KRITERIJUM STABILNOSTI DEFORMACIJE ELASTOPLASTIČNIH MATERIJALA

CONDITIONS FOR STABILITY OF DEFORMATION IN ELASTO-PLASTIC MATERIALS

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UVOD

Problem stabilnosti geomehaničkih materijala za tačno utvrđenu geometriju, granične uslove i karakter opterećenja, zahteva određivanje intervala parametara opterećenja, pri kojem naponsko-deformacijske veličine imaju konačne vrednosti. Gubitak stabilnosti može se posmatrati i kao početak loma materijala.

Od velikog praktičnog značaja jeste to da se odredi da li će posmatrana plastična deformacija - u trenutku kada je dostignuto ravnotežno stanje - povećati svoju dužinu za konačan (mali) inkrement i prestati s rastom (stabilan režim deformisanja) ili će nastaviti s nekontrolisanim rastom dok ne dođe do velikih priraštaja posmatranih deformacija (nestabilan režim deformisanja). Prelaz fizičkog (mehaničkog) sistema iz stabilnog režima deformisanja u nestabilan režim deformisanja često se naziva gubitak stabilnosti.

Granica ovog prelaza poznata je kao kritično stanje materijala, a njemu odgovarajuće opterećenje jeste kritično opterećenje. Potreban uslov za gubitak stabilnosti ponašanja materijala, gubitak jedinstvenosti rešenja i pojavu bifurkacije jeste promena znaka drugog izvoda funkcije rada na plastičnom deformisanju materijala (Drucker, 1950). S jedne strane, Drucker-ov postulat predviđa dovoljan uslov za stabilno ponašanje materijala, određena teorijska razmatranja ukazuju na to da je ovo možda samo potreban, ali ne i dovoljan uslov (Mroz, 1964). Ponašanje materijala postaje nestabilno kada počinje da se gubi eliptičnost tangente matrice krutosti. Ovaj početak nestabilnog ponašanja materijala povezan je s tačkom bifurkacije ili odstupanja od jedinstvenog rešenja na ravnotežnoj putanji (Mandel, 1964). Pojava

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INTRODUCTION

Stability of deformation for a geomechanical material under applied load, for defined geometry and boundary conditions, requires determination of the load interval where stress-strain properties have definite values. Loss of material stability may be considered as initiation of material failure.

From the practical point of view it is important to determine for the small increment in stress, if the plastic deformation increment was small (stable deformation occurred) or large (unstable deformation occurred). Losing the stability of deformation occurred at the boundary between these two regimes. The loading condition at the boundary is called the critical loading while material is called to be in a critical state.

The term "stability" is used here as a quasi-static response of a material under small increments of displacements. Necessary condition for the stability loss, as well as loss in unique relationship and appearance of bifurcation is the sign change in the second derivative of the work function (Drucker, 1950). Although from one point of view, Drucker's postulate provides a sufficient condition for material stability, some theoretical considerations have suggested that this is only a necessary condition (Mroz, 1964). Material instability occurs when ellipticity of the tangent stiffness matrix is lost. The beginning of unstable material behavior is connected with bifurcation point or deviation from the unique solution on the equilibrium path (Mandel, 1964). The onset of localized deformation is often associated with the satisfaction of the classical discontinuous bifurcation criterion. The localized deformation phenomena in

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lokalizovane deformacije, u klasičnom smislu diskontinuiteta, obično je povezana s pojavom tačke bifurkacije. Fenomen lokalizacije deformacije u granularnim materijalima veoma je interesantan kako sa eksperimentalne, tako i sa analitičke tačke gledišta. Klasični rezultati za elastoplastični kontinuum izraženi preko kritične vrednosti modula ojačanja i odgovarajuća orijentacija trake klizanja (*shear band*) prikazani su u radovima (Hill, 1950), (Rudnicki i sar., 1975) i (Rice, 1976). U ovim radovima definisani su statički i kinematički uslovi za početak klizanja u granularnom materijalu. Matematički precizan kriterijum za određivanje položaja kritične tačke izveden iz generalnog razmatranja svojstvenih vrednosti tangentne matrice krutosti (Borst, 1988). Ovako postavljen kriterijum za određivanje kritične tačke vrlo je pogodan sa stanovišta praktične primene. Više autora primenilo je ovu metodologiju na granularne materijale, s tim što su koristili Mohr-Coulomb-ov kriterijum kao funkciju tečenja.

U ovom radu prvo su prikazana generalna razmatranja odnosa napon-deformacija i kriterijum za određivanje kritične tačke deformacije, zatim tri ilustrativna primera, poređenje dobijenih rezultata sa eksperimentalnim vrednostima i naposletku - zaključak.

granular material are very interesting not only from the experimental point of view but numerical as well. Now classical results for elasto-plastic continuum derived from the critical values for the hardening modulus and corresponding orientation of the so called shear band were published by (Hill, 1950), (Rudnicki and Rice., 1975) and (Rice, 1976).

Static and kinematic conditions for the onset of slip (shear band) in granular materials were presented in those papers. Mathematically precise criteria to determine the critical point position is derived from general considerations of the values for the tangent of the material stiffness constants (Borst, 1988). Criteria established in such a way showed to be very useful for practical applications. Presented methodology was used by many authors, mostly employing the classical Mohr-Coulomb's type yield function.

In this paper, general points about the stress-strain relationship and the critical point deformation are given first; second two illustrative examples are given along with published experimental results; third analysis of the compared differences in critical loads and at the end are given conclusions.

KONSTITUTIVNE JEDNAČINE

U teoriji plastičnosti obično se zanemaruju uticaji brzine deformacije i temperaturnog polja. Tako dobijena teorija zove se izotermička teorija plastičnosti. Ograničićemo naša razmatranja na izotermički process $(T = cons. > 0)$. Pomeranje tačke tela označićemo sa $u(x)$, gde su sa x označene pravougle Dekartove koordinate posmatrane tačke. Usvajajući pretpostavku o malim pomeranjima, tenzor deformacije može se napisati u obliku:

CONSTITUTIVE EQUATIONS

In theory of plasticity, rate of deformation and temperature effects are usually neglected. Such considerations are called isothermal plasticity. In this work considerations are restricted to isothermal processes $(T = cons. > 0)$. The displacement of a point in the body is characterized by $u(x)$, where x are the rectangular Cartesian coordinates of the point. Following the assumption of small displacements, the strain-displacement relations are:

$$2\varepsilon_{ij} = u_{i,j} + u_{j,i} \tag{1}$$

gde zarez označava parcijalni izvod po odgovarajućoj koordinati. Veza između napona i elastične deformacije određena je Hukovim zakonom. Funkcija plastičnog potencijala za izotropan materijal može se napisati u obliku:

where comma followed by an index represents partial differentiation with respect to the corresponding coordinate. Hooks law is defining the relationship between the increments in stress and deformation. The yield potential for an isotropic material is given by the following function:

$$f = f(\sigma, q) \tag{2}$$

gde q označava određenu unutrašnju promenljivu stanja materijala (tj. njen tenzor). Uslov tečenja $f = f(\sigma, q) = 0$ definiše površ tečenja u (σ, q) prostoru. Neka veliko Q označava izvod površi tečenja u odnosu na napon, a veliko R označava izvod u odnosu na unutrašnju promenljivu q:

where q signifies suitable set of internal state variables for geomaterial (tensor). The yield condition $f = f(\sigma, q) = 0$ defines the yield surface in the (σ, q) space. Let Q denote the derivative of the yield surface with respect to the stress and R the derivative with respect to the internal variable q:

$$Q(\sigma, q) = \frac{\partial f}{\partial \sigma},\ R(\sigma, q) = \frac{\partial f}{\partial q} \tag{3}$$

Neka je P normala na površ tečenja koja označava pravac tečenja u odnosu na σ, onda ako je $P = Q$ plastično tečenje se naziva asocijativno, dok se za slučaj

If P is normal to the yield surface and describes the flow direction with respect to σ, then if $P = Q$ the plastic flow rule is called associative, where as if $P \neq Q$

gde $P \neq Q$ plastično tečenje naziva ne-asocijativno. Plastični deo priraštaja deformacije označen je kao:

the plastic flow is called non-associative. The plastic part of the strain rate is expressed as:

$$\dot{\varepsilon}^p = \lambda P(\sigma, q),\ \lambda > 0 \tag{4}$$

gde je λ arbitrarni faktor proporcionalnosti. Pozitivni znak za λ je usled činjenice da plastično tečenje uključuje gubitak energije. Tačka iznad promenljive označava puni izvod promenljive u odnosu na vreme, koji je usled pretpostavke malih brzina isti kao i parcijalni izvod u odnosu na vreme. Efekti disipacije u materijalu koji prate deformaciju mogu se uzeti u obzir preko unutrašnje promenljive stanja. Brzina promene unutrašnje promenljive q predstavljena je nelinearnom funkcijom h koja sadrži σ i q,

where λ is an arbitrary factor of proportionality. The positive sign of λ is due to the fact that plastic flow involves dissipation of energy. A dot above the variable denotes the total derivative of that variable with respect to time which, under the approximation of small velocities is the same as the partial derivative with respect to time. The dissipative effects which accompany deformation may be accounted for by the use of internal state variables. The rate of change of the q is represented with a non-linear function h which includes the state variable σ and the complete internal state q,

$$\dot{q} = \lambda h(\sigma, q) \tag{5}$$

Za infinitezimalne deformacije možemo izraziti tenzor deformacije kao zbir tenzora elastičnih i plastičnih deformacija:

The infinitesimal macro strain field may be divided in accordance with the following equation:

$$\varepsilon = \varepsilon^e + \varepsilon^p \tag{6}$$

gde indeks e označava elastičan deo, i indeks p označava plastičnu promenu u polju deformacija kada se unutrašnja koordinata q promeni u $q + \dot{q}$. Razmatrajući materijale kod kojih se konstitutivno ponašanje može idealizovati u segmentno-linearnoj formi:

where superscript e stands for the elastic part, and superscript p denotes inelastic change in a strain field during which q changes into $q + \dot{q}$. Considering the class of materials whose constitutive behaviour may be idealised as a piecewise-linear relation of the form:

$$\dot{\sigma} = L : \dot{\varepsilon} \tag{7}$$

gde je L konstitutivni tenzor materiala, (:) označava dijadu. Jedna forma konstitutivne jednačine (6) može se izraziti:

where L is the tangent-compliance tensor of the material, (:) signifies the dyadic product. One particular form of the constitutive rate relation (6) may be expressed as:

$$\dot{\sigma} = C : (\dot{\varepsilon} - \frac{1}{R \cdot h} P(Q : \dot{\sigma})) \tag{8}$$

gde je C tenzor inkrementa modula elastičnosti, h - brzina promene ojačavanja ili omekšavanja. Desna strana jednačine (7) može se napisati u odnosu na $\dot{\varepsilon}$, i poređenjem s jednačinom (6) dobija se sledeći odnos:

where C is the tensor of incremental elastic module, h is the rate of hardening or softening. The right-hand side of equation (7) may be written in terms of $\dot{\varepsilon}$, and compared with equation (6) to make the following evident:

$$L = C - \frac{(C : P)(Q : C)}{Q : C : P - R \cdot h} \tag{9}$$

Ako se funkcija f može razviti u Tejlorov red do drugog stepena oko trenutnih vrednosti za deformaciju i plastične promenljive, može se dobiti sledeći izraz:

If the function f can be expanded in Taylor series to second order around the current values of the strain and plastic variables, then it can be used to obtain the following:

$$\begin{aligned} f(\varepsilon + \dot{\varepsilon}^*, q + \dot{q}^*) &= f(\varepsilon, q) + \frac{\partial f}{\partial \sigma} : \left(L : \dot{\varepsilon}^*\right) + \frac{\partial f}{\partial q} \cdot \dot{q}^* + \\ &\frac{1}{2} \cdot \frac{\partial^2 f}{\partial \sigma \partial \sigma} : \left(L : \dot{\varepsilon}^*\right) : \left(L : \dot{\varepsilon}^*\right) + \frac{\partial^2 f}{\partial \sigma \partial q} : \left(L : \dot{\varepsilon}^*\right) \cdot \dot{q}^* + \frac{1}{2} \cdot \frac{\partial^2 f}{\partial q \partial q} \cdot \dot{q}^* \cdot \dot{q}^* \end{aligned} \tag{10}$$

gde su svi izvodi određeni na ε, q. Promene u $\varepsilon + \dot{\varepsilon}^*$ mogu se posmatrati kao varijacije u $\dot{\varepsilon}^*$, gde je $\dot{\varepsilon}^* = \varepsilon_{,u}(\delta u)$ prva promena u (nelinearnom) operatoru za deformaciju-pomeranje $\varepsilon(u)$ u odnosu na u. Sledeće veličine uvedene su da bi se jednačina (10) pojednostavila:

where all derivatives are evaluated at ε, q. The variations in $\varepsilon + \dot{\varepsilon}^*$ may be treated as variations in $\dot{\varepsilon}^*$, where $\dot{\varepsilon}^* = \varepsilon_{,u}(\delta u)$ is the first variation of the (non-linear) strain-displacement operator $\varepsilon(u)$ with respect to u. The following quantities are defined in order to express equation (10) in a simplified manner:

$$T(\sigma,q)=\frac{\partial^2 f}{\partial\sigma\partial\sigma},U(\sigma,q)=\frac{\partial^2 f}{\partial\sigma\partial q},V(\sigma,q)=\frac{\partial^2 f}{\partial q\partial q} \tag{11}$$

gde je T simetrični tenzor četvrtog stepena, U je simetrični tenzor drugog stepena, a V – scalar. Sada, jednačina (10) ima sledeći oblik:

where T is a symmetric fourth-order tensor, U is a symmetric second-order tensor and V is a scalar. Now, equation (10) may be written as:

$$\begin{aligned} f = f + Q:\left(L:\dot{\varepsilon}^*\right) + R\cdot\dot{q}^* + 1/2\cdot T:(L:\dot{\varepsilon}^*):(L:\dot{\varepsilon}^*) + \\ U:(L:\dot{\varepsilon}^*)\cdot\dot{q}^* + 1/2\cdot V\cdot\dot{q}^*\cdot\dot{q}^* \end{aligned} \tag{12}$$

Treći član u jednačini (12) zove se drugi osnovni kvadratni oblik površi tečenja.

The third member in the equation (12) is called the second basic quadratic form of yield surface.

ODREĐIVANJE KRITIČNE TAČKE

U analizi stabilnosti deformacije, određivanje kritične tačke jeste centralni problem. T,U,V, izrazi određeni relacijama (11), koeficijenti su druge osnovne kvadratne forme površi tečenja $f = f(\sigma, q)$. Izraz za normalnu krivinu posmatrane površi, možemo napisati u sledećem obliku:

CRITICAL POINT DETERMINATION

In the analysis of stability of deformation, the critical point determination is the central problem. T,U,V are the coefficient of the yield surface $f = f(\sigma, q)$ given by equations (11). The normal curvature of the considered surface could be written in accordance with (12) as:

$$\frac{1}{\rho} = \frac{T:(L:\dot{\varepsilon}^*):(L:\dot{\varepsilon}^*) + 2U:(L:\dot{\varepsilon}^*)\cdot\dot{q}^* + V\cdot\dot{q}^*\cdot\dot{q}^*}{\left[Q:\left(L:\dot{\varepsilon}^*\right) + R\cdot\dot{q}^*\right]^2} \tag{13}$$

Znak normalne krivine zavisi samo od znaka brojioca izraza na desnoj strani gornje jednačine. U proučavanju znaka normalne krivine, prema tome, mogu nastupiti tri slučaja:

Its sign depends only on the sign of the numerator because denominator is always positive. There are three possible cases:

$$1. \quad \delta = U^2 - T\cdot V < 0. \tag{14}$$

Imenilac izraza na desnoj strani uvek je pozitivna veličina, jer predstavlja kvadrat. Tada je brojilac potpuni kvadrat i ne menja znak, tj. krivina opet ostaje stalnog znaka u svim pravcima, ali postoji jedan pravac u kome je krivina jednaka nuli. Ovakve tačke površine zovu se eliptične, a za površinu se kaže da u takvoj tački ima eliptičnu krivinu. Znak krive uvek je pozitivan i prema koeficijentu T.

In the first case, the numerator does not change sign and curve has the same sign in all possible directions from the given point. In other word, normal vector to the possible cross-sections at a given point have the same directions. These points are called elliptical. The sign of the curve is always positive and towards the coefficient T.

$$2. \quad \delta = U^2 - T\cdot V = 0. \tag{15}$$

Brojilac ne menja znak i krivina normalnih preseka u svim pravcima ima isti znak, izuzev u tački gde je nula. Takve tačke površine zovu se paraboličke tačke.

The numerator still does not change sign and the curve has the same sign in all directions, except the one where it is equal to zero. These points are called parabolic.

$$3. \quad \delta = U^2 - T\cdot V > 0. \tag{16}$$

Brojilac menja znak, tj. u posmatranoj tački postoje glavne normale suprotnih smerova za normalne preseke raznih pravaca. Normalna krivina u istoj tački u tom slučaju za razne pravce može biti pozitivna i negativna, a postoje dva pravca u kojima je krivina nula. Takva tačka površi zove se hiperbolička, a površina u toj tački ima hiperboličku krivinu. Klasifikacija tačaka jedne površi, odnosno utvrđivanje oblika površi u okolini posmatrane tačke, jasno ističe značaj i druge osnovne kvadratne forme površi. Na ovaj način, posmatranjem ravnotežne putanje sistema, odnosno analizom njegovog ponašanja u okolini kritične tačke, direktno se formulišu uslovi za egzistenciju granične tačke. Kompletan nelinearan odgovor jednog elastoplastičnog sistema formuliše njegovu ravnotežnu putanju, određujući pri tome i odgovarajuće singularne vrednosti. Zadatak stabilnosti definisan je putem oblasti dopustivih deformacija posmatranog elastoplastičnog materijala. Sračunavanjem diskretnih tačaka ravnotežne krive, zajedno sa odgovarajućim vrednostima δ, definisane gornjim izrazima, nailazimo i na tačku u kojoj se menja znak ove veličine. Pri vrednosti funkcije $\delta > 0$, relacija postaje i potreban i dovoljan uslov za stabilnost ravnotežnog položaja posmatranog sistema za vreme opterećenja.

In this case, numerator changes sign at the given point for different normal vectors. The sign of the curve can be either positive or negative, with two directions where the sign is equal to zero. These points are called the hyperbolic points. This approach allows the determination of the value for the critical loading. The complete non-linear response of an elasto-plastic system defines its deformation path. Stability will define its allowed deformations. Calculation of discrete points along the stability path may lead to the change of the δ values in the normal curvature sign. The condition $\delta > 0$ becomes necessary and sufficient for the stability of deformation. With the analysis of the stability path around the critical points, the limiting conditions become directly formulated.

PRIMERI ZA ILUSTRACIJU

U radu (Lelović, 2012) razmatrane su različite naponske putanje u trijaksijalnom opitu, za slučaj primene konstitutivnih jednačina HISS modela (Desai i sar., 1987), za granularni materijal. Invarijante napona uvedene su prema sledećim izrazima (naponi i deformacije su pozitivni u kompresiji):

ILLUSTRATIVE EXAMPLES

Different loading paths and constitutive relations from the hierarchical single surface (HISS) [14] model for granular materials are described in this section. In order to describe the yield surface, the stress invariants are

$$I_1 = \sigma_{kk}, I_{2d} = \tfrac{1}{2} S_{ij} S_{ij}, I_{3d} = \tfrac{1}{3} S_{ij} S_{jk} S_{ki} \tag{17}$$

gde je I_1 prva invarijanta napona, a I_{2d} i I_{3d} – druga i treća invarijanta devijatora napona.

U praktičnim proračunima se druga invarijanta koristi za definisanje oktaedarskog smičućeg napona Toct, kao sto se može videti iz sledeće jednačine

where I_1 is the first invariant of the stress tensor and I_{2d} and I_{3d} are the second and third invariants of the deviatory stress tensor. For most practical calculations second invariant is used to define octahedral shear stress τ_{oct} as shown in the following relationship:

$$\tau_{\mathrm{oct}}^2 = 2/3 \cdot \mathrm{I}_{2\mathrm{d}} \tag{17a}$$

Funkcija tečenja opšteg tipa (Desai i sar., 1987), f za asocijativnu plastičnost sa izotropnim ojačanjem je oblika:

The model is based on general yield potential, f, for the associative, isotropic hardening plasticity is given by

$$f = p_a^{-2} I_{2d} - F_b \cdot F_s = 0 \tag{18a}$$

gde je:

where:

$$F_b = -\alpha \left(\frac{I_1}{p_a} \right)^n + \gamma \left(\frac{I_1}{p_a} \right)^2, \; F_s = \left(1 - \beta S_r \right)^m \tag{18b}$$

S_r je odnos napona dat u obliku $S_r = 3\sqrt{3}/2\, I_{2d}^{-\frac{3}{2}} I_{3d}$, α je funkcija ojačanja, γ i β su parametri materijala, a p_a je atmosferski pritisak. F_b je osnovna funkcija koja prikazuje oblik funkcije tečenja u

S_r is the stress ratio such that $S_r = \frac{3\sqrt{3}}{2} I_{2d}^{-\frac{3}{2}} I_{3d}$, α is hardening function, γ and β are material parameters, and p_a is atmospheric pressure. F_b is the basic function representing the shape of the yield prostoru $I_1 - \sqrt{I_{2d}}$, a F_s je funkcija oblika u oktaedarskoj ravni. Funkcija ojačanja je data izrazom:

function in the $I_1 - \sqrt{I_{2d}}$ space, F_s is the shape function which represents the shape in the octahedral plane. The hardening relation is given by

$$\alpha = a_1 \xi^{-\eta} \tag{19}$$

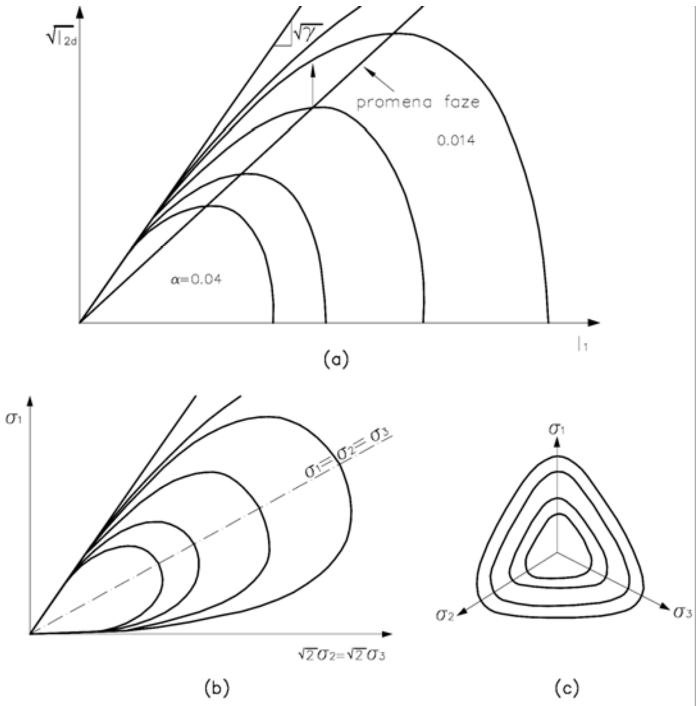
gde su a_1 i η materijalni parametri za model izotropnog ojačanja, ξ je putanja plastične deformacije. Ukupna efektivna plastična deformacija ξ definisana je kao preko promene

where a_1 and η are material parameters for the hardening behaviour, ξ is the trajectory of the plastic strains. The accumulated effective plastic strain, ξ, is defined by the rate

$$\dot{\xi} = \sqrt{{}^p\dot{e}_{ij}\,{}^p\dot{e}_{ij}} \tag{20}$$

gde se $m = -0.5$ uzima za većinu (geotehničkih) materijala. Ovaj model uključuje osam materijalnih konstanti, od kojih su dve konstante elastične.

With $m = -0.5$ for most (geotechnical) materials. This model involves eight constants including two elastic. Factors such as non associative response, anisotropy, cycling loading, damage and softening are not considered.



Slika 1. Funkcija tečenja HISS (Desai,1986): (a) u prostoru $I_1 - \sqrt{I_{2d}}$; (b) u trijaksijalnoj ravni; (c) u devijatorskoj ravni

Figure 1. Yield function HISS (Desai,1986): (a) in space $I_1 - \sqrt{I_{2d}}$; (b) triaxial plane; (c) deviatoric plane

Iz relacije (18) eksplicitni izrazi za gradijente funkcije tečenja dobijaju se diferenciranjem

From relationship (18) the explicit expressions for the gradients of yield function are found by differentiation as follows

$$\frac{\partial f}{\partial \sigma_{ij}} = p_a^{-1} F_s \left[n\alpha \left(I_1/p_a \right)^{n-1} - 2\gamma \left(I_1/p_a \right) \right] \delta_{ij} + \left(p_a^{-2} - 1.5 m F_s^{\frac{m-1}{m}} F_b S_r I_{2d}^{-1} \right) S_{ij} + m F_s^{\frac{m-1}{m}} F_b A_{ij} \tag{21a}$$

$$\frac{\partial f}{\partial \xi} = -a_1 \eta \xi^{-(\eta+1)} \left(I_1/p_a\right)^n F_s \tag{21b}$$

$$\begin{aligned}
&\frac{\partial f}{\partial \sigma_{ij}\partial \sigma_{kl}} = p_a^{-2} Fs\left[n\left(n-1\right)\alpha\left(I_1/p_a\right)^{n-2} - 2\gamma\right]\delta_{ij}\delta_{kl} + \\
&+p_a^{-1} mFs^{\frac{m-1}{m}}\left[n\alpha\left(I_1/p_a\right)^{n-1} - 2\gamma\left(I_1/p_a\right)\right]\cdot\left[3/2\, SrI_{2d}^{-1}\left(\delta_{ij}S_{kl} + S_{ij}\delta_{kl}\right) - \left(\delta_{ij}A_{kl} + A_{ij}\delta_{kl}\right)\right] + \\
&+3/4\, mFs^{\frac{m-1}{m}} FbI_{2d}^{-1}\left\{SrI_{2d}^{-1}\left[5 - 3\left(m-1\right)Fs^{-\frac{1}{m}}Sr\right]S_{ij}S_{kl} + 2\left(Fs^{-\frac{1}{m}}Sr - 1\right)\left(S_{ij}A_{kl} + A_{ij}S_{kl}\right)\right\} - \\
&-m\left(m-1\right)Fs^{\frac{m-2}{m}} FbA_{ij}A_{kl} + \left(p_a^{-2} - 3/2\, mFs^{\frac{m-1}{m}} FbSrI_{2d}^{-1}\right)\left(\delta_{ik}\delta_{jl} - \tfrac{1}{3}\delta_{ij}\delta_{kl}\right) + \\
&+\tfrac{3\sqrt{3}}{2} mFs^{\frac{m-1}{m}} Fb\beta I_{2d}^{-\frac{3}{2}}\left[\delta_{ik}S_{jl} + S_{ik}\delta_{jl} - 2/3\left(S_{ij}\delta_{kl} + \delta_{ij}S_{kl}\right)\right]
\end{aligned} \tag{21c}$$

$$\frac{\partial^2 f}{\partial \xi \partial \sigma_{ij}} = -a_1 \eta \xi^{-(\eta+1)}\left[p_a^{-1} n\left(I_1/p_a\right)^{n-1} Fs\delta_{ij} + \left(I_1/p_a\right)^n mFs^{\frac{m-1}{m}}\left(3/2\, SrI_{2d}^{-1}S_{ij} - A_{ij}\right)\right] \tag{21d}$$

$$\frac{\partial^2 f}{\partial \xi^2} = a_1 \eta(\eta+1)\xi^{-(\eta+2)}\left(I_1/p_a\right)^n Fs \tag{21e}$$

gde je uvedena sledeća oznaka:

Where the following parameter is introduced:

$$A_{ij} = \tfrac{3\sqrt{3}}{2}\beta I_{2d}^{-\frac{3}{2}}\left(S_{ik}S_{kj} - \tfrac{2}{3} I_{2d}\delta_{ij}\right) \tag{21f}$$

Pretpostavili smo da nema drugih napona izuzev normalnih, tako da su normalni naponi u ovom slučaju i glavni. Ako sa σ_{xx} označimo aksijalni napon, a sa σ_{yy} i σ_{zz} bočne napone, onda ćemo odgovarajuće veze izraziti prema ovim oznakama. Materijal je prvo opterećen hidrostatičkim pritiskom od $\sigma_0 = 90\left[kPa\right]$, pri atmosferskom pritisku $p_a = 101\left[kPa\right]$. U ovom slučaju, jednačina (15) postaje

The assumption made was that only normal stresses exist in this case. σ_{xx} designates axial stresses and σ_{yy} and σ_{zz} designate side stresses. First, the material was loaded under hydrostatic pressure of $\sigma_0 = 90\left[kPa\right]$, when the atmospheric pressure was $p_a = 101\left[kPa\right]$. In this case equation (15) becomes

$$\overline{\delta} = 2\gamma\left[n^2 \frac{\eta}{\eta+1} + n\left(n-1\right) - 1\right] \tag{22}$$

Materijalne konstante za pesak – Leighton Buzzard Sand (Desai, 1989) date su u Tabeli I.

Material constants for sand (Leighton Buzzard Sand) are given in Table I (Desai, 1989).

Tabela I. Parametri za HISS model
Table I. Parameters for HISS model

$E\left[MPa\right]$	ν	β	γ	n	a_1	η	m
103.8	0.29	0.442	0.089	3	0.00018	0.85	-0.5

Ilustracija # 1: Konvencionalni trijaksijalni pritisak (CTC). Za CTC stanje komponente napona i devijatora napona date su sledećim izrazima

Case # 1: Conventional triaxial compression (CTC) state. For the CTC state of stress components of the stress tensor and the stress deviator are as follows

$$\sigma^{CTC} = \begin{bmatrix} \sigma_0 + \sigma_{xx} & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}, \quad S^{CTC} = \frac{\sigma_{xx}}{3}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{23}$$

Uvodeći jednačinu (23) u (21) i zamenjujući u (15), dobija se sledeći izraz:

Introducing (23) into (21) and substituting this equation in the (15), leads to:

$$\delta=\left(1-2k\right)^{2} n^{2} \frac{p_a^{-2} a_1^{2} \eta^{2} \xi^{-2(\eta+1)}}{(1-\beta)} \left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{2(n-1)} + \frac{p_a^{-2} a_1 \eta(\eta+1) \xi^{-(\eta+2)}}{(1-\beta)} \left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n} \cdot$$

$$\left\{\left[\frac{2\sqrt{1-\beta}}{3}+n\left(n-1\right)\alpha\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n-2}-2\gamma\right]-\right.$$

$$-4k\left[-\frac{\sqrt{1-\beta}}{3}+n\left(n-1\right)\alpha\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n-2}-2\gamma\right]+ \tag{24a}$$

$$\left.2k^{2}\left[\frac{\sqrt{1-\beta}}{3}+2n\left(n-1\right)\alpha\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n-2}-4\gamma\right]\right\}$$

gde je:

where:

$$k=\frac{\left[\left(c_1^{2}+1/3c_1-2/9\right)-\nu c_2\right]}{\left[\left(c_1+2/3\right)^{2}+c_2\right]}\cdot\frac{\left\{\left[2c_1^{2}+2/3\cdot c_1+\frac{5-4\nu}{9(1+\nu)}\right]+\left(1-\nu\right)c_2\right\}}{\left[2\left(c_1-1/3\right)^{2}+\left(1-\nu\right)c_2\right]} \tag{24b}$$

$$c_1=\left(\frac{3\sigma_0}{\sigma_{xx}}+1\right)\left[n\alpha\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n-2}-2\gamma\right]\frac{1}{\sqrt{1-\beta}} \tag{24c}$$

$$c_2=\frac{a_1\eta p_a^{2}}{\xi^{(\eta+1)}E\sigma_{xx}}\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n}\cdot\sqrt{\left(\frac{3\sigma_0}{\sigma_{xx}}+1\right)^{2}\left[n\alpha\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n-2}-2\gamma\right]^{2}\frac{3}{\left(1-\beta\right)^{3}}+\frac{2}{3(1-\beta)}} \tag{24d}$$

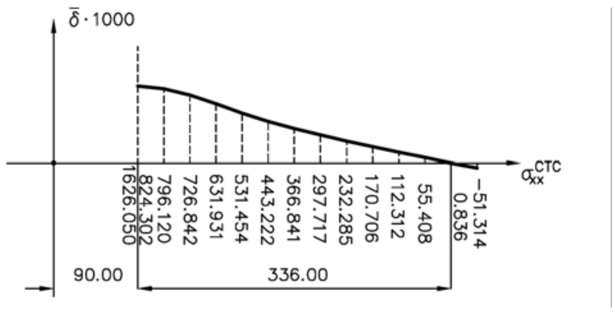
Nakon nekoliko transformacija, jednačina (24) se može prikazati u formi

After few simple transformations the equation (24) can be written in the form

$$\bar{\delta}=\left[n^{2}\frac{\eta}{\eta+1}+n\left(n-1\right)\right]\alpha\left(\frac{3\sigma_0+\sigma_{xx}}{p_a}\right)^{n-2}-2\gamma+\left(\frac{1+k}{1-2k}\right)^{2}\frac{2\sqrt{1-\beta}}{3}=0 \tag{25}$$

Rešenje karakteristične jednačine stabilnosti (25) prikazano je na slici 2. Najnepovoljnije ravni - u pogledu kritičnog naponskog stanja - biće one kod kojih ugao između totalnog napona i normale na ravan ima najveću vrednost.

Solution to the characteristic equation of stability of deformation (25) is shown in Fig. 2. The least favorable planes, from the critical stress point of view, are those at which the angle between the total stress and normal vector to the plane has the maximum value.



Slika 2. Konvencionalna trijaksijalna kompresija - rešenje jednačine stabilnosti
Figure 2. Case of conventional triaxial compression - solution for stability equation

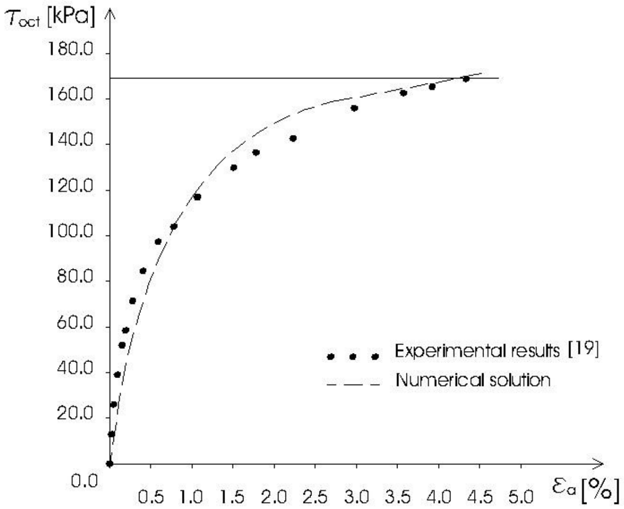
Kao što se vidi s dijagrama, nagib krive je negativan, ali je vrednost δ pozitivna (uslov dat izrazom (16)) i postoji ravnomernost u opadanju u tom pravcu ka vrednosti gde je krivina jednaka nuli. U tački gde je $\delta=0$ (15), ta tačka površine je nazvana parabolička. Na osnovu prikazanog dijagrama, jasno se vidi da je kritična vrednost sračunata numeričkim postupkom $\sigma_{xx}^{CTC}=336\left[kPa\right]$. Eksperimentalni podaci prikazani su na slici 3. (na osnovu podataka prikazanih u Referenci

As can be seen from the diagram, slope of the curve is negative but δ is positive (condition given by the equation $\delta=U^{2}-T\cdot V\succ 0$) with relative equality in stress increment towards the value where y-axis approaches zero. At the point where y-axis is zero, that point is called parabolic ($\delta=U^{2}-T\cdot V=0$). As clearly stated on the diagram, critical stress value for the numerical model $\sigma_{xx}^{CTC}=336\left[kPa\right]$.

Experimental data is shown in Figure 3. (based on

[Desai, 1989]).

the results published in Reference 3).



Slika 3. Poređenje eksperimentalnih i numeričkih rezultata za CTC test.
Figure 3. Comparing experimental and numerical results for CTC case

Iz priloženih podataka, uočava se da je kritična vrednost oktaedarskog napona smicanja $\tau_{oct}^{CTC} = 168,90\left[kPa\right]$, odnosno koristeći izraz $\tau_{oct} = \sqrt{2/3 \cdot I_{2d}}$ može se odrediti vrednost kritičnog normalnog napona $\sigma_{xx}^{CTC} = 358,30\left[kPa\right]$.

Ilustracija # 2: Test trijaksijalne kompresije (TC). Za test trijaksijalne kompresije komponente napona i komponente devijatora napona mogu se predstaviti u obliku:

From these results follows that critical value for octahedral shear stress was $\tau_{oct}^{CTC} = 168.90\left[kPa\right]$, which leads to critical normal stress by substituting into equation (17a), $\sigma_{xx}^{CTC} = 358.30\left[kPa\right]$.

Case # 2: Triaxial compression state (TC). For the TC state of stress components of the stress tensor and the stress deviator are as follows

$$\sigma^{TC} = \begin{bmatrix} \sigma_0 + \sigma_{xx} & 0 & 0 \\ 0 & \sigma_0 - 1/2\,\sigma_{xx} & 0 \\ 0 & 0 & \sigma_0 - 1/2\,\sigma_{xx} \end{bmatrix}, S^{TC} \Rightarrow \frac{\sigma_{xx}}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad (26)$$

Uvodeći jednačinu (26) u (21) i zamenjujući u (15), dobija se sledeći izraz:

Introducing (26) into (21) and substituting this equation in the (15), leads to:

$$\begin{aligned} \delta = &\left(1-2k\right)^2 n^2 \frac{p_a^{-4} a_1^2 \eta^2 \xi^{-2(\eta+1)}}{(1-\beta)^{\frac{3}{2}}}\left(\frac{3\sigma_0}{p_a}\right)^{2(n-1)} + \frac{p_a^{-4} a_1 \eta(\eta+1)\xi^{-(\eta+2)}}{(1-\beta)^{\frac{3}{2}}}\left(\frac{3\sigma_0}{p_a}\right)^{n} \cdot \\ &\left\{\left[\frac{2\sqrt{1-\beta}}{3} + n\left(n-1\right)\alpha\left(\frac{3\sigma_0}{p_a}\right)^{n-2} - 2\gamma\right] - \left[\frac{2\sqrt{1-\beta}}{3} + n\left(n-1\right)\alpha\left(\frac{3\sigma_0}{p_a}\right)^{n-2} - 2\gamma\right]\right. \\ &\left. +2k^2\left[\frac{\sqrt{1-\beta}}{3} + 2n\left(n-1\right)\alpha\left(\frac{3\sigma_0}{p_a}\right)^{n-2} - 4\gamma\right]\right\} \end{aligned} \qquad (27a)$$

gde je:

where:

$$k = \frac{\left[\frac{3}{2}(2c_1-1) - \left(1+\nu\right)c_2\right]\left[2c_1^2 + c_1 + \frac{5-4\nu}{4(1+\nu)} + \left(1-\nu\right)c_2\right]}{\left[3(c_1+1) + 2\left(1+\nu\right)c_2\right]\left[2c_1^2 - 2c_1 + \frac{1}{2} + \left(1-\nu\right)c_2\right]} \qquad (27b)$$

$$c_1 = \frac{3\sigma_0}{\sigma_{xx}}\left[n\alpha\left(\frac{3\sigma_0}{p_a}\right)^{n-2} - 2\gamma\right]\frac{1}{\sqrt{1-\beta}} \qquad (27c)$$

$$c_2 = \frac{a_1 \eta p_a^2}{\xi^{(\eta+1)} E \sigma_{xx}} \left(\frac{3\sigma_0}{p_a}\right)^n \sqrt{\left(\frac{3\sigma_0}{\sigma_{xx}}\right)^2 \left[n\alpha\left(\frac{3\sigma_0}{p_a}\right)^{n-2} - 2\gamma\right]^2 \frac{3}{(1-\beta)^3} + \frac{3}{4(1-\beta)}} \tag{27d}$$

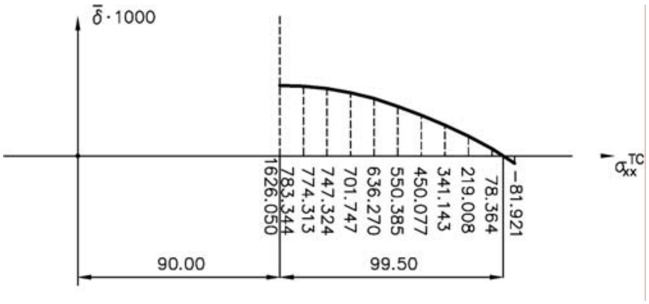
Nakon transformacija, jednačina (27) može se prikazati u formi

After few simple transformations the equation (27) can be written in the form

$$\bar{\delta} = \left[n^2 \frac{\eta}{\eta+1} + n(n-1)\right]\alpha\left(\frac{3\sigma_0}{p_a}\right)^{n-2} - 2\gamma + \left(\frac{1+k}{1-2k}\right)^2 \frac{2\sqrt{1-\beta}}{3} = 0 \tag{28}$$

Rešenje karakteristične jednačine stabilnosti (28) prikazano je na slici 4.

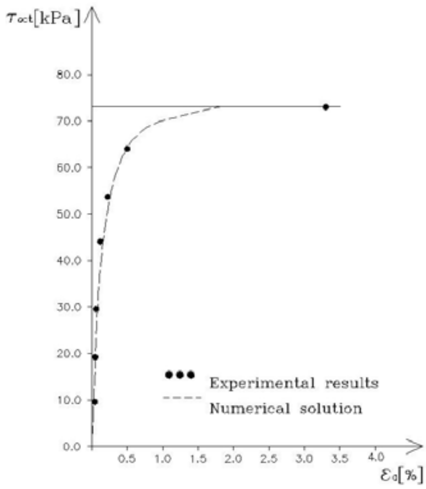
Solution to the characteristic equation of stability of deformation (28) is shown in Fig. 4.



Slika 4. Primer trijaksijale kompresije -Rešenje jednačine stabilnosti
Figure 4. Case of triaxial compression - solution for stability equation

U tački gde je $\delta = 0$ (15), na osnovu prikazanog dijagrama, kritična vrednost aksijalnog napona sračunata numeričkim postupkom jeste $\sigma_{xx}^{TC} = 99,50\left[kPa\right]$. Eksperimentalni podaci prikazani su na slici 5. (na osnovu podataka prikazanih u ref. [Desai, 1989]). Iz priloženih podataka se vidi da je kritična vrednost oktaedarskog napona smicanja $\tau_{oct}^{TC} = 73,10\left[kPa\right]$, odnosno, korišćenjem izraza $\tau_{oct} = \sqrt{2/3 \cdot I_{2d}}$, može se odrediti vrednost kritičnog normalnog napona $\sigma_{xx}^{TC} = 103,36\left[kPa\right]$.

At the point where $\delta = 0$ (15) (for zero y-axis) as clearly shown on the diagram, critical stress value calculated using the numerical model is $\sigma_{xx}^{TC} = 99.50\left[kPa\right]$. Experimental data is shown in Figure 5 (based on published results from reference Desai 1989). From these results follows that critical value for the octahedral shear stress was $\tau_{oct}^{TC} = 73.10\left[kPa\right]$, which leads to critical normal stress of $\sigma_{xx}^{TC} = 103.36\left[kPa\right]$.



Slika 5. Poređenje eksperimentalnih i numeričkih rezultata za TC test
Figure 5. Comparing experimental and numerical results for TC case

Ilustracija # 3: Test trijaksijalne ekstenzije (TE). Za stanje TE komponentalni naponi i devijator napona dati su sledećim izrazima

Case # 3: Triaxial extension state (TE). For the TE state of stress components of the stress tensor and the stress deviator are as follows

$$\sigma^{TE} = \begin{bmatrix} \sigma_0 - \sigma_{xx} & 0 & 0 \\ 0 & \sigma_0 + \sigma_{xx}/2 & 0 \\ 0 & 0 & \sigma_0 + \sigma_{xx}/2 \end{bmatrix}, \; S^{TE} = \frac{\sigma_{xx}}{2}\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{29}$$

Uvođenjem jednačine (29) u (21) i zamenom u jednačinu (15), dobija se:

Introducing equation (29) into (21) and substituting this equation in the (15), leads to:

$$\begin{aligned} \delta = {} & \left(1-2k\right)^2 n^2 \frac{p_a^{-2} a_1^2 \eta^2 \xi^{-2(\eta+1)}}{(1+\beta)} \left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{2(n-1)} + \frac{p_a^{-2} a_1 \eta(\eta+1) \xi^{-(\eta+2)}}{(1+\beta)} \left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{n} \cdot \\ & \left\{\left[\frac{2\sqrt{1+\beta}}{3} + n\left(n-1\right)\alpha\left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{n-2} - 2\gamma\right] - 4k\left[-\frac{\sqrt{1+\beta}}{3} + n\left(n-1\right)\alpha\left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{n-2} - 2\gamma\right] \right. \\ & \left. + 2k^2\left[\frac{\sqrt{1+\beta}}{3} + 2n\left(n-1\right)\alpha\left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{n-2} - 4\gamma\right]\right\} \end{aligned} \tag{30a}$$

gde je:

where:

$$k = \frac{\left[\left(c_1^2 - \frac{1}{3}c_1 - \frac{2}{9}\right) - \nu c_2\right]}{\left[\left(c_1 - \frac{2}{3}\right)^2 + c_2\right]} \cdot \frac{\left\{\left[2c_1^2 - \frac{2}{3}c_1 + \frac{5-4\nu}{9(1+\nu)}\right] + \left(1-\nu\right)c_2\right\}}{\left[2\left(c_1 + \frac{1}{3}\right)^2 + \left(1-\nu\right)c_2\right]} \tag{30b}$$

$$c_1 = \left(\frac{3\sigma_0}{\sigma_{xx}} - 1\right)\left[n\alpha\left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{n-2} - 2\gamma\right]\frac{1}{\sqrt{1+\beta}} \tag{30c}$$

$$c_2 = \frac{a_1 \eta p_a^2}{\xi^{(\eta+1)} E \sigma_{xx}}\left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^n \cdot \sqrt{\left(\frac{3\sigma_0}{\sigma_{xx}} - 1\right)^2\left[n\alpha\left(\frac{3\sigma_0-\sigma_{xx}}{p_a}\right)^{n-2} - 2\gamma\right]^2 \frac{3}{(1+\beta)^3} + \frac{2}{3(1+\beta)}} \tag{30d}$$

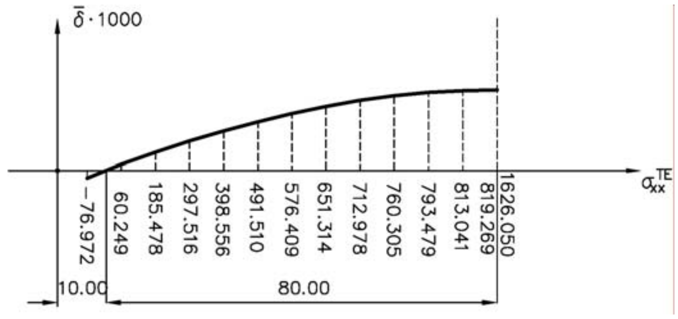
Nakon nekoliko jednostavnih transformacija, (30) može se napisati u formi:

After some simple transformation the equation (30) can be written in the form:

$$\bar{\delta} = \left[n^2 \frac{\eta}{\eta+1} + n\left(n-1\right)\right]\alpha\left(\frac{3\sigma_0 - \sigma_{xx}}{p_a}\right)^{n-2} - 2\gamma + \left(\frac{1+k}{1-2k}\right)^2 \frac{2\sqrt{1+\beta}}{3} = 0 \tag{31}$$

Rešenje karakteristične jednačine (31) prikazano je na slici 6.

Solution to the characteristic equation of stability of deformation (31) is shown in Figure 6.



Slika 6. Primer trijaksijale ekstenzije - Rešenje jednačine stabilnosti
Figure 6. Case of triaxial extension - solution for stability equation.

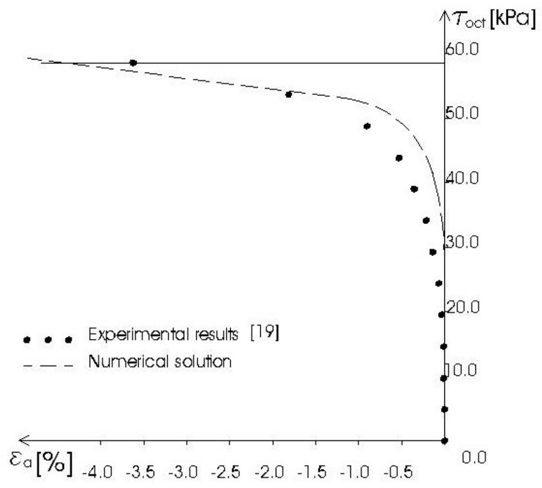
Kao što se vidi s dijagrama, nagib krive je pozitivan (uslov dat izrazom $\delta \succ 0$).

Na osnovu prikazanog dijagrama, numerički sračunata kritična vrednost napona jeste $\sigma_{xx}^{TE} = 80\left[kPa\right]$.

As can be seen from the diagram, slope of the curve is positive (condition $\delta = \; U^2 - T \cdot V \succ 0$). As clearly stated on the diagram, critical stress value for the numerical model $\sigma_{xx}^{TE} = 80\left[kPa\right]$. Experimental data is

Eksperimentalni podaci prikazani su na slici 7. (Desai, 1989). Kritična vrednost oktaedarskog napona smicanja jeste $\tau_{oct}^{TE} = 58,54 \left[kPa \right]$, odnosno može se odrediti vrednost kritičnog normalnog napona $\sigma_{xx}^{TE} = 82,80 \left[kPa \right]$.

shown in Figure 7. (based on published results Desai, 1989). From these results follows that critical value for octahedral shear stress was $\tau_{oct}^{TE} = 58,54 \left[kPa \right]$, which leads to critical normal stress by substituting into equation (17a) $\sigma_{xx}^{TE} = 82,80 \left[kPa \right]$.



Slika 7. Poređenje eksperimentalnih i numeričkih rezultata za TE test
Figure 7. Comparing experimental and numerical results for TE case.

ANALIZA REZULTATA

Očigledno je da postoji dobro slaganje između rezultata dobijenih numeričkim postupkom i eksperimentalnih rezultata. Razlika između kritičnog napona određenih numeričkim postupkom i odgovarajućih eksperimentalnih vrednosti za TE test jeste 3.4%, za TC test je 3.7%, dok je za CTC razlika 6.2%.

Prikazan numerički kriterijum za određivanje kritične tačke pogodan je sa stanovišta praktične primene za analizu stabilnosti deformacije geomehaničkih materijala u zavisnosti od geometrije, graničnih uslova i karaktera opterećenja.

Klasični rezultati prikazani u radovima (Hill, 1950), (Rudnicki i sar., 1975) i (Rice, 1976) idu i korak dalje i definišu ugao linije nestabilnosti tečenja. Ugao linije nestabilnosti tečenja možda se može i zamisliti kao ugao analogan lokalizovanju plastične deformacije i pojavi vrata u testu zatezanja u metalnim uzorcima. Međutim, takva analogija primenjena na granularne materijale ne može se funkcionalno povezati sa uglom unutrašnjeg trenja u Mohr-Coulomb-ovom kriterijumu tečenja. U ovom radu ugao linije nestabilnosti tečenja, kao parameter od interesa, nije definisan.

ANALYSIS OF RESULTS

Experimental and calculated numerical modulation values for critical stress are shown in Table II. It is obvious that there is good agreement between them. Percent difference between critical values predicted with numerical model and experimental results for TE test is only 3.4 %, for TC it is 3.7 % while for CTC it is 6.2 %.

Presented numerical criterion to determine critical loading point is useful from the practical point of view for the analysis of the deformation stability for geomechanical material for defined geometry, boundary conditions and loading path.

Classical results presented in work by Hill, Rudnicki and Rice, go a step further into defining the angle of instability for plastic flow. It is possible to think of this angle as analogous to the onset of localization of plastic deformation and beginning of necking in metallic samples. However, in that analogy for metal samples, angle at which macroscopic slip occurs and the beginning of plastic deformation and angle of necking and the beginning of local deformation are not functionally related. If analogy is applied to granular material, it means that angle of plastic instability is not be related to the angle of internal friction in a Mohr-Coulomb approach to plastic flow. In the presented paper, angle of plastic deformation instability for granular material was not considered to be the parameter of interest.

ZAKLJUČAK

Prikazana je analiza uslova stabilnosti za granularni materijal. Rezultati dobijeni numeričkom analizom - korišćenjem HISS konstitutivnog modela - upoređeni su s publikovanim dijagramima za pesak (Leighton Buzzard Sand) u eksperimentalnim uslovima trijaksijalne kompresije i zatezanja. Vrednosti za kritični napon u numeričkom testu niže su za 3-6 % od vrednosti u odgovarajućem eksperimentalnom testu. Nedovoljno široka primena HISS modela možda proističe iz potrebe za relativno velikim brojem materijalnih parametara (osam), kao i zbog njihove ograničene zastupljenosti u literaturi.

CONCLUSION

Criterion for stability of deformation in geomechanical materials is presented. Results obtained using HISS model for numerical analysis were compared with experimental results for particular sand material (Leighton Buzzard Sand) under triaxial compression and extension. Values for critical stress were 3-6 % lower in the numerical prediction compared with experimental data. That small difference in critical values may be considered as a good agreement. However, the main drawback of the HISS model may be its requirement for eight material parameters and limited availability of those parameters in the published literature.

LITERATURA
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REZIME

KRITERIJUM STABILNOSTI DEFORMACIJE ELASTOPLASTIČNIH MATERIJALA

Selimir V. LELOVIČ

Određivanje kritične tačke moguće je svesti na određivanje onog stanja pri kome dolazi do promene znaka drugog izvoda funkcije tečenja. U ovom radu određeno je kritično opterećenje geomehaničkog materijala numeričkim postupkom, korišćenjem inkrementalno iterativnog algoritma za različite putanje napona. Konstitutivne jednačine za hijerarhijski model (*hierarchical single surface model* - HISS) primenjene su da bi se opisalo ponašanje granularnog materijala u toku deformacije. Rezultati dobijeni numeričkom analizom upoređeni su sa eksperimentalnim rezultatima za pesak (Leighton Buzzard Sand).

Ključne reči: stabilnost, gubitak eliptičnosti, kriva površi tečenja, kritično opterećenje, numerička metoda

SUMMARY

CONDITIONS FOR STABILITY OF DEFORMATION IN ELASTO-PLASTIC MATERIALS

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It is possible to look at the critical point determination as a sign change in the second derivative of the yield function. Numerical method in determining critical stress in geomechanical material using an iterative algorithm for different loading paths is presented. The constitutive relationships of the hierarchical single surface (HISS) model were used to describe the granular material during deformation. Critical stresses under applied triaxial compression and extension obtained numerically were compared with published experimental results for Leighton Buzzard Sand

Key words: stability, loss of ellipticity, curvature of yield surface, critical loading, numerical solution