitch moment *M*. Based on these forces steady coefficients (or force coefficients) for drag C_D, lift C_L and moment C_M are obtained as:

$$C_D = \frac{D}{\frac{1}{2}\rho U_\infty^2 B L_B} \quad C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 B L_B} \quad C_M = \frac{M}{\frac{1}{2}\rho U_\infty^2 B^2 L_B} \tag{1}$$

gde *ρ* predstavlja gustinu vazduha, *U* srednju brzinu vazduha, *B* i L_B su širina preseka mosta i dužina. Pošto ovi koeficijenti zavise od geometrijskog oblika preseka, najčešće se eksperimentalno određuju na osnovu standardnih testova u aerotunelu i tada su izraženi kao funkcija napadnog ugla *α* (slika 1). Ovi bezdimenzionalni

where *ρ* represents the air density, *U* the mean wind velocity, *B* and L_B are the bridge deck width and length. Since these coefficients depend on geometrical shape of the cross-section, they are usually obtained experimentally from standard wind tunnel tests as a function of angle of attack *α* (Figure 1). These non-dimensional

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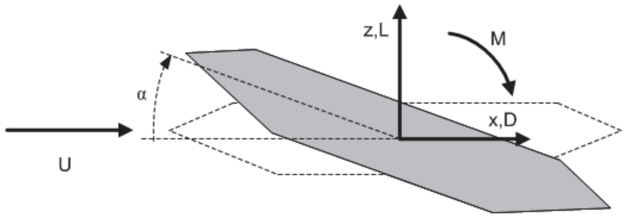
koeficijenti koriste se za prenos sila s modela u aerotunelu na model mosta s realnim dimenzijama koji se koristi pri projektovanju. Ovaj ustaljeni model opterećenja prikladan je za određivanje statičkih sila koje deluju na poprečni presek mosta i može se takođe nazivati i kvazistatički model opterećenja.

coefficients are used to transfer the forces from the wind tunnel model to design model of the bridge with real dimensions. This steady load model is appropriate for obtaining the static forces on the bridge deck, and it can be called also the quasi-static load model.



Slika 1. Usvojena konvencija za sile od vetra
Figure 1. Adopted convection for wind forces

Međutim, posmatranje potpuno nepokretne konstrukcije ne predstavlja ispravan pristup sagledavanju opterećenja od vetra. Naime, fleksibilnost mostova se ne može zanemariti, pošto stvara potencijal za generisanje složene interakcije između fleksibilne konstrukcije i vetra koji je opstrujava. Mehanizam interakcije se može opisati na sledeći način. Sile nastale usled opstrujavanja vetra izazivaju pomeranja i/ili deformacije konstrukcije. Ukoliko su ta pomeranja i deformacije dovoljno veliki, oni utiču na način opstrujanja vetra oko konstrukcije i samim tim izazivaju promenu samih sila. Ova interakcija između fluida i konstrukcije se naziva aeroelastičnost i može dovesti do različititih aeroelastičnih fenomena.

Kao sledeći korak, može se uzeti u obzir aproksimacija vezana za kvaziustaljeni model kao dodatak na ustaljeni pristup. Ovaj model tretira pomeranje poprečnog preseka mosta. Ali, u ovom slučaju, ustaljeni model opterećenja proširen je na dinamiku, te se u svakom trenutku dejstvo vetra može modelovati pomoću ustaljenih izraza (jednačine (1)) za trenutnu konfiguraciju poprečnog preseka. Na ovaj način se zanemaruje memorija fluida. Ipak, ovo snažno pojednostavljenje pod određenim uslovima može dovesti do dobre aproksimacije sila.

Međutim, nezgode kod visećih mostova koje su u prošlosti izazvane vetrom, kao što je katastrofa mosta Tacoma Narrows, pokazuju da su ove aproksimacije nedovoljne za opisivanje mehanizma interakcije i kao naredni korak je usledilo razvijanje neustaljenih modela sila.

2 FLATER

Flater predstavlja dinamičku nestabilnost gde energija uzeta iz strujanja vetra povećava energiju oscilovanja mosta. Može dovesti do snažnih oscilacija s povećanjem amplituda, a time i do kolapsa konstrukcije.

Klasičan flater je aeroelastični fenomen kod kog se dva dominantna stepena slobode konstrukcije, naime rotacija i vertikalna translacija, sprežu u nestabilnu oscilaciju na koju utiče opstrujavanje vetra. Tipični poprečni preseci koji su podložni ovakvoj nestabilnosti su aeroprofili i mostovi aerodinamičnog preseka. Kretanje je karakterisano silama vetra koje tokom jednog ciklusa oscilovanja dodaju energiju u sistem. Ova

However, this consideration of perfectly motionless structure does not present a correct consideration of the wind loads. Namely, the flexibility of the bridge decks cannot be neglected, since it creates a potential in generating a complex interaction between flexible structure and circumfluent wind. The interaction mechanism can be described as follows. Forces produced from the surrounding flow are inducing displacements and/or deformations of a structure. If these displacements and deformations are large enough, they influence the flow field around the structure and consequently the forces change. This fluid-structure interaction is regarded as aero elasticity and can lead to different aero elastic phenomena.

As a next step of approximation a quasi-steady load model can be taken into account as an extension of steady approach. This model considers the motion of bridge cross-section. But in this particular case, steady load model is extended to dynamics, by imagining that at each instant the wind action can be modelled by using the steady expressions (Eq.(1)) related to the current configuration of the cross-section at that instant. In this way the fluid-memory is neglected. Still, this strong simplification under certain conditions can result in a good approximation of forces.

However, wind-induced accidents concerning the suspension bridges in the past, such as the famous collapse of the Tacoma Narrows bridge, proved that these approximations are insufficient to describe interaction mechanism and as a next step, development of unsteady force models followed.

2 FLUTTER

Flutter is a dynamic instability where the energy drawn from the wind flow increases the energy of the bridge deck oscillations. It can lead to violent oscillations with increase of amplitudes and therefore to the collapse of the structure.

Classical flutter is an aero elastic phenomenon, in which the two dominant degrees of freedom of the structure, namely rotation and vertical translation, couple in a flow-driven unstable oscillation. Typical cross-sections which are prone to this instability are airfoils and streamlined bridge decks. The motion is razmena energije je vođena razlikom u fazi između vertikalnih i torzionih oscilacija ([1], [9]) i suprotstavlja se energiji koja se troši u prigušenju konstrukcije. Kritičan uslov ostvaruje se pri određenoj brzini vetra, koja se naziva kritična brzina vetra, i koja je vezana za izjednačavanje ukupnog prigušenja s nulom, odnosno konstrukcijskog i aerodinamičnog zajedno. Ovaj efekat je takođe povezan s promenom frekvencije oscilovanja. Naime, konstrukcija osciluje sa istom frekvencijom pri fleksionim i torzionim vibracijama – što se naziva kritičnom frekvencijom.

Odvajanje vrtloga nije neophodno za nastanak flatera, što uz činjenicu da se ovaj fenomen javlja pri brzini vetra koja je iznad kritične brzine vetra nastale usled odvajanja vrtloga, jasno izdvaja flater od rezonantnih problema ([3]). Na kritično stanje, nastalo usled flatera, može se uticati delovanjem na geometriju preseka, takođe na prigušenje i povećavanjem odnosa između svojstvenih frekvencija.

Mehanizam flatera proučavan je od strane [17] i [18], s ciljem određivanja svojstvenih oblika konstrukcije koji su odgovorni za flater, a nastalih usled modifikacije pomoću aeroelastičnih efekata. Ovi svojstveni oblici konstrukcije takođe se nazivaju i granama flatera.

Poprečni preseci koji nemaju aeroelastičan oblik podložni su jakom odvajanju struje vetra koja vodi ka nestabilnosti koja je izražena pomoću jednog torzionog stepena slobode i koja se naziva torzioni flater. Praktično u blizini kritičnog stanja, strujanje vetra predaje energiju uglavnom torzionom tonu.

U ovom radu akcenat je na klasičnom flateru i metodama za rešavanje flater-problema. Prikazani numerički primer takođe je vezan za tipičan poprečni presek mosta koji pripada grupi aerodinamičnih poprečnih preseka, gde je klasični flater značajan.

3 PRISTUP U FREKVENTNOM DOMENU

3.1 Model za flater kod mostova

S pretpostavkom da ravna ploča podleže malim harmonijskim oscilacijama pri vertikalnoj translaciji i rotaciji sa istom kružnom frekvencijom (kritičan uslov za nastanak flatera), neustaljene sile vetra su izvedene u zatvorenoj formi u frekventnom domenu - Theodorsen [35]. Nažalost, slične analitičke funkcije koje daju zatvoreno rešenje za neustaljene sile vetra, koje deluju na uobičajene poprečne preseke mostova, nije moguće odrediti. Razlog je u vezi sa opstrujavanjem vazduha oko oscilujućeg poprečnog preseka mosta, koje je znatno kompleksnije u poređenju sa opstrujavanjem oko ravne ploče, pre svega usled kompleksnih fizičkih fenomena kao što su masivno odvajanje strujanja, ponovno prijanjanje, odvajanje vrtloga i tako dalje. Ipak, analogna formulacija onoj koju je prezentovao Theodorsen, u smislu frekventno zavisnih parametara, zadržana je i u slučaju modela flatera kod mostova.

Scanlan [33] izveo je metod u kome su aerodinamički parametri - flater derivati primenjeni za definisanje neustaljenih sila vezanih za uobičajene mostove. Flater derivati identifikovani su putem eksperimenata i koriste se za procenu sila vetra nastalih usled kretanja konstrukcije (takođe se nazivaju i aeroelastične ili samopobuđujuće sile). S tim u vezi, aeroelastični uzgon i momenat po jedinici dužine mosta mogu se izraziti u proši-

characterized by the fluid forces feeding energy into the system during one cycle of its oscillation. This exchange of energy is driven by the phase shift between the vertical and torsional oscillations ([1], [9]) and it counteracts the energy absorption by structural damping. The critical condition is reached by the certain wind speed, called critical wind velocity, related to the total zero damping, i.e. structural and aerodynamic damping together. This effect is also coupled with a variation of a frequency of oscillation. Namely, the structure oscillates with the same frequency in bending and torsion – called critical frequency.

Flow separation is unnecessary for the occurrence of flutter and also the fact that this phenomenon occurs at flow velocity above the critical vortex shedding one, clearly distinguishes the flutter from resonance problem ([3]). The critical state, related to flutter, can be influenced upon by acting on the geometry of the section, also on the damping and by increasing the ratio between natural frequencies.

Mechanism of flutter has been studied in [17] and [18], with the purpose of understanding which structural modes are responsible for the instability, as being modified by aero elastic effects. These structural modes are also called flutter branches.

Relatively bluffer cross-sections undergoing strongly separated flow are prone to the single degree of freedom torsional instability, which is called the torsional flutter. Basically in the neighbourhood of the critical condition the flow tends to insert the energy mainly in a torsional mode.

In this paper, classical flutter and its solutions are of main concern. Presented numerical example is also related to the typical bridge cross-section which belongs to the group of streamlined cross-sections, where classical flutter is of a main concern.

3 FREQUENCY-DOMAIN APPROACH

3.1 Bridge flutter model

Assuming that the flat plate undergoes small harmonic oscillations in heave and pitch with the same circular frequency (critical condition for the onset of flutter), the unsteady wind forces given in the frequency domain are derived in a closed form by Theodorsen [35]. Unfortunately, similar analytical functions giving a closed form expressions for the unsteady wind forces acting on a common bridge decks are impossible to obtain. The reason is related to the air flow around an oscillating bridge deck which is much more complicated than around a simple flat plate, primarily due to the complex physical phenomena such as massive separations, reattachment, shedding of eddies, etc. Nevertheless, an analogous formulation to the one presented by Theodorsen, in terms of frequency-dependent parameters, is kept also in the case of the bridge flutter models.

Scanlan [33] derived a method in which aerodynamic parameters - flutter derivatives are applied to define an unsteady forces related to the common bridge deck. The flutter derivatives are identified by experiments and used to estimate the occurring motion-induced forces (also called aero elastic or self-excited forces). Thus, the aero elastic lift and moment forces per unit length of span, can be expressed in the extended force model from [34] renom modelu sila [34] pomoću diferencijalnih relacija:

by the differential relations:

$$L_{ae} = \frac{1}{2}\rho U^2 B\left[KH_1^*\frac{\dot{z}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{z}{B}\right] \quad (2)$$

$$M_{ae} = \frac{1}{2}\rho U^2 B^2\left[KA_1^*\frac{\dot{z}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{z}{B}\right] \quad (3)$$

U ovim relacijama, *K=Bω/U* je redukovana frekvencija, a H_i^*, A_i^*(*i=1..4*) jesu flater derivati, dok *ρ* predstavlja gustinu vazduha, *U* srednju brzinu vetra, *B* širinu poprečnog preseka mosta. Obično se za određeni poprečni presek mosta određuje set flater derivata i svaki derivat se predstavlja kao bezdimenzionalna funkcija redukovane frekvencije.

Aeroelastični model sila predstavljen jednačinama (2) i (3) baziran je na dvema pretpostavkama. Prva je da samopobuđujuća sila uzgona i moment mogu biti opisani kao linearna funkcija pomeranja konstrukcije i njene rotacije (z; α) i njihovih prvih i drugih izvoda ($\dot{z},\dot{\alpha},\ddot{z},\ddot{\alpha}$), kao što se često koristi i kako je predstavljeno i u radu [11], kao:

In these equations, *K=Bω/U* is the reduced frequency and H_i^*, A_i^*(*i=1..4*) are the flutter derivatives, while *ρ* represents the air density, *U* the mean wind velocity, *B* is the bridge deck width. Usually, a set of flutter derivatives is evaluated for a specific cross-sectional shape of a bridge deck and each derivative is a dimensionless function of the reduced frequency.

The aero elastic force model presented in Eq.(2) and Eq.(3) is based on two assumptions. The first assumption is that the self-excited lift force and moment can be described as a linear function of the structural displacements and rotation (z; α) and their first and second order derivatives ($\dot{z},\dot{\alpha},\ddot{z},\ddot{\alpha}$), as commonly used and presented in [11], as:

$$F = F(z,\alpha,\dot{z},\dot{\alpha},\ddot{z},\ddot{\alpha}) = P_z z + P_\alpha \alpha + P_{\dot{z}}\dot{z} + P_{\dot{\alpha}}\dot{\alpha} + P_{\ddot{z}}\ddot{z} + P_{\ddot{\alpha}}\ddot{\alpha} \quad (4)$$

gde *F* predstavlja ili aeroelastičnu silu uzgona *L* ili aeroelastični momenat *M*, a P_i (i = z; α) jesu aeroelastični parametri sile. Validnost ove pretpostavke vezana je za ograničene amplitude oscilacija pri nastanku flatera ([34]). Uvodeći drugu pretpostavku o postojanju harmonijskog kretanja s jedinstvenom frekvencijom pri nastanku flatera, pomeranje i njegov prvi i drugi izvod mogu se izraziti kao:

where *F* represents either the aero elastic lift force *L* or the aero elastic moment *M* and P_i (i = z; α) are aero elastic force parameters. The validity of this assumption is related to limited amplitudes of oscillations at the onset of flutter ([34]). Introducing a second assumption of the existence of harmonic motions with a single frequency at the onset of flutter, the displacement and its first- and second-order derivatives can be expressed as:

$$x = \hat{x}e^{i\omega t}, \dot{x} = \hat{x}i\omega e^{i\omega t}, \ddot{x} = -\hat{x}\omega^2 e^{i\omega t} \quad (5)$$

gde je $\hat{x}$ amplituda pomeranja (x = z; α) i ω je kružna frekvencija kretanja. Iz jednačina (4) i (5) može se uočiti da se članovi koji se odnose na pomeranja i ubrzanja mogu kombinovati, što je u skladu s prikazom datim u jednačinama (2) i (3). Ovo omogućava interpretaciju flater derivata kao delova samopobuđujućih sila, koji se u dinamici konstrukcija vide kao aeroelastično prigušenje, pomoću derivata ($H_1^*, H_2^*, A_1^*, A_2^*$) i spregnute aeroelastične krutosti i mase, pomoću derivata ($H_3^*, H_4^*, A_3^*, A_4^*$).

Validnost ovog linearnog modela samopobuđujućih sila vezanih za poprečni presek mosta predstavlja važnu temu. Jedan od važnih efekata jeste zavisnost flater derivata od amplitude kretanja ([23]).

Pored prikazane konvencije za flater derivate postoje takođe i druge. Primeri se mogu pronaći kod [15] i [38].

3.2 Identifikacija flater derivata

Flater derivati se obično određuju eksperimentalno u aerotunelu za pojedinačne geometrije poprečnog preseka mosta. Za tu svrhu postoje dve glavne eksperimentalne strategije: metod slobodnih vibracija i metod prinudnih vibracija. Kod eksperimenata sa slobodnim vibracijama poprečni presek je elastično oslonjen pomoću opruga i eventualno prigušivača i

where $\hat{x}$ is the amplitude of the displacement (x = z; α) and ω is the circular frequency of motion. From Eq.(4) and Eq.(5) it can be observed that terms related to the displacements and accelerations can be combined, which is consistent with the representation in Eq.(2) and Eq.(3). This allows the interpretation of flutter derivatives as parts of self-excited forces, which feed back into the structural dynamics as aero elastic damping, through derivatives ($H_1^*, H_2^*, A_1^*, A_2^*$) and coupled aero elastic stiffness and masses, through derivatives ($H_3^*, H_4^*, A_3^*, A_4^*$).

The validity of this linear model for bridge deck self-exited forces is an important issue. One of the important effects is the dependence of flutter derivatives on the amplitude of motion ([23]).

Besides presented convention, there also exist other conventions for flutter derivatives. Examples could be found in [15] and [38].

3.2 Identification of flutter derivatives

Flutter derivatives are usually determined experimentally in wind tunnel tests for individual bridge deck geometries. For this purpose, two major experimental strategies exist: the free vibration method and forced vibration method. In the free vibration postavljen je u aerotunel. Identifikacione tehnike za izdvajanje flater derivata mogu se naći u [7], [2], [29]. U slučaju testova prinudnih vibracija, potrebni su motor i kinematički mehanizam da pokreću model harmonijski u svojim stepenima slobode. Samopobuđujuće sile mogu se dobiti direktno iz merenja sila ili iz pritisaka. Ovakvi primeri identifikacije flater derivata vezanih za pravougaone prizme mogu se naći u [17], [19] i [12]. Poređenje ove dve eksperimentalne tehnike – slobodnih i prinudnih vibracija – na primeru pravougaonog poprečnog preseka može se naći u [36] i [37]. Sveobuhvatnije poređenje metoda vezano za poprečne preseke koji se kreću od pravougaonih prizmi do aerodinamičnih preseka prikazano je u [28]. Izvori odstupanja eksperimentalnih rezultata i nepouzdanosti vezane za eksperimentalne metode istaknute su i analizirane. Implikacije uočenih razlika na nastanak flater nestabilnosti analizirane su u [5].

Za potrebe ove studije, primenjen je metod prinudnih vibracija s predviđenim harmonijskim kretanjima i sile su direktno merene. Za takav identifikacioni metod je ključno odvajanje slabih signala vezanih za aeroelastične sile koje deluju na poprečnom preseku mosta od jakih signala vezanih za inercijalne sile samog modela. Rešenje je da se izvedu dva seta merenja. Referentno merenje s prinudnim vibracijama bez strujanja vazduha neophodno je za identifikaciju mehaničkog sistema modela. Nakon toga, merenja se ponavljaju sa identičnom frekvencijom oscilovanja i amplitudom pod dejstvom opterećenja vetra u aerotunelu. Budući da je primenjeno prinudno harmonijsko kretanje, ove merene sile takođe se pretpostavljaju kao harmonijske. Na ovaj način, merene sile bez strujanja vazduha F_0 i usled strujanja vetra F_w mogu se izraziti kao:

experiments a section model is elastically supported by springs and eventually a damper and mounted in a wind tunnel. Identification techniques for extracting the flutter derivatives can be found in [7], [2], [29]. In the case of forced vibration tests, a motor and a kinematic mechanism are necessary to drive the model harmonically in its degrees of freedom. Self-excited forces can be obtained directly through either force or pressure measurements. Such examples of identifying flutter derivatives related to the rectangular prisms using the pressure measurements can be found in [17], [19] and [12]. A comparison of both experimental techniques – free and forced vibration - on the rectangular cross-section can be found in [36] and [37]. More comprehensive comparisons of methods related to cross-sections ranging from rectangular prisms to streamlined sections are given in [28]. Sources of discrepancies of experimental results and uncertainties related to the experimental methods are pointed out and analyzed. Implications of these discrepancies to the onset of flutter instability have been analyzed in [5].

For the purpose of this study the forced vibration method with prescribed harmonic motions is applied and the forces are directly measured. For such an identification method the separation of the small signals of the aero elastic forces acting on the bridge deck model from the larger signals due to inertial forces of the model itself is crucial. A solution strategy is to perform two sets of measurements. A reference measurement with forced vibrations in still air is required in order to identify the mechanical system of the model. Then, the measurement is repeated with an identical oscillation frequency and amplitude under the action of the wind tunnel flow. Considering the applied forced harmonic motion these measured forces are also assumed harmonic. In this way, measured forces in still air F_0 and under the action of the wind F_w can be expressed as:

$$F_0 = \hat{F}_0 e^{i(\omega t+\varphi_0)}, F_w = \hat{F}_w e^{i(\omega t+\varphi_w)} \tag{6}$$

gde su $\hat{F}_0$ i $\hat{F}_w$ amplitude sila, a φ_0 i φ_w fazna pomeranja ostvarena u odnosu na primenjeno kretanje dato jednačinom (5), vezano za merenja bez strujanja vazduha i usled opterećenja vetrom (slika 2), respektivno. Samopobuđujuće sile se dobijaju izračunavanjem razlike između ova dva seta merenja prema [14], sa slike 2:

where $\hat{F}_0$ and $\hat{F}_w$ are the force amplitudes and φ_0 and φ_w are the phase shifts related to the applied motion given in Eq.(5), regarding the measurements in still air and under the action of the wind (refer to Figure 2), respectively. The self-excited forces are obtained by calculating the difference between these two sets of measurements by [14], in Figure 2:

$$\Delta F = F_w - F_0 \tag{7}$$

Može se pokazati da se flater derivati vezani za torziona kretanja dobijaju iz:

It can be shown that flutter derivatives related to the torsional motion can be obtained from:

$$\frac{1}{2}\rho K^2 U^2 B \hat{\alpha}\left[H_3^*(K) + iH_2^*(K)\right] = \Delta L^{\alpha}(K) \tag{8}$$

$$\frac{1}{2}\rho K^2 U^2 B^2 \hat{\alpha}\left[A_3^*(K) + iA_2^*(K)\right] = \Delta M^{\alpha}(K) \tag{9}$$

a vezano za vertikalna kretanja iz:

and related to the vertical motion from:

$$\frac{1}{2}\rho K^2 U^2 \hat{z}\left[H_4^*(K) + iH_1^*(K)\right] = \Delta L^{z}(K) \tag{10}$$

$$\frac{1}{2}\rho K^2 U^2 B\hat{z}\left[A_4^*(K) + iA_1^*(K)\right] = \Delta M^{z}(K) \tag{11}$$

ΔL^x i ΔM^x (x=z,α) jesu pomenute razlike vezane za aeroelastični uzgon i aeroelastični momenat, respektivno, koje treba da se dobiju iz eksperimenata bez strujanja vazduha i pod dejstvom vetra. U ovom radu, flater derivati su definisani prateći konvenciju po kojoj su sila uzgona i verikalno pomeranje definisani pozitivno na gore, dok su aeroelastični momenat kao i torzione deformacije pozitivne sa smerom kad je prednji kraj poprečnog preseka orijentisan na gore, kao što je prikazano na slici 1. Prema jednačini (6), samopobuđujuće sile su takođe harmonijske u vremenu, ali s faznom razlikom u poređenju sa zadatim kretanjem preseka. Ovo svojstvo dozvoljava određivanje karakteristika sile kao što su amplituda i fazna razlika merenih signala. Mehaničke nepravilnosti u kinematičkom mehanizmu kao i odvajanje vrtloga od preseka mogu da poremete signal, zbog čega su naročito neophodni posebno stabilni algoritmi identifikacije - videti [22].

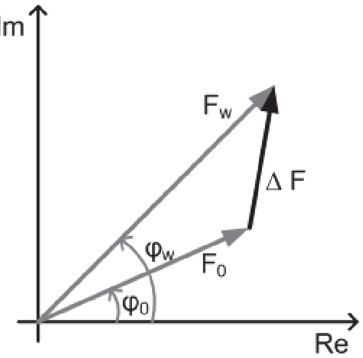
Prema tome, postupak se može sažeti kao:

- izvršiti testove s prinudnim oscilacijama bez strujanja vazduha i usled dejstva vetra pri vertikalnom kretanju ili torzionom;
- izračunati najbolje uklopljen harmonik iste prinudne frekvencije da bi se dobili koeficijent amplitude sile i fazna razlika vezana za primenjeno kretanje, jednačina (6);
- izračunati razliku između dva merenja, jednačina (7);
- izračunati derivate na osnovu jednačina (8)-(11).

ΔL^x and ΔM^x (x=z,α) are the mentioned differences of the aero elastic lift and the aero elastic moment, respectively, which should be obtained from the experiments in still air and under the action of wind. In this paper, flutter derivatives are defined following a convention after which the lift force and the heaving displacement are positive upwards, while the aerodynamic moment and the pitching rotation are positive nose-up, as it is shown in Figure 1. According to Equation (6) the self-excited forces are also harmonious in time, but with a phase shift compared to the prescribed motion of the deck. This characteristic allows determining force properties such as amplitude and phasing from the measured signals. As mechanical imperfections in the kinematic mechanism or vortex shedding from the section can disturb the signal, specifically stable identification algorithm are needed, see [22].

Thus, the procedure can be summarized as:

- perform forced oscillation tests in still air and under the action of the wind in either vertical (heave) or torsional (pitch) motion,
- calculate a best-fit harmonic of the same forcing frequency to obtain the force amplitude coefficients and phase shifts related to the applied motion, Eqs.(6)
- calculate the differences between two measurements, from Eq.(7)
- calculate the derivatives from Eqs.(8)-(11)



Slika 2. Identifikacija aeroelastičnih sila u kompleksnoj ravni ([20])
Figure 2. Identification of the aero elastic forces in the complex plane ([20])

3.3 Rešenje jednačina flatera

Kada su aeroelastične sile utvrđene (jednačine (2) i (3)), može se dobiti kritični uslov, odnosno kritična brzina vetra za nastanak flatera. Najjednostavniji način za određivanje kritične brzine vetra jeste da se posmatra krut model poprečnog preseka mosta sa dva stepena slobode (2DOF model); naime, posmatraju se vertikalno i torziono kretanje; 2DOF jednačine kretanja po jedinici dužine mogu se napisati kao:

3.3 Solution of flutter equations

Once the aero elastic forces are established (Eq.(2) and Eq.(3)), the critical condition, i.e. critical wind velocity for the onset of flutter, can be calculated. The simplest way to establish critical wind velocity is to consider a rigid section model of the bridge deck with two degrees of freedom (2DOF model), namely vertical *z* (heave) and torsional *α* (pitch) motion are considered. The 2DOF equation of motion per unit span length can be written as follows:

$$m\ddot{z} + c_z \dot{z} + k_z z = L_{ae} \qquad I\ddot{\alpha} + c_\alpha \alpha + k_\alpha \alpha = M_{ae} \tag{12}$$

gde su L_{ae} i M_{ae} samopobuđujuće sile predstavljene u jednačinama (2) i (3), *m* je masa, a *I* maseni moment inercije po jedinici dužine i k_z i k_α su krutosti, a c_z i c_α su

where L_{ae} and M_{ae} are self-excited forces presented by Eq.(2) and Eq.(3), *m* is mass and *I* mass moment of inertia per unit length and k_z and k_α are stiffnesses and koeficijenti prigušenja za respektivne stepene slobode.

Za slučaj kritičnog uslova kod flatera, vertikalno i torziono pomeranje može se posmatrati kao harmonijsko kretanje sa istom kružnom frekvencijom:

c_z and c_α damping coefficients, for respective degrees of freedom.

For the case of flutter critical condition, heave and pitch can be considered as harmonic motion with the same circular frequency:

$$z(t) = \bar{z}e^{i\omega t}, \alpha(t) = \bar{\alpha}e^{i\omega t} \tag{13}$$

Posle zamene jednačina (2), (3) i (13) u jednačinu (12), formulisan je problem svojstvenih vrednosti stabilnosti kretanja, s frekvencijom flatera i kritičnom brzinom vetra kao nepoznatim:

After the substitution of Eqs.(2), (3) and (13) into Eqs.(12) eigenvalue problem of stability of motion is formulated with flutter frequency and the critical wind speed as unknowns:

$$\left[2\gamma_m\left(-1+\frac{1}{X^2}+i2\varsigma_z\frac{1}{X}\right)-\left(H_4^*+iH_1^*\right)\right]\frac{\bar{z}}{B}-\left(H_3^*+iH_2^*\right)\bar{\alpha}=0 \tag{14}$$

$$-\left(A_4^*+iA_1^*\right)\frac{\bar{z}}{B}+\left[2\gamma_I\left(-1+\frac{2\gamma_\omega^2}{X^2}+i2\varsigma_z\frac{\gamma_\omega}{X}\right)-\left(A_3^*+iA_2^*\right)\right]\bar{\alpha}=0 \tag{15}$$

gde su $\gamma_m = m/(\rho B^2)$ i $\gamma_I = I/(\rho B^4)$ bezdimenzionalne vrednosti mase i momenta inercije mase, respektivno, $\gamma_\omega = \omega_\alpha / \omega_z$ jeste odnos svojstvenih frekvencija, dok je $X = \omega/\omega_z$ nepoznata spregnuta frekvencija, normalizovana u odnosu na svojstvenu frekvenciju vertikalnog oscilovanja.

Kako jednačine (14) i (15) predstavljaju homogen linearan sistem jednačina po $\bar{z}$ i $\bar{\alpha}$, za dobijanje netrivijalnih rešenja determinanta mora da bude jednaka nuli. Kako jednačine (14) i (15) predstavljaju sistem jednačina s kompleksnim brojevima, oba dela determinante, i realan i imaginaran, moraju nestati, što dovodi do novog sistema jednačina:

where $\gamma_m = m/(\rho B^2)$ and $\gamma_I = I/(\rho B^4)$ are the nondimensional values of the mass and mass moment of inertia, respectively, $\gamma_\omega = \omega_\alpha / \omega_z$ is the frequency ratio of natural frequencies, while $X = \omega/\omega_z$ is the unknown coupling frequency, nondimensionalized regarding the heaving natural frequency.

Since Eq.(14) and Eq.(15) represent linear homogeneous system of equations of $\bar{z}$ and $\bar{\alpha}$, to obtain nontrivial solutions, the determinant must be equal to zero. Since Eq.(14) and Eq.(15) presents system of complex number equations, both, real and imaginary part of the determinant must vanish, leading to the new system of equations:

$$R_4X^4 + R_3X^3 + R_2X^2 + R_1X + R_0 = 0 \tag{16}$$

$$I_3X^3 + I_2X^2 + I_1X + I_0 = 0 \tag{17}$$

gde je:

where:

$$\begin{aligned}
R_4 &= 1+\frac{1}{2\gamma_m}H_4^*+\frac{1}{2\gamma_I}A_3^*+\frac{1}{4\gamma_m\gamma_I}\left(H_4^*A_3^*-H_1^*A_2^*-H_3^*A_4^*+H_2^*A_1^*\right)\\
R_3 &= \varsigma_\alpha\frac{\gamma_\omega}{\gamma_m}H_1^*+\varsigma_z\frac{1}{\gamma_I}A_2^*\\
R_2 &= -1-\gamma_\omega^2-4\varsigma_z\varsigma_\alpha\gamma_\omega-\frac{\gamma_\omega^2}{2\gamma_m}H_4^*-\frac{1}{2\gamma_I}A_3^*\\
R_1 &= 0\\
R_0 &= \gamma_\omega^2\\
I_3 &= \frac{1}{2\gamma_m}H_1^*+\frac{1}{2\gamma_I}A_2^*+\frac{1}{4\gamma_m\gamma_I}\left(H_4^*A_2^*-H_1^*A_3^*-H_3^*A_1^*+H_2^*A_4^*\right)\\
I_2 &= -2\varsigma_z-2\varsigma_\alpha\gamma_\omega-\varsigma_\alpha\frac{\gamma_\omega}{\gamma_m}H_4^*-\varsigma_z\frac{1}{\gamma_I}A_3^*\\
I_1 &= -\frac{\gamma_\omega^2}{2\gamma_m}H_1^*-\frac{1}{2\gamma_I}A_2^*\\
I_0 &= 2\varsigma_z\gamma_\omega^2+2\varsigma_\alpha\gamma_\omega
\end{aligned} \tag{18}$$

Na ovaj način je dobijen sistem jednačina s dve nepoznate. Nepoznate su *X*, koja sadrži spregnutu (flater) frekvenciju i kritična redukovana brzina U_{cr}, od

In this way a system of equations with two unknowns is obtained. The unknowns are *X*, containing the coupled (flutter) frequency, and the critical reduced velocity U_{cr}, koje zavise flater derivati. Rešenje se dobija grafičkim prikazom realnih rešenja *X* vezanih za obe jednačine u odnosu na U_{red}. Presek ovih krivih vodi ka rešenju flatera.

Generalno gledano, uočeno je da više stepeni tonova kod trodimenzionalnih konstrukcija može biti uključeno u flater nestabilnost. U tom slučaju, jednostavan 2DOF model nije dovoljan. Proračun se može obaviti na dva načina: primenjujući aeroelastične sile direktno na trodimenzionalni model mosta pomoću metode konačnih elemenata, što se naziva direktan metod, ili posmatrajući odgovor konstrukcije, uzimajući u obzir samo odgovarajući broj svojstvenih tonova, što se naziva multimodalni metod. Diskusija u vezi s multimodalnim metodom prikazana je u [31]. Neki primeri implementacije sa odgovarajućim aplikacijama mogu se naći u [8] i [25]. U radu [10] direktan metod za flater analizu predstavljen je i upoređen s multimodalnim metodom. S jedne strane, direktan metod obezbeđuje učešće svih svojstvenih tonova, ali s druge strane, zahteva veću računarsku snagu.

4 PRISTUP U VREMENSKOM DOMENU

Flater derivati nisu pogodni za proračune u vremenskom domenu, jer su izraženi kao funkcija frekvencije. Kao pandan flater derivatima u vremenskom domenu mogu se izvesti funkcije koje nisu analitičke. Ovakve funkcije opisuju vremenski razvoj sila usled naglog infinitenzimalnog pomeranja konstrukcije i ove funkcije nazivaju se indicijalne funkcije. Prvi relevantni rad u kom je spomenuta mogućnost primene indicijalnih funkcija zabeležen je u [32].

Kako bi se definisale samopobuđujuće sile, istorija kretanja se posmatra kao niz ovih infinitenzimalnih inkremenata. Pod pretpostavkom linearnosti opterećenja, samopobuđujuće sile u vremenskom domenu (pandani jednačinama (2) i (3)) mogu se izraziti pomoću konvolucije ovih indicijalnih funkcija:

that flutter derivatives depend on. The solution is obtained by plotting the real *X* solutions of both equations against U_{red}. The intersection of these curves leads to the flutter solution.

Generally speaking, it is observed that more modes of the three-dimensional structures can be involved in flutter instability. In this case, simple 2DOF model is insufficient. The calculation can be done in two ways: to apply aero elastic forces directly to the three-dimensional finite element model of the bridge, which is called a direct method, or to consider the structural response taking into account an adequate number of natural modes, called multimode method. The multimode method was discussed in [31]. Some examples of implementation with related applications can be found in [8] and [25]. In [10] a direct method for flutter analysis is presented and compared to the multimode method. Thus, direct method provides participation of all natural modes, but accordingly, it demands higher computational power as well.

4 TIME-DOMAIN APPROACH

The flutter derivatives are inadequately suited for the time domain calculations, due to being expressed as a function of frequency. As a counterpart to flutter derivatives in time domain, specific non-analytical functions can be derived. Such functions describe the time development of the forces due to the sudden infinitesimal structural motions and these functions are called the indicial functions. The first relevant work mentioning the possibility of using indicial functions is noted in [32].

In order to define the self-excited forces, the history of motion can be seen as a series of these infinitesimal step-wise increments. Under the assumption of linearity of load, the self-excited forces in time domain (counterparts of Eq. (2) and Eq.(3)) may be expressed as convolutions of these indicial functions:

$$L_{ae}(s) = qBC_L' \left[\Phi_{L\alpha}(0)\alpha(s) + \Phi_{Lz}(0)\dot{z}(s) + \int_0^s \dot{\Phi}_{L\alpha}(s-\tau)\alpha(\tau)d\tau + \int_0^s \dot{\Phi}_{Lz}(s-\tau)\dot{z}(\tau)d\tau \right] \tag{19}$$

$$M_{ae}(s) = qB^2C_M' \left[\Phi_{M\alpha}(0)\alpha(s) + \Phi_{Mz}(0)\dot{z}(s) + \int_0^s \dot{\Phi}_{M\alpha}(s-\tau)\alpha(\tau)d\tau + \int_0^s \dot{\Phi}_{Mz}(s-\tau)\dot{z}(\tau)d\tau \right] \tag{20}$$

Aeroelastične sile u vremenskom domenu (jednačine (19) i (20)) izražene su u funkciji bezdimenzionalnog vremena *s=2Ut/B*. Kao što se može uočiti, četiri indicijalne funkcije – Φ_{il} koriste se za opisivanje aeroelastičnih sila, gde indeks *i* identifikuje komponentu opterećenja *L* za uzgon ili *M* za aeroelastični momenat, a indeks *l* komponentu pomeranja koja se menja sukcesivno u koracima, *z* ili *α*.

Ove funkcije se obično određuju na osnovu odgovarajućih (merenih) flater derivata ([4], [26]). To se radi tako što se uzima tipična aproksimacija, predstavljajući indicijalne funkcije u vidu sume *m* eksponencijalnih grupa (filtera):

Aero elastic forces in the time domain (Eq.(19) and Eq.(20)) are expressed as a function of a dimensionless time *s=2Ut/B*. As it may be observed, four indicial functions - Φ_{il} are used to describe the aero elastic forces, where subscript *i* indentifies the load component *L* for lift and *M* for aerodynamic moment and subscript *l* the motion component that experiences the step change *z* or *α*.

Usual practice to determine these functions is from the corresponding (measured) flutter derivatives ([4], [26]). This is done by taking the typical approximation, by representing the indicial function as a sum of *m* exponential groups (filters):

$$\Phi_{il}(s) = 1 - \sum_{k=1}^{m} a_{ilk} \exp(-b_{ilk}s) \tag{21}$$

S ciljem uspostavljanja veza između indicijalnih funkcija i flater derivata, zadaje se harmonijsko kretanje u prethodno spomenutim konvolucionim integralima (jednačine (19) i (20)). Na ovaj način, aeroelastično opterećenje, dato konvolucionim integralima, izraženo je u frekventnom domenu, i u ovoj formi se može uporediti sa opterećenjem baziranim na flater derivatima (jednačine (2) i (3)), što dovodi do ovih relacija:

In order to establish the relationships between indicial functions and flutter derivatives, harmonic motions are imposed into the previously mentioned convolution integrals (Eq.(19) and Eq.(20)). In this way the aero elastic load given by convolution integrals is expressed in the frequency domain, and in this form it can be compared to the load based on flutter derivatives (Eq.(2) and Eq.(3)), providing these relationships:

$$\begin{aligned}
&\frac{2\pi}{U_{red}}H_1^* = -\left(\frac{dC_L}{d\alpha}+C_D\right)\left[1-\pi^2\sum_k a_{Lzk}\frac{1}{U_{red}^{*2}b_{Lzk}^2+\pi^2}\right] && \frac{2\pi}{U_{red}}A_1^* = -\frac{dC_M}{d\alpha}\left[1-\pi^2\sum_k a_{Mzk}\frac{1}{U_{red}^{*2}b_{Mzk}^2+\pi^2}\right]\\
&\frac{4\pi}{U_{red}^3}H_2^* = -\frac{dC_L}{d\alpha}\left[\sum_k a_{L\alpha k}\frac{b_{L\alpha k}}{U_{red}^{*2}b_{L\alpha k}^2+\pi^2}\right] && \frac{4\pi}{U_{red}^3}A_2^* = \frac{dC_M}{d\alpha}\left[\sum_k -a_{M\alpha k}\frac{b_{M\alpha k}}{U_{red}^{*2}b_{M\alpha k}^2+\pi^2}\right]\\
&\frac{4\pi^2}{U_{red}^2}H_3^* = -\frac{dC_L}{d\alpha}\left[1-\pi^2\sum_k a_{L\alpha k}\frac{1}{U_{red}^{*2}b_{L\alpha k}^2+\pi^2}\right] && \frac{4\pi^2}{U_{red}^2}A_3^* = \frac{dC_M}{d\alpha}\left[1-\pi^2\sum_k a_{M\alpha k}\frac{1}{U_{red}^{*2}b_{M\alpha k}^2+\pi^2}\right]\\
&\frac{2}{U_{red}^2}H_4^* = \left(\frac{dC_L}{d\alpha}+C_D\right)\left[\sum_k a_{Lzk}\frac{b_{Lzk}}{U_{red}^{*2}b_{Lzk}^2+\pi^2}\right] && \frac{2}{U_{red}^2}A_4^* = -\frac{dC_M}{d\alpha}\left[\sum_k a_{Mzk}\frac{b_{Mzk}}{U_{red}^{*2}b_{Mzk}^2+\pi^2}\right]
\end{aligned} \tag{22}$$

S obzirom na prirodu ovih relacija, indicijalne funkcije (s bezdimenzionalnim koeficijentima a_{ilk} i b_{ilk} kao nepoznatim) mogu se identifikovati uz pomoć nelinearne optimizacije najmanjih kvadrata. Detalji u vezi s metodom koja je praćena u ovom radu, opisani su u [27].

Važno je napomenuti da je direktna eksperimentalna identifikacija indicijalnih funkcija takođe teorijski moguća i jedan primer je zabeležen u [5].

Due to the nature of these relationships, the indicial functions (with non-dimensional coefficients a_{ilk} and b_{ilk} as unknowns) can be then identified by means of nonlinear least-square optimisation. Further details of the method, which is followed within this work, are described in [27].

It is also worth of mentioning that direct experimental identification of indicial functions is theoretically also possible and one example has been noted in [5].

5 KVAZIUSTALJENA APROKSIMACIJA

Kao što je već napomenuto, uobičajeno je da se aeroelastične sile mere u aerotunelu na umanjenim modelima mostova. Posle toga se ove sile prenose na realni model mosta. Parametar sličnosti koji omogućava ovaj prenos naziva se redukovana brzina vetra $U_{red} = U/Bf$.

U radu [20] ovaj se parametar sličnosti objašnjava posmatranjem vazduha iza pomerljivog poprečnog preseka mosta. Naime, usled prinudnih oscilacija poprečnog preseka mosta, vazduh iza tela takođe ispoljava kretanje sa istom frekvencijom, slika 3. Uzimajući u obzir dolazeću brzinu vetra U, talasna dužina vazduha, koji je pod uticajem, može se proceniti kao $L_W = U\,T$, gde je T period prinudnih oscilacija. Tada se redukovana brzina može predstaviti kao:

5 QUASI-STEADY APPROXIMATION

As already mentioned, the aero elastic forces are usually measured in the wind tunnel on the scaled bridge deck models. Later these forces are to be transferred to the design model of the real bridge. A similarity parameter which enables this transfer is called the reduced wind velocity $U_{red} = U/Bf$.

In [20] this similarity parameter is considered by observing the air behind moving bridge deck. Namely, due to the forced motion of the bridge deck, the air behind the body is also experiencing a motion with the same frequency, Figure 3. Taking into account the approaching wind velocity U, the wavelength of affected air can be estimated as $L_W = U\,T$, where T is the forced motion period. Then the reduced velocity can be presented as:

$$U_{red} = \frac{U}{fB} = \frac{UT}{B} = \frac{L_W}{B} \tag{23}$$

Shodno tome, efekti memorije fluida postaju manji kada talasna dužina L_W raste, bilo povećavanjem brzine, ili smanjivanjem frekvencije oscilovanja. Za ove više redukovane brzine, strujanje vetra se približava stanju koje je dobijeno u slučaju nepokretnog poprečnog preseka. U tom slučaju, aeroelastične sile mogu biti aproksimirane kvaziustaljenim pristupom (pomoću koeficijenata sila (jednačina (1)). Redukovana brzina od oko $U_{red}{\approx}20$ ([13]) smatra se tačkom od koje kvaziustaljeni pristup može da se primenjuje umesto

Consequently, the fluid-memory effects become smaller when the wavelength L_W is increasing, either by raising the velocity, or decreasing the oscillation frequency. For these higher reduced velocities the wind flow field approach conditions obtained in a case of a fixed cross-section. In this case the aero elastic forces can be approximated by quasi-steady approach (using the force coefficients (Eq. (1)). As a transition point for application of the quasi-steady approach instead of the unsteady one is found at the reduced velocity $U_{red}{\approx}20$

neustaljenog. Aeroelastične sile uz usvojenu pretpostavku kvaziustaljenosti mogu se izvesti kao:

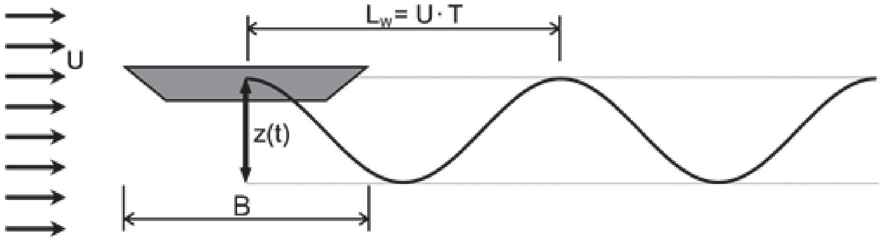
([13]). Aero elastic forces based on quasi-steady assumption can be derived as:

$$L_{ae}^{qs} = qB\left[-\left(\frac{dC_L}{d\alpha} - C_D\right)\frac{\dot{z}}{U} + \frac{dC_L}{d\alpha}\alpha + \left(\frac{dC_L}{d\alpha} - C_D\right)\beta_z \frac{B}{U}\dot{\alpha}\right] \tag{24}$$

$$M_{ae}^{qs} = qB\left[-\frac{dC_M}{d\alpha}\frac{\dot{z}}{U} + \frac{dC_M}{d\alpha}\alpha + \frac{dC_M}{d\alpha}\beta_\alpha \frac{B}{U}\dot{\alpha}\right] \tag{25}$$

C_i su koeficijenti sila iz jednačina (1) a $dC_i/d\alpha$ su njihovi prvi izvodi. Bezdimenzionalni parametar β_i predstavlja parametar ekscentričnosti ([27]). Izvođenje jednačina (24) i (25) može se naći u [27].

C_i are force coefficients from Eqs.(1) and $dC_i/d\alpha$ are its first derivatives. The dimensionless parameter β_i represents the eccentricity parameter ([27]). The derivation of Eq.(24) and Eq.(25) can be found in [27].



Slika 3. Talasna dužina L_{W}, *prema [20]*
Figure 3. Wavelength L_W *from [20]*

6 NUMERIČKI PRIMER

6.1 Eksperimentalna postavka

Model simetričnog, aerodinamički optimizovanog jednoćelijskog preseka nosača mosta (slika 4 levo) testiran je u aerotunelu s graničnim slojem na Univerzitetu u Bohumu (Ruhr – Universität Bochum). Aerotunel s nepovratnom vazdušnom strujom ima ukupnu dužinu od 9,4 m, širinu od 1,8 m i visinu od 1,6 m (slika 4 sredina). Turbulentna mreža se nalazi na ulazu u tunel, i proizvodi intenzitet turbulencije od oko 3–4%, a integralna skala turbulencije je oko 0.03 m.

Drveni model ima širinu od 0.36 m, visinu od 0.06 m i dužinu od 1.8 m. Ukupna masa modela je oko 4.9 kg. Model je horizontalno postavljen u aerotunelu (slika 4 desno). Na početku su sprovedeni testovi na fiksnom modelu. Model je postavljen na dva balansa sila, opremljena mernim trakama (koje mere sile) i koji se nalaze na bočnim stranama aerotunela. Merenja su realizovana pri različitim napadnim uglovima (-10° to 10°) s Reynolds-ovim brojem od oko 10^5 (ovo je određeno na osnovu širine preseka mosta).

Pored toga, flater derivati su dobijeni sprovođenjem testova s prinudnim vibracijama. Zbog toga, motor i kinematički mehanizam pokreću model mosta periodično u dva stepena slobode (vertikalno i torziono kretanje). U slučaju testova s prinudnim vibracijama, posebna pažnja se mora uzeti u obzir da bi se aeroelastične sile razdvojile od inercijalnih sila nastalih usled mase modela. Za tu svrhu se obavljaju dva seta merenja: jedno referentno merenje s prinudnim vibracijama bez strujanja vazduha i jedno merenje pod dejstvom vetra, kao što se pominje u odeljku 3.2. Testovi s prinudnim vibracijama sprovedeni su koristeći Reynolds-ove broje- ve u opsegu od $0.6*10^5$ do $3.5*10^5$. Amplitude prinudnih

6 NUMERICAL EXAMPLE

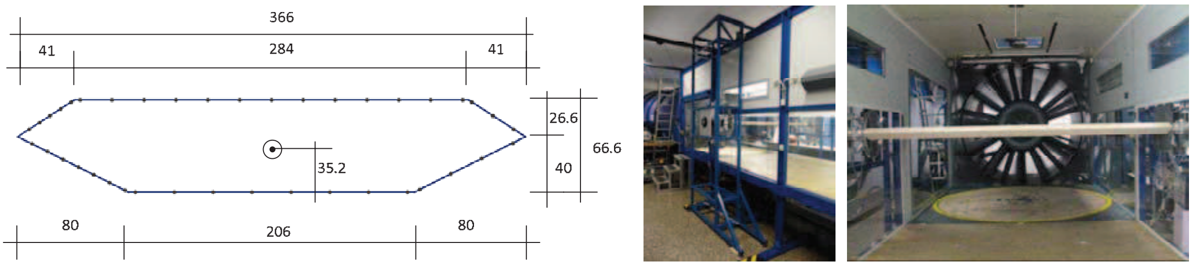
6.1 Experimental set-up

The model of a symmetric, aerodynamically optimized single-box section of a bridge girder (Figure 4 left) has been tested in the boundary layer wind tunnel at Ruhr-Universität Bochum. The open circuit wind tunnel has a total length of 9.4m, 1.8m width and 1.6m height (Figure 4 middle). A honeycomb grid is located at the inlet of the tunnel, generating turbulence intensity of around 3-4%, and having an integral turbulence length scale of around 0.03m.

The wooden model has a width *B* of 0.36m, a height *H* of 0.06m and a length *L* of 1.8m. The total mass of the model is about 4.9kg. The model is horizontally placed in the wind tunnel (refer to Figure 4 right). First, tests at the fixed model are carried out. The model is mounted on two force balances equipped with strain gauges (measuring the forces) which are located at each side of the wind tunnel. The measurements are realized at various angles of flow attack (-10° to 10°) with a Reynolds number of around 10^5 (based on the deck width of the bridge).

Furthermore, flutter derivatives are obtained performing forced vibration tests. Therefore a motor and a kinematic mechanism are driving the bridge deck model periodically in two degrees of freedom (vertical and torsional motion). In the case of forced vibration tests, special care has to be taken into account in order to separate the aero elastic forces from the inertial forces caused by the model's mass. For that purpose two sets of measurements are performed: one reference measurement with forced vibrations in still air and one measurement under the action of the wind, as it is also mentioned in section 3.2. The forced vibration tests are performed using Reynolds numbers in the range of vibracija su oko 4 mm u slučaju vertikalnog kretanja i oko 1° za torziono kretanje. Opseg frekvencija vibracija za testove je od 1.0 do 6.6 Hz. Ostali datalji u vezi s merenjem u aerotunelu mogu se naći u [30], a u pogledu eksperimentalne platforme u [22].

$0.6*10^5$ to $3.5*10^5$. The forced vibration amplitudes are around 4mm in the case of heaving motion and around 1° for the torsional motion. The vibration frequency range for the test is 1.0 to 6.6Hz. Further details regarding the wind tunnel measurements can be found in [30] and concerning the used experimental rig in [22].



Slika 4. Model poprečnog preseka mosta postavljen na eksperimentalnu platformu
Figure 4. Model of the bridge deck section placed in the experimental rig

6.2 Rezultati i diskusija

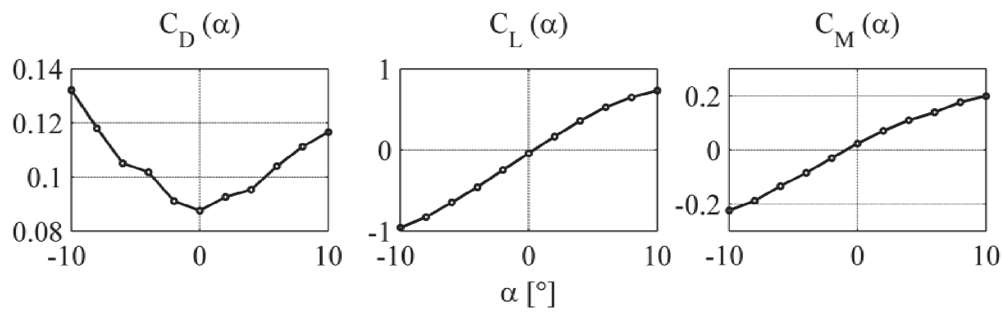
Dobijeni koeficijenti sila prikazani su na slici 5 u funkciji napadnog ugla. Brzina vetra koji prilazi konstrukciji je 4 m/s. Koeficijenti C_i, $i=D, L$ i M iz jednačine (1) i gradijenti C'_i, neophodni za izražavanje aeroelastičnih sila pri kvaziustaljenoj aproksimaciji (jednačine (24) i (25)) pri osrednjenom napadnom uglu od α = 0, izvedeni su aproksimiranjem funkcije u obliku polinoma mernim tačkama sa slike 5. Odgovarajuće vrednosti su tada definisane kao vrednosti i gradijenti aproksimirane funkcije (funkcije u obliku polinoma) pri osrednjenom napadnom uglu od α = 0. Zbog prirode krivih otpora, uzgona i momenta sa slike 5, koeficijenti sila uzgona i momenta C_L i C_M aproksimirani su s linearnom funkcijom, a koeficijent sile otpora C_D aproksimiran je s polinomom drugog reda. Aproksimacije su izvedene za -4° ≤ α ≤ 4°. Vrednosti određenih koeficijenata i njihovi prvi izvodi dati su u tabeli 1.

6.2 Results and discussion

Obtained force coefficients are plotted in Figure 5 as a function of the angle of the flow attack. The oncoming wind velocity is around 4 m/s. The coefficients C_i, $i=D, L$ and M from Eq.(1) and the gradients C'_i, necessary for expressing the aero elastic forces using the quasi-steady approximation (Eq.(24) and Eq.(25)), at the mean angle of attack α = 0, are derived by fitting a polynomial function to the measured points presented in Figure 5. The respective values are then defined as the values and gradients of the approximation function at α = 0 (polynomial function). Due to the nature of the drag, lift and moment curves in Figure 5, the lift and the moment force coefficients C_L and C_M are approximated with a linear function and the drag force coefficients C_D is approximated by a polynomial of second order. The approximations are performed for -4° ≤ α ≤ 4°. The values of respective coefficients and its first derivatives are given in Table 1.

Tabela 1: Koeficijenti sile i njihovi prvi izvodi pri osrednjenom napadnom uglu od α = 0
Table 1: Force coefficients and its first derivatives at the mean angle of attack α = 0

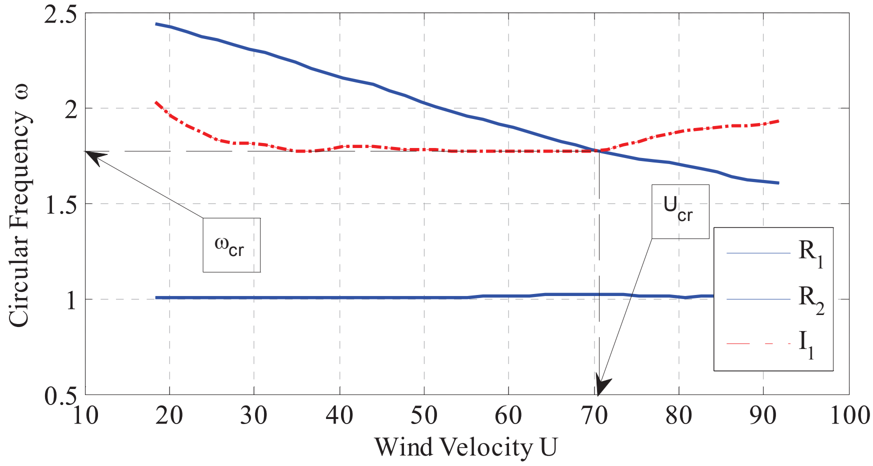
	C_D	C'_D	C_L	C'_L	C_M	C'_M
Statički eksperiment Static Experiment	0.0886	-0.0329	-0.0442	5.8513	0.0179	1.3984



Slika 5. Ustaljeni koeficijenti sila
Figure 5. Steady force coefficients

Svih osam flater derivata korišćenih u jednačinama (2) i (3) predstavljeni su na slici 7. Oni su mereni u opsegu redukovanih brzina do U_{red}=30 (gde je U_{red} =U/Bf=$2\pi/K$). Na osnovu ovih vrednosti, aeroelastično opterećenje može se odrediti pomoću jednačina (2) i (3). U ovom slučaju, konstruktivne karakteristike posmatranog mosta prikazane su u tabeli 2 i koristeći 2DOF model, opisan u odeljku 3.4, moguće je odrediti kritičnu brzinu. Stoga su na slici 6 prikazana realna rešenja X, realne i imaginarne jednačine (jednačine (16) i (17)) u funkciji U_{red}. Prvi presek ovih krivih vodi ka flater rešenju. Kao što se može primetiti sa slike 6, dobijena je kritična brzina od oko U_{cr}=70.46m/s.

All eight flutter derivatives used in Eq.(2) and Eq.(3) are presented in Figure 7. They are measured for the range of reduced velocities till U_{red}=30 (where U_{red} =U/Bf=$2\pi/K$). Based on these values, aero elastic loads can be evaluated by the Eq.(2) and Eq.(3). In this case, the structural properties of the used bridge deck are given in Table 2 and using the 2DOF model described in section 3.4 critical velocity can be estimated. Therefore, in Figure 6 the real X solutions of real and imaginary equations (Eq.(16) and Eq.(17)) against U_{red} are plotted. The first intersection of these curves leads to the flutter solution. As it may be observed from Figure 6, the critical velocity around U_{cr}=70.46m/s is obtained.



Slika 6. Određivanje kritične brzine i kritične frekvencije
Figure 6. Determination of critical velocity and critical frequency

Kao dodatak flater derivatima koji su određeni iz testova u aerotunelu, na slici 7 su takođe prikazane i kvaziustaljene aproksimacije derivata. One su određene poređenjem koeficijenata koji se nalaze pored pomeranja i njihovih prvih izvoda koji su uzeti u obzir u dvema aeroelastičnim formulacijama: aeroelastične sile koje su bazirane na derivatima (jednačine (2) i (3)) i kvaziustaljenoj aproksimaciji (jednačine (24) i (25)). Kao što se može uočiti, nemaju svi flater derivati svoje pandane pri kvaziustaljenoj aproksimaciji. Oni koji nedostaju su derivati H^*_4 i A^*_4, koji i nemaju odlučujuću ulogu kod praktičnih primera aerodinamike mostova. Može se primetiti da aproksimacije prate isti trend. Još jedna nepoznata u slučaju kvaziustaljene aproksimacije u vezi je sa izborom parametra ekscentričnosti β_i. Ovi parametri imaju veliki uticaj na najvažnije derivate vezane za prigušenje H^*_2 i A^*_2. Naime, parametri β_i opisuju pozicije neutralnih tačaka odgovarajućih komponenata sile. U opštem slučaju poprečnog preseka mosta, zajednička neutralna tačka ne postoji ([26], [21]). Moguće rešenje bilo bi da se usvoje pozicije neutralnih tačaka u vezi s poprečnim presecima gde su one poznate, kao što je primer aeroprofila. Ipak, usled velikog uticaja na važne derivate H^*_2 i A^*_2, β_i bi trebalo, ako je moguće, da budu određeni na osnovu dinamičkih testova (iz flater derivata). U ovom radu, prateći proceduru koja je opisana u [21], parametri su određeni na osnovu izmerenih H^*_2 i A^*_2 krivih, što vodi do βz=1.761 i $\beta\alpha$=-1.378. Na osnovu kvaziustaljenih flater

In addition to flutter derivatives evaluated from the wind tunnel tests, Figure 7 also shows quasi-steady approximations of derivatives. They are evaluated comparing the coefficients which stand beside considered displacements and their first derivatives in two aero elastic formulations: aero elastic forces based on the derivatives (Eq.(2) and Eq.(3)) and quasi-steady approximation (Eq.(24) and Eq.(25)). As may be observed, not all flutter derivatives have their counterparts in quasi-steady approximation. The missing ones are H^*_4 and A^*_4 which are not decisive related to practical examples of bridge aerodynamics. It can be remarked that the approximations are following the same trend. Another unknown in the case of quasi-steady approximation is related to the choice of eccentricity parameters β_i. They have strong influence on the most important damping derivatives H^*_2 and A^*_2. Namely, parameters β_i describe the position of the neutral points for the respective force components. In general case of the bridge section a common neutral point does not exists ([26], [21]). One possible solution could be to adopt the positions of neutral points related to the certain cross-section where those positions are known, such as airfoil. Still due to the strong influence on important H^*_2 and A^*_2 derivatives, β_i parameters should be, if possible, evaluated from dynamic tests (from flutter derivatives). In this work, following the procedure described in [21] parameters are evaluated from measured H^*_2 and A^*_2 curves which leads to βz=1.761 and $\beta\alpha$=-1.378. Based derivata sa slike 7, određena je kritična brzina kao konzervativnija vrednost od U_{cr} =66.97m/s u odnosu na kritičnu brzinu prethodno dobijenu s neustaljenim pristupom.

Kao što je već pomenuto u odeljku 4, koeficijenti indicijalnih funkcija se mogu identifikovati na osnovu nelinearne optimizacije pomoću metode najmanjih kvadrata. Kao primer će biti određeni nepoznati koeficijenti iz jednačine (21), koji su vezani za indicijalnu funkciju $\Phi_{L\alpha}$ koja opisuje silu uzgona usled rotacionog kretanja. Na osnovu uspostavljenih relacija između indicijalnih funkcija i flater derivata koje su prikazane u jednačinama (22), nepoznati koeficijenti se mogu odrediti na osnovu derivata H^*_2 i H^*_3. Koristeći izmerene aeroelastične derivate H^*_2 i H^*_3 sa svojim diskretnim vrednostima pri redukovanim brzinama U^m_{red}, $m = 1,\ldots,M$, funkcija greške $\varepsilon_{L\alpha}$ koja je potrebna da bude minimizovana može biti data kao:

on the quasi-steady flutter derivatives from Figure 7 critical velocity is evaluated as a more conservative value U_{cr} =66.97m/s when compared to the critical velocity obtained from previously shown unsteady approach.

As already mentioned in section 4, indicial functions coefficients may be identified by means of a nonlinear least-square optimization. As an example unknown coefficients from Eq.(21) related to the indicial function $\Phi_{L\alpha}$ which describe the lift force due to the pitch motion are going to be identified. Based on established relationships between indicial functions and flutter derivatives shown in Eqs.(22), unknown coefficients should be evaluated from the derivatives H^*_2 and H^*_3. Using the measured aero elastic derivatives H^*_2 and H^*_3 at discrete values of the reduced wind velocity U^m_{red}, $m = 1,\ldots,M$, the error function $\varepsilon_{L\alpha}$ can be established, which is to be minimized:

$$\varepsilon_{L\alpha}(\mathbf{p}_{L\alpha}) = \sum_{m=1}^{M}\left[\frac{(D_{L\alpha}(\mathbf{p}_{L\alpha},U^m_{red}) - \overline{D}^m_{L\alpha})^2}{\sigma^2_{\overline{D}^m_{L\alpha}}} + \frac{(E_{L\alpha}(\mathbf{p}_{L\alpha},U^m_{red}) - \overline{E}^m_{L\alpha})^2}{\sigma^2_{\overline{E}^m_{L\alpha}}}\right] \qquad (26)$$

gde je $D_{L\alpha} = -K^2H^*_3$ a $E_{L\alpha} = -KH^*_2$ i K je redukovana frekvencija. Reprezentacija neustaljenih koeficijenata pomoću $D_{L\alpha}$ i $E_{L\alpha}$ pogodnija je zbog identifikacione procedure u odnosu na klasičnu reprezentaciju pomoću H^*_2 i H^*_3, gde su vrednosti pri nižim redukovanim brzinama umanjene. Stoga bi vrednosti pri većim redukovanim brzinama imale veći uticaj na totalnu grešku, te bi se kao rezultat dobila slabija aproksimacija pri nižim redukovanim brzinama, gde je neustaljenost izraženija ([26]). Vektor $\mathbf{p}_{L\alpha}$, iz jednačine Eq.(26), sažima sve nepoznate parametre koji treba da budu određeni optimizacionom procedurom:

where $D_{L\alpha} = -K^2H^*_3$ and $E_{L\alpha} = -KH^*_2$ and K is reduced frequency. The representation of unsteady coefficients in terms of $D_{L\alpha}$ and $E_{L\alpha}$ is more suitable for this identification procedure than the classical representation in terms of H^*_2 and H^*_3 where the values at low reduced velocities are scaled down. Therefore the values at high reduced velocities would be weighted too much in the total error, resulting in a poor approximation at low reduced velocities, where unsteadiness is more important ([26]). Vector $\mathbf{p}_{L\alpha}$, from Eq.(26), collects the unknown parameters which have to be determined through the optimisation procedure:

$$\mathbf{p}_{L\alpha} = \left[a_{L\alpha 1},\ldots,a_{L\alpha N_{L\alpha}},b_{L\alpha 1},\ldots,b_{L\alpha N_{L\alpha}}\right]^T \qquad (27)$$

gde su $a_{L\alpha i}$ i $b_{L\alpha i}$, $i=1-N_{L\alpha}$ bezdimenzionalni koeficijenti i $N_{L\alpha}$ je broj izabranih članova za aproksimaciju indicijalne funkcije $\Phi_{L\alpha}$. Ukoliko su ustaljeni koeficijenti kao i njihovi prvi izvodi nepoznati moguće je i njih tretirati kao nepoznate parametre.

Optimizacija se izvodi na osnovu algoritma ‚pouzdane oblasti' (□trust-region' algoritma), koji je implementiran u Matlab-u i koristi analitičke izraze za gradijente greške $\partial\varepsilon_{il}/\partial p_{il}$ i $\partial^2\varepsilon_{il}/\partial^2 p_{il}$ koji su izvedeni u radu [26]. Slične funkcije greške koriste se za identifikaciju ostalih indicijalnih funkcija. Sve četiri sračunate indicijalne funkcije za posmatrani poprečni presek prikazane su na slici 8.

Za aerodinamične poprečne preseke, s ravnomernim izgledom svih derivata, upotreba jedne ([3]) ili dve (kao kod Jones-ove aproksimacije Theodorsen-ove funkcije date u [16]) grupe eksponencijalnih članova dovoljna je da prikaže globalno ponašanje. U ovom slučaju, jedna eksponencijalna grupa je iskorišćena za opisivanje ponašanja svih indicijalnih funkcija. Za proveru kvaliteta identifikovanih indicijalnih funkcija, flater derivati mogu biti određeni na osnovu bezdimenzionalnih koeficijenata sadržanih u $\mathbf{p}_{il}$ i jednačina (22). Stoga su na slici 9 takođe predstavljeni odgovarajući flater derivati koji su određeni na osnovu identifikovanih bezdimenzionalnih

where $a_{L\alpha i}$ and $b_{L\alpha i}$, $i=1-N_{L\alpha}$ are nondimensional coefficients and $N_{L\alpha}$ is the number of terms chosen to approximate the indicial function $\Phi_{L\alpha}$. If the steady coefficients and their first derivatives are unknown it is possible to treat them as additional unknown parameters.

The optimization is performed by using a trust-region algorithm which is implemented in Matlab and using analytical expressions for the error gradients $\partial\varepsilon_{il}/\partial p_{il}$ and $\partial^2\varepsilon_{il}/\partial^2 p_{il}$ developed by [26]. Similar error functions are used to identify other indicial functions. All four resulting indicial functions for the considered bridge deck are presented in Figure 8.

For streamlined cross-sections, with 'uniform' trends in all derivatives, the use of one ([3]) or two (Jones' approximation of Theodorsen's function given in [16]) groups of exponential terms is sufficient to capture the general behaviour. In this case one exponential group is used to describe the behaviour of all indicial functions. As a quality check for identified indicial functions, flutter derivatives can be evaluated based on the non-dimensional coefficients contained in $\mathbf{p}_{il}$ and Eqs.(22). Therefore, Figure 9 also include the corresponding flutter derivatives evaluated based on identified non-dimensional coefficients a_{il} and b_{il}, related to the indicial koeficijenta a_{il} i b_{il}, koji odgovaraju indicijalnoj funkciji $\Phi_{L\alpha}$ (na slici 9 obeleženo je sa ‚optimized') i štaviše, pokazuju zadovoljavajuće poklapanje s merenim flater derivatima. Slična provera je izvršena i za ostale identifikovane indicijalne funkcije.

Ove funkcije, kroz formu konvolucionih integrala, mogu biti iskorišćene za određivanje kritične brzine vetra. Naime, jednačine kretanja predstavljene u jednačini (12) mogu biti rešene u slučaju različitih brzina vetra, samo u ovom slučaju, pomoću aeroelastičnih sila izraženih u vremenskom domenu (jednačine (19) i (20)). Povećavanjem brzine, kritična brzina se može odrediti kao brzina koja izaziva nestabilne-divergentne oscilacije. Uprkos tome, zbog ekvivalentnosti ova dva pristupa, frekventnog i vremenskog, oba rešenja moraju da konvergiraju. No ipak, neke prednosti u izboru jednog u odnosu na drugi pristup, trebalo bi uzeti u obzir. Naime, zabeleženo je u [27] da je, sa stanovišta računara, analiza stabilnosti obimnija u vremenskom domenu, pogotovu kada se uzmu u obzir komplikovaniji modeli od 2DOF, i stoga je analiza u frekventnom domenu poželjnija. Isti autori takođe navode da bi metod u vremenskom domenu trebalo da se koristi kod analize mostova kada je pristup u frekventnom domenu komplikovaniji (na primer, spregnuta analiza flatera i uticaja turbulencije, analiza koja uključuje lokalizovane prigušivače), ili kada nije moguć (na primer, analiza koja uključuje konstruktivne nelinearnosti, nelinearne prigušivače).

function $\Phi_{L\alpha}$ (in Figure 9 marked as 'optimized') and moreover they show satisfying agreement with measured flutter derivatives. Similar check is also performed for other identified indicial functions.

These functions, in the form of convolution integrals can also be used to estimate critical wind velocity. Namely, the equations of motion presented in Eq.(12) can be solved for different wind velocities, only in this case, with the aero elastic forces expressed in the time domain (Eq.(19) and Eq.(20)). By increasing the velocity, the critical velocity can be estimated as one causing unstable, divergent oscillations. Nevertheless, due to the equivalency of these two approaches, namely frequency and time, both solutions should converge. Still, some preferences in choosing one or the other approach should be taken in consideration. Namely, it is noted in [27] that from the computational point of view, stability analysis in the time domain is more extensive, especially when considering more complicated models then 2DOF, and therefore frequency-domain analysis is preferable. The same authors also mention that the time-domain method should be used for bridge analyses where the frequency-domain approach is more complicated (e.g. coupled buffeting analysis, analyses including localized damping devices), or where it is inapplicable (e.g. analyses including structural nonlinearities, nonlinear damping devices).

Tabela 2: Konstruktivne karakteristike posmatranog mosta [1]
Table 2: Structural properties of considered bridge[2]

B[m]	m_z[kg/m]	m_α[kg/m]	f_z[Hz]	f_α[Hz]	ζ_z[-]	ζ_α[-]
18.3	12820	426000	0.142	0.355	0.006	0.005

7 ZAKLJUČCI

Dinamičke sile vetra koje deluju na fleksibilne mostovske nosače nastaju usled turbulencije koja dolazi do konstrukcije, zatim koja je uzrokovana samom konstrukcijom, te vrtložnim tragom iza konstrukcije, kao i usled interakcije između konstrukcije i vetra koji je opstrujava. Poslednji (aeroelastični) tip sila deluje kao dodatni dinamički uticaj na poprečni presek mosta. Te sile imaju potencijal da generišu aeroelastični mehanizam samopobuđujućih oscilacija nosača, i mogu da dovedu konstrukciju do dinamičke divergencije, stvarajući aeroelastični fenomen poznat kao flater.

Glavni cilj ovog rada je da predstave različite metode koje mogu da se koriste za rešenje problema flatera kod mostova. Kao prvi metod je predstavljen najčešće primenjivan pristup u frekventnom domenu u kom se koriste frekventno zavisni aerodinamični parametri poznati kao flater derivati. Razmatran je 2DOF model i definisan je problem svojstvenih vrednosti, koji kao rezultat daje kriti-

[1] Vrednosti su uzete iz [24], gde je sličan poprečni presek mosta posmatran pomoću multimodalne analize. Dva tona, vezana za savijanje i torziju, odgovaraju predstavljanim glavnim spregnutim tonovima u pomenutom članku.

7 CONCLUSIONS

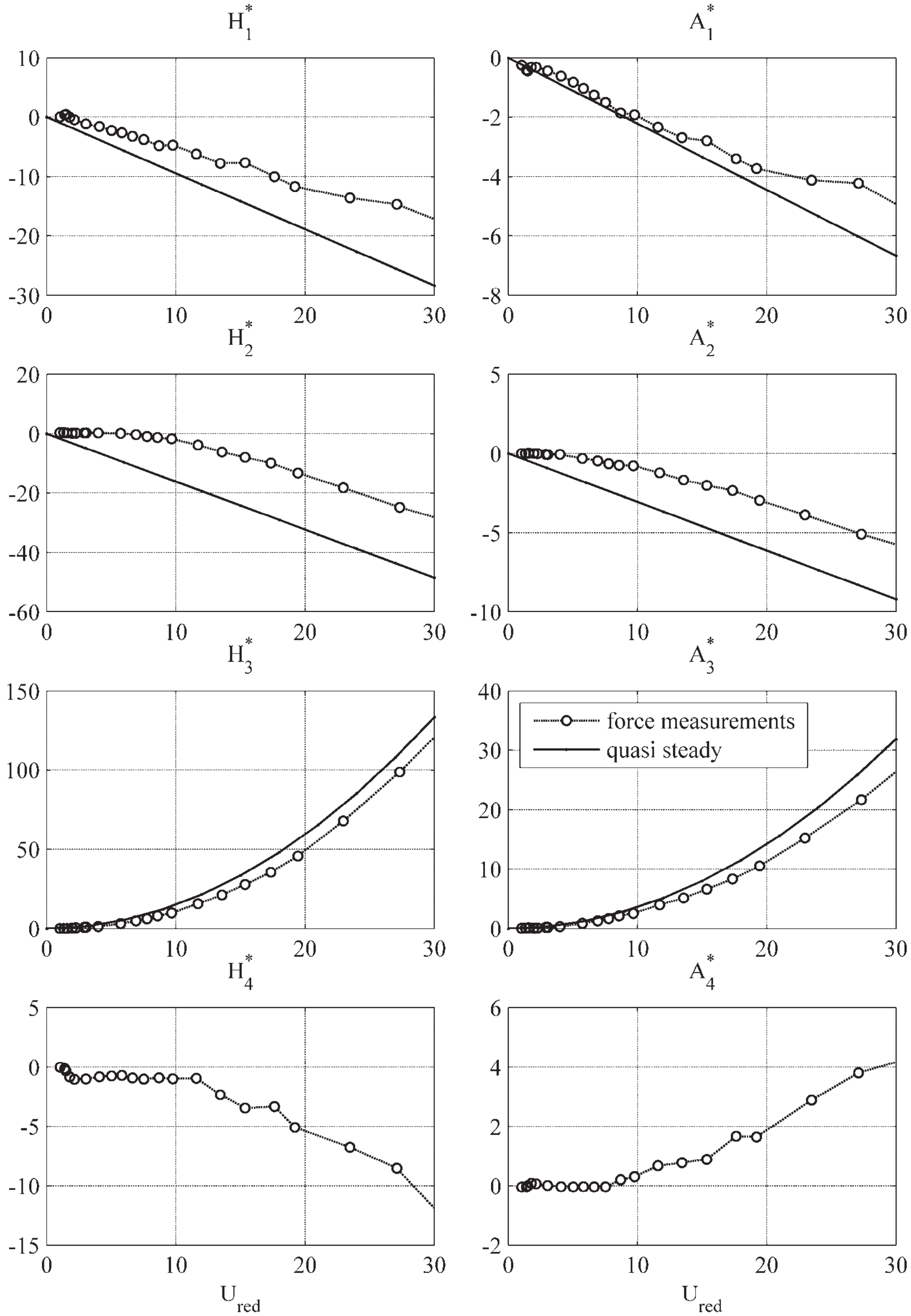
Dynamic wind forces upon flexible bridge girders evolve from the action of the incident, body- and wake-induced turbulence and from the interaction between the motion of the structure and the circumfluent wind. The latter (aero elastic) type of forces acts as the additional dynamic effect upon the girder cross-section. It has the potential to generate an aero elastic mechanism of self-excitation of girder oscillations, and it can bring the structure to dynamic divergence, creating aero elastic phenomenon called flutter.

The main objective of this paper is to present different bridge flutter methods which can be used to solve the flutter problem. At first, the most commonly used frequency-domain approach is presented in which frequency dependent aerodynamic parameters called flutter derivatives are applied. The 2DOF model is considered and eigenvalue problem is defined giving as the outcome critical wind speed - the main critical condition

[2] Values are taken from [24], where the similar bridge deck section is considered with the use of multimode analisys. Two modes, for heave and pitch, are corresponding to the presented main coupled modes from the related paper.

čnu brzinu vetra - glavni kritični uslov za nastanak flatera. Kao sledeći korak, ekvivalentni pristup u vremenskom domenu, koji je baziran na indicijalnim funkcijama, sumiran je i zaključno je predstavljena aproksimacija vezana za kvaziustaljenu teoriju.

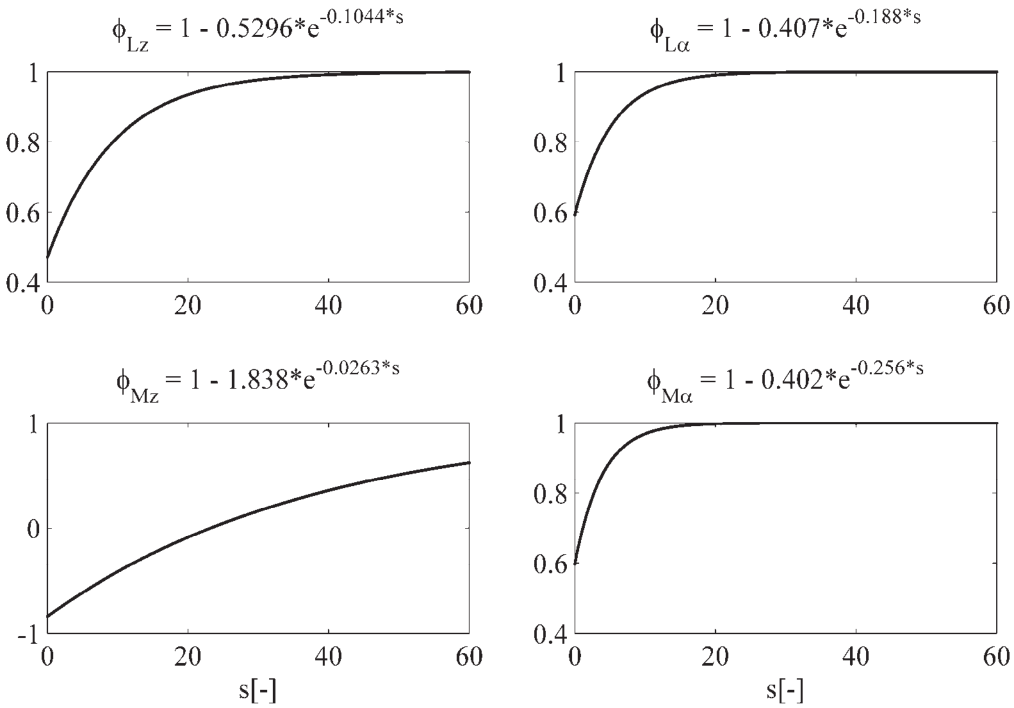
for the onset of flutter. As a next step, equivalent approach in time-domain, based on the indicial functions, is summarized and finally the approximation based on the quasi-steady theory is presented.



Slika 7. Flater derivati dobijeni direktno na osnovu merenja i korišćenjem kvaziustaljene aproksimacije
Figure 7. Flutter derivatives obtained directly from the measurements and using quasi-steady approximation

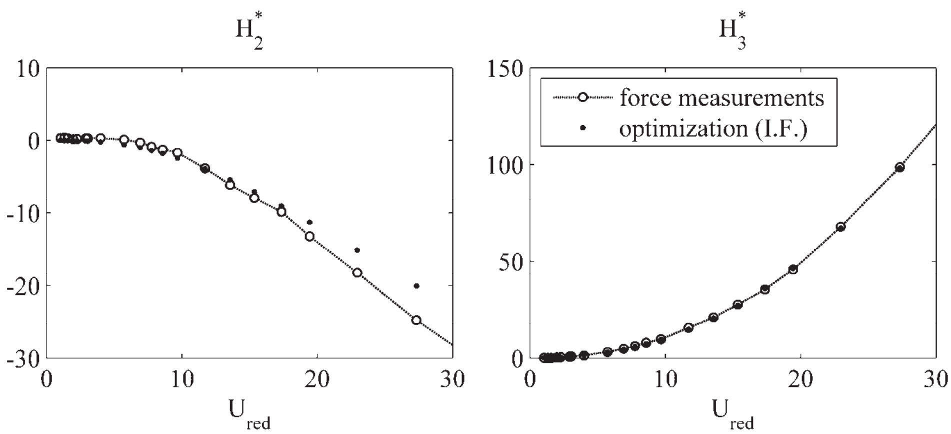
Prikazan je numerički primer jednog tipičnog poprečnog preseka mosta. S tom svrhom, serija eksperimenata je sprovedena u aerotunelu na fiksnom modelu postavljenom s različitim napadnim uglovima, kao i pomoću mehanizma za prinudne vibracije. Prikazane su identifikacione tehnike vezane za frekventno zavisne koeficijente – flater derivate i vremenski zavisne – indicijalne funkcije. Prednosti i mane prezentovanih pristupa su navedene.

A numerical example is offered related to one typical bridge deck cross-section. For that purpose, series of wind tunnel experiments conducted upon a rigid model placed under different angles of flow attack and by operating a forced vibration mechanism are performed. Identification techniques related to the frequency dependent coefficients – flutter derivatives and time dependent functions – indicial functions are provided. Advantages and disadvantages of the presented approaches are discussed.



Slika 8. Indicijalne funkcije

Figure 8. Indicial functions



Slika 9. Izabrani flater derivati dobijeni direktno na osnovu merenja u poređenju sa optimizovanim vrednostima dobijenim na osnovu procene indicijalne funkcije $\Phi_{L\alpha}$ sa slike 8

Figure 9. Selected flutter derivatives obtained directly from the measurements compared with optimized values obtained from indicial function estimation of the $\Phi_{L\alpha}$ from Figure 8

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REZIME

METODE ANALIZE FLATERA U FREKVENTNOM I VREMENSKOM DOMENU

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Fenomen flatera mostova predstavlja važan kriterijum stabilnosti, koji mora biti uzet u obzir tokom procesa projektovanja mosta. U ovom radu su prikazane različite metode koje se mogu koristiti pri rešavanju problema flatera. Najčešće korišćeni pristup je u frekventnom domenu i baziran je na formulaciji aeroelastičnih sila putem frekventno zavisnih koeficijenata koji se nazivaju flater derivati. Na osnovu ovako izraženih aeroelastičnih sila, određuje se kritična brzina vetra, kao glavni uslov za nastanak flatera. Aeroelastične sile mogu se takođe izraziti i u vremenskom domenu, pomoću takozvanih indicijalnih funkcija. Ove funkcije su najčešće određene iz odgovarajućih flater derivata. U slučajevima kada su efekti memorije fluida zanemarljivi, kvaziustaljena teorija može se koristiti za aproksimaciju aeroelastičnih sila. Numerički primer tipičnog poprečnog preseka mosta prati prikazane pristupe.

Ključne reči*:* Flater, rešenje flatera, modeli opterećenja, flater derivati, indicijalne funkcije

SUMMARY

FREQUENCY- AND TIME-DOMAIN METHODS RELATED TO FLUTTER INSTABILITY PROBLEM

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Bridge flutter phenomenon presents an important criterion of instability, which should be considered in the bridge design phase. This paper presents different bridge flutter methods which can be used to solve the flutter problem. Most commonly used frequency-domain approach is based on the formulation of aero elastic forces with frequency dependent coefficients called flutter derivatives. The critical wind speed, as the main critical condition for the onset of flutter is obtained based on these aero elastic forces. Aero elastic forces can be also expressed in the time-domain, using so-called indicial functions. These functions are usually determined from the corresponding flutter derivatives. In situations when fluid-memory effects tend to become small the quasi-steady theory can be used as an approximation of aero elastic forces. A numerical example related to the typical bridge cross-section follows presented approaches.

Key words: Flutter, flutter solution, load models, fultter derivatives, indicial functions