

# MODELLING SHAPE OF ARCHITECTURAL STRUCTURE - ELLIPTIC HYPERBOLOID OF ONE SHEET

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*The combination of straight lines of constructive elements and curved contours of structural shapes is always challenging in the engineering practice. The 2<sup>nd</sup> order surface – elliptic hyperboloid of one sheet (ELHY) provides such combination. Given that in the architectural surroundings, arts, or other scientific fields ELHY is less common, than the other representatives of the same family of ruled surfaces, it is worth of attention and research. Here presented constructive geometry approach resulted in Auto CAD application for generating 3D wire-frame and triangulated net model of ELHY surface and some examples of structures – prototypes of structural shapes, designed by using ELHY fragments.*

**Key words:** elliptic hyperboloid of one sheet (ELHY), 3D geometric modelling, structures, Auto CAD application.

## INTRODUCTION

The study of architectural shapes revealed that geometric surface of the 2<sup>nd</sup> order – hyperboloid of one sheet, has been well known and very common architectural shape ever since the 19<sup>th</sup> century and still is. In 1890s, famous Russian engineer Vladimir Shukov constructed first lattice – hyperboloid shaped structure, tower in Polibino, Russia. The other aspect of employment of hyperboloid was provided in 1957 by architect Eduardo Torroja (Antuna, 2010), applying concrete shell for the water tank Fedala Reservoir, in Morocco. Later, this kind of surface has become recognizable around the world in industrial architecture (power cooling towers), or in high special purpose structures.

At the end of 20<sup>th</sup> and the beginning of the 21<sup>st</sup> century, the hyperboloid steel lattice structures have been 'clothed' with glass either outside or inside and brought some other functionalities. Thus, the *Corporation Street Bridge*, designed by arch. Hodder & Partners, in the central part of Manchester (England), is horizontal

communication made of steel and glass. On the other hand, very tall buildings, such as *Aspire Tower*, also known as *Torch Hotel*, built for the Asian Games in Qatar in 2006 and designed by Hadi Simaan and AREP architects, exhibits combinations of concrete core and outer steel mesh structure, with high-tech illuminations.

Modern architecture and computer modelling of the 21<sup>st</sup> century brought some innovations, regarding this surface. Thus, dynamic shapes have been employed in projects of *Al Masdar Headquarters* in Abu Dhabi, by A. Smith and G. Gill architects; *BMW Welt*, 2007 in München, by arch. Wolf D. Prix and Copp Himmelblau corporation etc. (Vrančić, 2010)

All the mentioned structures are examples of the hyperboloid of revolution surface. Although elliptic one sheeted hyperboloid (ELHY) is the representative of the same family of hyperboloids, it is significantly less common and familiar architectural shape than hyperboloid of revolution. Very tall high-tech building, *Canton tower* in Guangzhou, China, is one of the rare representatives of lattice structures, presenting the important characteristic of

ELHY, characterized by straight lines, as generatrices of the architectural shape.

The main goal of this manuscript is to offer designers a tool i.e. computer application and examples of prototypes of ELHY structures to stimulate its wider application in building environment.

## 3D MODELLING OF ELHY

When discussing the ELHY surface, the aspects of its generating, or modelling are of particular importance. One straight line, i.e. generatrix is ruling along three straight directrices generating the surface. These three directrices are skewed, finite lines  $d_1$ ,  $d_2$ ,  $d_3$ , presented in Figure 1. For the easier constructive treatment,  $d_1$ ,  $d_2$ ,  $d_3$  are presented

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as diagonals, or edges of the sides of the prism ABCDEFGH (Čučaković, 2008). Likewise, three directrices  $r_1, r_2, r_3$ , of the other system of generatrices are inscribed in the sides of the same prism. Orthogonal projections of  $d_1', d_2', d_3'$ , onto the plane normal to  $d_1$ , were used for constructing spatial positions of generatrices. In such settings, projections of the generatrices are radial lines through point  $D \equiv d_1'$ , which intersect directrices  $d_2'$  and  $d_3'$  in points  $1'-15'$ .

Since this surface has two systems of generatrices, the same procedure was applied for parallel directrices  $r_1, r_2, r_3$ , of the opposite sides of the prism ABCDEFGH, in the second system of ELHY generatrices (Figure 1).

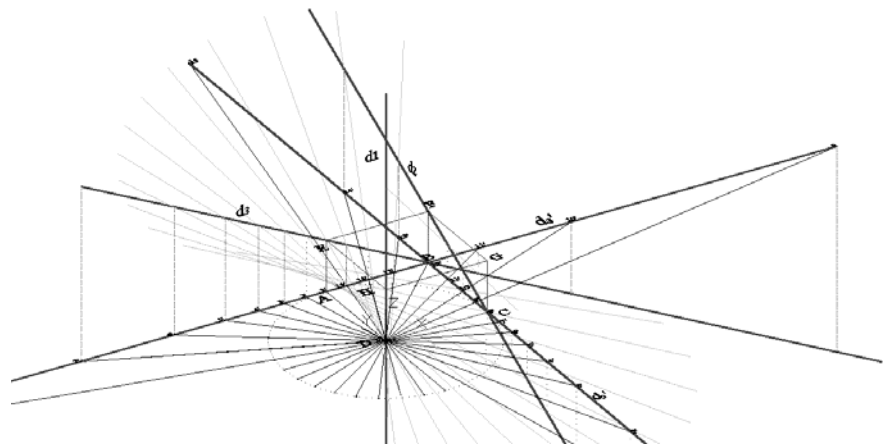


Figure 1. The first system of generatrices of ELHY

In order to visualize the surface constructed 'inside' auxiliary prism, ELHY is cut with two planes, parallel to the prism's base ABCD, symmetrically towards the center of the surface, deriving two identical ellipses (Dragović, 2013).

**Auto CAD application for the modelling of ELHY surface**

The relevant relation of the ELHY surface and its asymptotic cone (Dragović, 2013), where one asymptotic cone is mutual for infinite number of ELHY surfaces, affected creative approach to CAD application (written in *Auto Lisp*), for modelling both the wire-frame net and triangulated surface model of ELHY. In this non-standard approach, the circular sections of ELHY were considered as the surface generating curves.

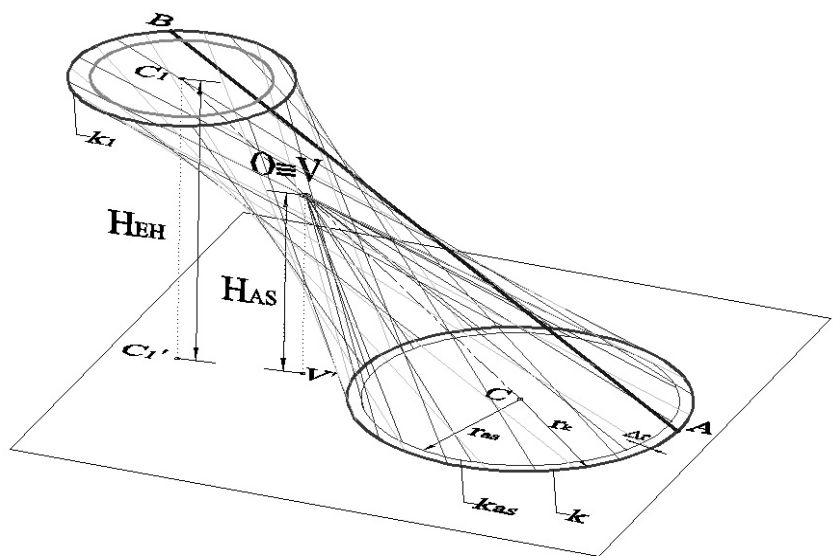


Figure 2. The disposition of geometric parameters for ELHY

Hence, asymptotic cone of ELHY is given with generating curve  $k_{as}$  and vertex  $V$ , where the centre point  $C \neq V$ . The center  $O$  of the ELHY surface is identical to the vertex  $V$  of its asymptotic cone. Besides, generatrices of ELHY are parallel with generatrices of its asymptotic cone. Base curve  $k$  of ELHY has to satisfy the condition  $r_k > r_{as}$  (Figure 2).

The first part of the application for obtaining the net of ELHY's generatrices, has two options (construction modes) given in algorithm in Figure 3, requesting below listed responses:

a) *Left side algorithm option:*

- construction mode - 1 (circle),
- selection of the circle –  $k_{as}$  or  $k$ , drawn in WCS (World coordinate system),
- confirmation whether the selected circle belongs to asymptotic cone, or to ELHY,
- $V'$  position (orthogonal projection of vertex  $V$ ) in WCS,
- asymptotic cone height -  $H_{AS}$  (z coordinate of  $V$ ),
- radius difference  $\Delta r$  of the circles ( $k, k_{as}$ ),
- ELHY height -  $H_{EH}$  and
- generatrices number – n.

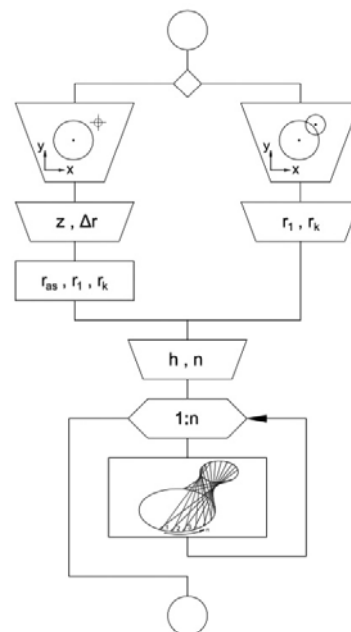


Figure 3. Algorithm for CAD application

b) *Right side algorithm option:*

- construction mode: 2 (circles)
- selection of the base ( $k$ ) and top ( $k_1$ ) circles, drawn in WCS,
- selection of the start point ( $A$ ) and end point ( $B$ ) of generatrix in WCS,
- ELHY height -  $H_{EH}$  and
- number of generatrices –  $n$ .

The results are symmetrical or asymmetrical shapes of ELHY nets. It must be noted that when setting the starting parameters, the spatial prediction of ELHY shape is very important (Dragović, 2013). This includes setting of: circles - cross sections of ELHY, i.e.  $H_{EH}$  (mode 2); start ( $A$ ) and end ( $B$ ) points of generatrix (mode 2), etc.

Both options of CAD application offer export data file for creating ELHY triangulated surface model (Figure 4), as well as the sets of circular sections, incorporated in the net of generatrices. Benefits of the structure triangulation concerning various unfavourable influences, such as snow, wind, temperature decrease, etc., were confirmed in various studies (Nenadović, 2010).

**PROTOTYPES OF HYPERBOLOID STRUCTURES**

Bearing in mind the property of ELHY, related to straight lines – generatrices of the surface, the possibilities of their use in appropriate structural (building) systems, applicable in architectural design and practice, have been considered (Nestorović, 2000). In such a way, strained cables of prestressed thin concrete shells, as well as guidelines for the elements of mesh structures (carrying construction, or facades) could represent skewed ELHY lines.

Regarding this, here are presented geometrical models of structures – prototypes of architectural shapes, offering both possibilities: thin shells and meshes.

**Prototypes of structures – combinations of thin concrete shells**

*Shells with curved edges*

In the following prototypes, fragments of the main shape – ELHY are cut out from the solid model of surface by Boolean operations between ELHY and two stellated pyramids (6-pointed star) derived 6 saddle shaped fragments – shells.

The element of structure - saddle shaped fragment of ELHY, is incorporated in triangular, square and pentagonal scheme, obtaining 3D model, presented in Figure 5. Structures contain supplementary planar elements, according to the system of wedging of ELHY.

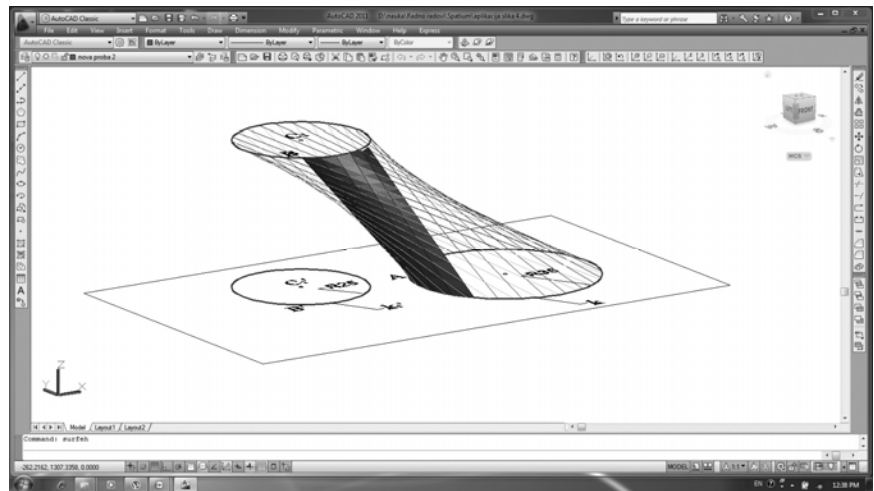


Figure 4. Generating process for triangulated surface model of ELHY (option "a") in Auto-CAD

	SCHEME	TOP VIEW	3D MODEL VIEW
a) Triangular			
b) Squared			
c) Pentagonal			

Figure 5. Fragments of ELHY incorporated in three polygonal schemes

**Shells with straight edges and circular sections**

ELHY belongs to a group of surfaces with three planes of symmetry and two sets of circular sections being related to the planes of symmetry (Dragović, 2013). This property enables surface slicing in fragments adjustable to various structures containing rotational symmetry (Jadrešin-Milić, 2008). Using slicing method of ELHY, with two symmetrically positioned planes (with respect to the plane of symmetry  $\varphi_1$ ) through chosen generatrices of the surface and two circular sections, a fragment of the surface can be obtained (Figure 6), adequate for combinations in polar array patterns.

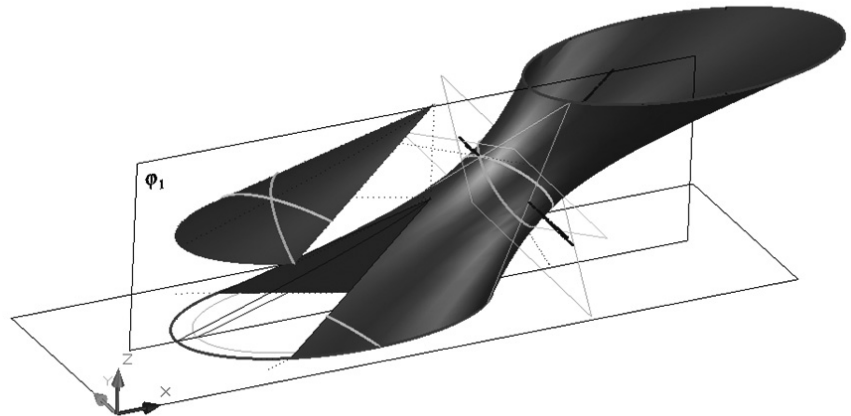


Figure 6. Fragment of ELHY with two straight edges

The elements of the structure - ELHY fragments, are incorporated in the triangular, square and pentagonal scheme (Figure 7), forming 3D model. Following the same principles, structures contain supplementary planar elements.

**Mesh structure models**

This part of our study discusses several issues: employment of diagrid structural system and triangulation, as well as, revolving and twisting with double curved surface:

- ELHY, although containing straight generatrices, is actually a double curved surface. This property makes it possible to employ diagrid (triangular-grid structures with diagonal support beams), as structural system in design of buildings, which has its attractive and practical values (Volner, 2011). One of these properties was reported by Boake (2013:578) in his article stating: 'Buildings whose diagrids support more curvilinear forms tend to use triangulated windows as these more easily adapt to the shape'.

- With respect to ELHY, our study also deals with 'spiral', or 'twisted', shapes in order to follow modern trends in architecture. Namely, it can be noticed that in the past decade numerous tall spiral/twisted buildings have been built worldwide. One of the remarkable examples of 'stepped twisters' (according to Vollers, 2009) is *Gakuen spiral tower* in Nagoya, built with all the characteristics of modern high rise design. Its facade has triangulated configuration of protruding wings (Vollers, 2009). When it comes to 'revolver' sub-category of generating the surface (Ibid.) the elliptic one sheeted hyperboloid can be employed.

Two systems of generatrices and circular sections derive triangular net – mesh structure. Double curved surface is tessellated with flat panes, where geometry of diagrid structures fits. Diagrid optimization for tall buildings was reported by Moon (2007) and Boake (2013), whereby the building's elementary geometry is prism like.

	SCHEME	TOP VIEW	3D MODEL VIEW
a) Triangular			
b) Squared			
c) Pentagonal			

Figure 7. ELHY fragments with straight edges, incorporated in three polygonal schemes

Since ELHY cross sections are ellipses (perpendicular to the inner principal axis) and hyperboles (sections containing inner principal axis), the other possibilities of mesh grid are also acceptable.

Two prototypes of ELHY shapes, presented in Figure 8 (top view, front view and 3D model), are 3D triangulated surface structures modelled in Auto CAD application.

## CONCLUSIONS

Practical engagement of ELHY shape in modern engineering design (architecture, civil engineering, art, science, etc.) should follow important geometric characteristics of the surface itself. Bearing in mind that the surface itself is curved but with the straight lines as generatrices, it seems likely to expect its employment in imaginative structures in a variety of patterns.

ELHY-shaped 3D model structures, presented and designed in Auto CAD software application, combining straight lines and circular sections in triangular net, can provide a variety of possibilities in modelling of structural shapes.

Such structures could be viewed from several aspects: objects dimensions, applicable constructive systems, functionality, energy efficiency and cost-effectiveness. Casale *et al.* (2013) showed such opportunities when it comes to the shape of hyperbolic-paraboloid, a close 'relative' of ELHY. The possibilities of 'transformation' of ELHY into the form of a folded structure (Nestorović, 2000; Šekularac, 2012) could be of interest in the further study of this interesting geometric shape and its implementation in modern architectural design.

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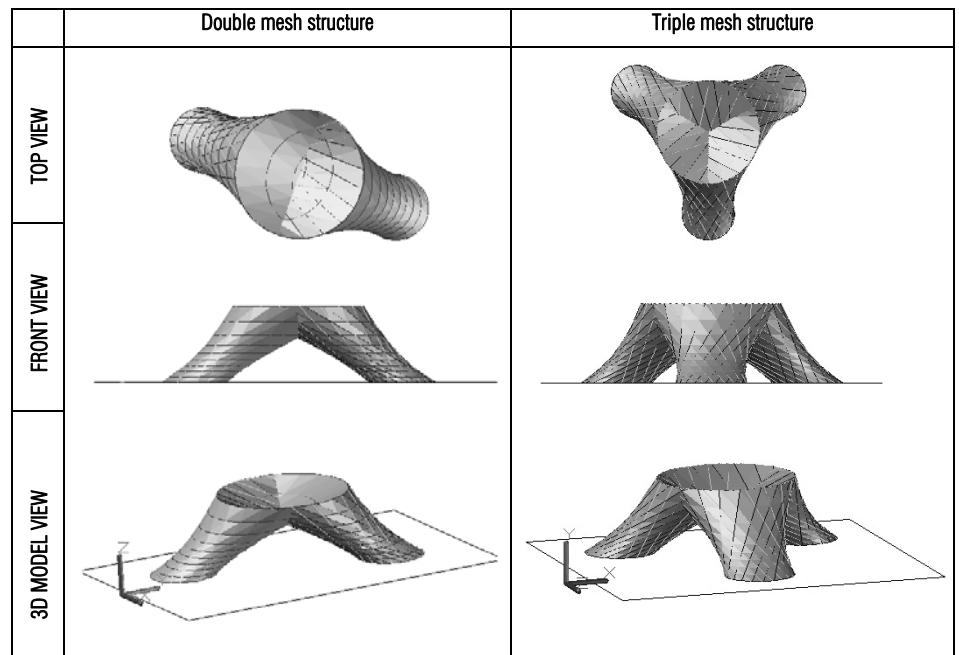


Figure 8. Two types of mesh structures ELHY shaped

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