VALIDACIJA I IMPLEMENTACIJA *HASP* KONSTITUTIVNOG MODELA ZA PREKONSOLIDOVANE GLINE

VALIDATION AND IMPLEMENTATION OF *HASP* CONSTITUTIVE MODEL FOR OVERCONSOLIDATED CLAYS

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1 UVOD

Značajan deo u oblasti konstitutivnog modeliranja tla predstavlja opisivanje naponsko-deformacijskih relacija prekonsolidovanih glina. Prekonsolidovane gline su u prošlosti bile opterećene vertikalnim efektivnim naponom koji je veći od tekuće veličine vertikalnog efektivnog napona. Prekonsolidacija može biti i posledica izvođenja različitih građevinskih radova na tlu i u tlu. U poređenju s normalno konsolidovanim glinama, imaju manji koeficijent poroznosti i veću smičuću čvrstoću. U prirodi su najčešće ispucale, što dovodi do nehomogenog polja deformacija. Iz tog razloga, ispoljavaju složen oblik ponašanja pri lomu.

Veliki broj konstitutivnih modela za prekonsolidovane gline razvijen je koristeći koncept kritičnog stanja [35, 38] i Modifikovani Cam Clay (MCC) model [36]. MCC model se može, pri monotonom opterećenju, koristiti s velikom pouzdanošću za normalno konsolidovane i lako prekonsolidovane gline. Za jako prekonsolidovane gline, MCC model precenjuje smičući napon pri lomu i predviđa nagli prelaz iz elastične oblasti u elasto-plastičnu oblast, što nije u skladu sa eksperimentalnim podacima koji pokazuju postepeno smanjenje krutosti prilikom opterećivanja.

Za prevazilaženje nedostataka MCC modela, korišćeni su različiti koncepti. Zienkiewicz i Naylor [52] u relacije konstitutivnog modela uveli su matematički opis površi Hvorsleva, što su u svojim modifikacijama sledili i

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1 INTRODUCTION

A significant part in the area of constitutive soil modelling is the description of the stress-strain relationships of overconsolidated clays. In the past, overconsolidated clays were exposed to the vertical effective stress that is greater than the current magnitude of vertical effective stress. Overconsolidation can also be a consequence of carrying out various construction works on the soil and in the soil. Compared to the normally consolidated clays, they have a lower void ratio and higher shear strength. In nature, they are mostly cracked, leading to a nonhomogeneous field of strains. For this reason, they exhibit a complex form of shear failure.

A large number of constitutive models for overconsolidated clays has been developed using the critical state concept [35, 38] and Modified Cam Clay (MCC) model [36]. The MCC model can be used for normally consolidated and lightly overconsolidated clays under monotonic load, with great certainty. For heavily overconsolidated clays, the MCC model overestimates the failure shear stress and predicts a sudden transition from elastic to elastic-plastic region, which is not in accordance with experimental data that indicate a gradual stiffness reduction during loading.

Different concepts were used to overcome the deficiencies of the MCC model. Zienkiewicz and Naylor [52] have incorporated the mathematical description of

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drugi autori [15, 25, 50, 46, 37]. Na taj način se modifikuje granica mogućih naponskih stanja iznad linije kritičnog stanja i realnije opisuje veličina smičućeg napona pri lomu u dreniranim i nedreniranim uslovima.

Pored toga, razvijen je koncept s više površi tečenja – *Multi Surface Plasticity* – MSP [16, 26], koji adekvatnije opisuje zakon ojačanja materijala, postepen prelaz iz elastične u plastičnu oblast, ponašanje prekonsolidovanog tla, kao i ponašanje tla pri cikličnom opterećenju. Predstavljao je generalni okvir u kome su razvijeni mnogi konstitutivni modeli. Koncept granične površi – *Bounding Surface Plasticity* – BSP [8, 9, 22] je zasnovan na MSP konceptu i predstavljao je poboljšanje u opisivanju postepenog prelaza iz elastične oblasti u elasto-plastičnu oblast. Osnovna ideja je da se – umesto klasične površi tečenja kod Cam Clay modela koja ograničava elastični region – definiše granična površ unutar koje je dozvoljen razvoj plastične deformacije. Prednost ovog koncepta jeste uzimanje u obzir prethodne istorije opterećivanja. Takođe, omogućena je simulacija ponašanja tla pod cikličnim opterećenjem, jer površ popuštanja koja ograničava elastični region može da se translatorno pomera unutar granične površi. Brojni konstitutivni modeli za prekonsolidovano tlo zasnovani su na MSP ili BSP konceptima: Bubble model [2], MIT-E3 [48], 3 SKH model [45], Two Kinematic Hardening Constitutive Models [12], Modified 3 SKH model [24], SANICLAY model [10], UH-model [50]. Navedeni modeli, u matematičkom smislu, složeniji su od MCC modela i imaju veći broj materijalnih parametara. Matematička složenost zahteva napredne numeričke metode i odgovarajući softver, što u današnje vreme ne predstavlja veliki problem, jer su takvi komercijalni softveri dostupni inženjerima u praksi. Znatno veći problem za primenu ovih modela u praksi jeste to što se dodatni materijalni parametri uglavnom ne mogu dobiti iz standardnih laboratorijskih opita. Upravo zahvaljujući jednostavnosti i lakoj identifikaciji parametara modela, MCC model se još uvek najčešće koristi u analizi geotehničkih problema, iako predviđanja naponsko-deformacijskih relacija ne odgovaraju realnom ponašanju prekonsolidovanih glina. Jedan od načina da se unapredi konstitutivni model, a da se ne povećava broj materijalnih parametara, jeste da se koriste unutrašnje promenljive koje adekvatno definišu stanje tla – kao bitnu odrednicu njegovog mehaničkog ponašanja. Jedna od takvih promenljivih je parametar stanja (*state parameter*) koji se još uvek ne koristi dovoljno u konstitutivnom modeliranju.

2 KONCEPT PARAMETRA STANJA

Koncept parametra stanja prvi su predstavili Been i Jefferies [4] za opisivanje ponašanja peska. Umesto koeficijenta poroznosti koji se koristio kao bitna karakteristika za ponašanje peska, predloženo je korišćenje parametra stanja kao fundamentalne promenljive. Veličina srednjeg normalnog efektivnog napona p' značajno utiče na ponašanje tla, tako da se krupnozrno tlo za dati koeficijent poroznosti pri velikoj vrednosti srednjeg efektivnog napona ponaša kao rastresito, dok se za manje vrednosti srednjeg efektivnog napona ponaša kao zbijeno. To znači da je za karakterizaciju krupnozrnog tla – pored koeficijenta poroznosti – neophodna i veličina srednjeg efektivnog

the Hvorslev surface, which was followed by other authors in their modifications [15, 25, 50, 46, 37]. That imposes a more realistic limit to possible stress states above the critical state line and gives a more realistic description of peak shear stress value in drained and undrained conditions.

In addition, the concept of Multi Surface Plasticity – MSP [16, 26] has been developed, which describes more specifically the hardening rule, a gradual transition from elastic to elastic-plastic region, mechanical behaviour of overconsolidated soil, as well as soil behaviour at cyclic loads. It was a general framework in which many constitutive models were developed. The boundary surface concept – Bounding Surface Plasticity – BSP [8, 9, 22] is based on the MSP concept and has been an improvement in describing the gradual transition from elastic to elastic-plastic region. The basic idea is to define, instead of the classic Cam Clay yield surface that limits the elastic region, the boundary surface within which development of plastic strain is allowed. The advantage of this concept is taking into account previous history of loads. Also, simulation of the soil behaviour under a cyclic load is made possible, since the yield surface that limits the elastic region can be moved within the boundary surface. Numerous constitutive models for overconsolidated soil are based on MSP or BSP concepts: Bubble model [2], MIT-E3 [48], 3 SKH model [45], Two Kinematic Hardening Constitutive model [12], Modified 3 SKH model [24], SANICLAY model [10], UH-model [50]. These models are mathematically more complex than the MCC model and have a greater number of material parameters. The mathematical complexity requires advanced numerical methods and appropriate software, which is not a problem because such commercial software is available to engineers in practice. Much greater problem for practical application of these models is that additional material parameters mostly cannot be obtained from standard laboratory tests. Due to the simplicity and easy identification of model parameters, the MCC model is still most often used in analysis of geotechnical problems, although the prediction of stress-strain relations do not correspond to the real behaviour of overconsolidated clays. One way to improve the constitutive model, without increasing the number of material parameters, is to use internal variables that adequately define soil state as an essential determinant of its mechanical behaviour. One such variable is state parameter, which is used insufficiently in constitutive modelling.

2 STATE PARAMETER CONCEPT

The state parameter concept was first introduced by Been and Jefferies [4] to describe the behaviour of sand. Instead of the void ratio that was used as an essential characteristic of the sand behaviour, they suggested to use the state parameter as the fundamental variable. The size of the mean normal effective stress p' significantly influences the behaviour of the soil, so that the coarse-grained soil for the given void ratio, at a large value of the mean effective stress behaves as loose, while for lower values of the mean effective stress behaves compacted. This means that besides the void ratio, the magnitude of the mean effective stress is also necessary for the characterization of the coarse-grained napona. Parametar stanja predstavlja razliku izmeču trenutnog koeficijenta poroznosti e i koeficijenta poroznosti e_c na liniji referentnog (kriticnog) stanja, pri istom srednjem normalnom efektivnom naponu (Slika 1a):

soil. The state parameter is the difference between the current void ratio e and void ratio e_c on the reference state (critical) line at the same mean effective stress (Figure 1a):

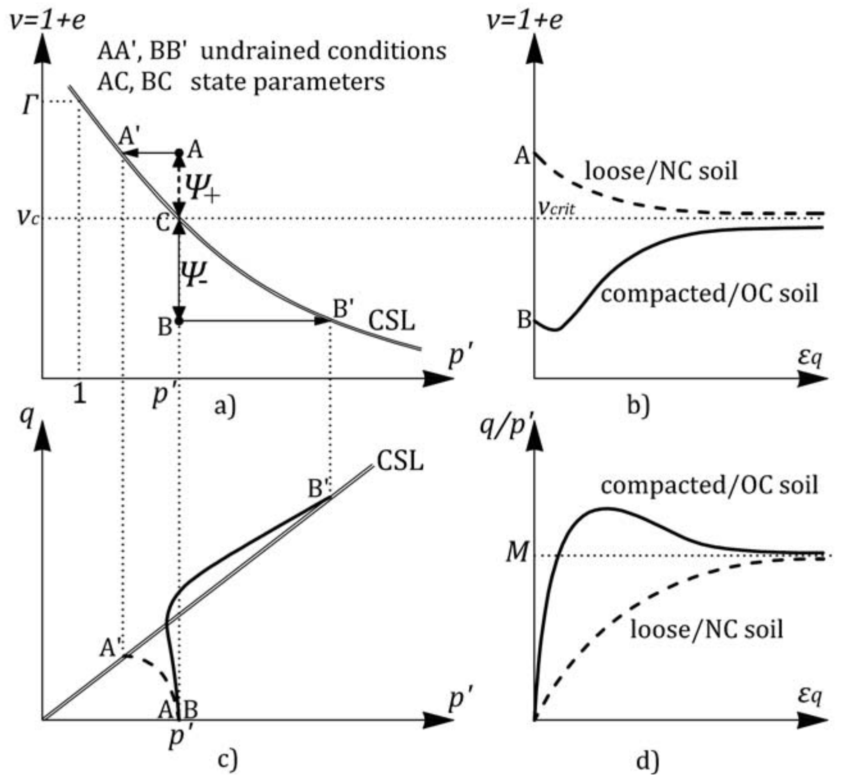
$$\Psi = e - e_c \tag{1}$$

Ovakav koncept podrazumeva da postoji referentno stanje (*steady state condition*) koje treba da ima jedinstvenu strukturu. Za konstitutivne modele, definisane u okviru teorije kriticnog stanja, referentno stanje jeste upravo kriticno stanje, kada se smicu´e deformacije razvijaju bez promene zapremine i efektivnog napona. Takoče, mora biti ispunjen uslov da je linija kriticnog stanja CSL u *v–p′* ravni jedinstvena, gde je *v* specificna zapremina tla.

Za inicijalnu vrednost parametra stanja ve´u od nule, karakteristicnu za rastresita i normalno konsolidovana tla, tacka A na Slici 1a, zapremina tla se smanjuje (kontrakcija) sve do dostizanja kriticnog stanja (Slika 1b). Dolazi do plasticnog smicu´eg loma bez pojave vršne vrednosti (Slika 1d). Ako je inicijalna vrednost parametra stanja manja od nule, kao što je slucaj sa zbijenim i prekonsolidovanim tlom – tacka B na Slici 1a – tlo ´e nakon pocetne kompresije težiti da pove´ava zapreminu (Slika 1b). Tlo ispoljava krto plasticni lom koji podrazumeva pove´anje smicu´eg napona do maksimalne velicine (vršna smicu´a cvrsto´a), a zatim opadanje smicu´eg napona (omekšanje) pri daljem deformisanju do konstantne velicine (Slika 1d). U nedreniranim uslovima, karakteristicne putanje efektivnih napona prikazane su na Slici 1c.

Such concept implies that there is a steady state condition that needs to have a unique structure. For the constitutive models defined within the critical state theory, the reference state is the critical state, when shear strains develop without changing the volume and effective stresses. Also, the condition that the critical state line CSL in *v–p′* plane is unique (where *v* is the specific soil volume) must be fulfilled.

For the initial value of the state parameter greater than zero, characteristic for loose and normally consolidated soil, point A in Figure 1a, the soil volume is decreasing (contraction) until the critical state is reached (Figure 1b). This leads to plastic shear failure (Figure 1d). If the initial value of the state parameter is less than zero, as it is the case with compacted and overconsolidated soil, point B in Figure 1a, after the initial compression the soil will tend to increase the volume (Figure 1b). The soil exhibits a brittle failure, which implies an increase in the shear stress up to the maximum value (peak shear strength), and then decrease in shear stress (softening) during further deformation to the constant value (Figure 1d). In undrained conditions, characteristic effective stress paths are shown in Figure 1c.



Slika 1. a) parametar stanja; b) promena koeficijenta poroznosti tla; c) putanje efektivnih napona u nedreniranim uslovima; d) naponsko-deformacijske krive

Figure 1. a) State parameters b) Change of the void ratio c) Effective stress paths in undrained conditions d) Stress-strain relations

Konstitutivni modeli za pesak – nastali iz koncepta parametra stanja – jesu: Nor-Sand model [17], Severn-Trent sand model [11], model koji su razvili Li & Dafalias [23].

Može se uspostaviti analogija između ponašanja zbijenih granularnih materijala i ponašanja prekonsolidovane gline, odnosno između zbijenosti i stepena prekonsolidacije, tako da se parametar stanja može uspešno koristiti i za opisivanje ponašanja prekonsolidovanih glina. Jedan od takvih modela je CASM model (Clay And Send Model) [51].

Constitutive models for sand formulated from the state parameter concept are: Nor-Sand model [17], Severn-Trent sand model [11], model developed by Li & Dafalias [23].

An analogy can be established between the behaviour of compacted granular materials and behaviour of overconsolidated clay, i.e. between compactness and overconsolidation ratio, so that the state parameter can also be used successfully to describe the behaviour of overconsolidated clays. One such model is the CASM model (Clay and Send Model) [51].

3 FORMULACIJA HASP MODELA

U okviru koncepta parametra stanja, razvijen je i HASP (*HArdening State Parameter*) model [18]. Polazna tačka za formulisanje novog konstitutivnog modela je Modifikovani Cam Clay model. U okviru koncepta granične površi [9], izvršena je modifikacija zakona ojačanja koristeći parametar stanja. Granična površ (*bounding surface*) je MCC površ čiju veličinu definiše vrednost maksimalnog srednjeg efektivnog napona $\bar{p}'_0$ (Slika 2). Ova površ može se nazvati i površ normalne konsolidacije:

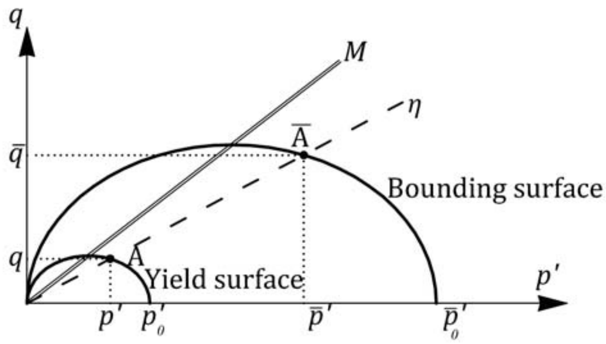
3 FORMULATION OF THE HASP MODEL

The HASP (HArdening State Parameter) model [18] was developed within the state parameter concept. The starting point for formulating a new constitutive model is the Modified Cam Clay model. Within the bounding surface concept [9] a modification of the hardening rule was made by using the state parameter. The bounding surface is the MCC surface, the size of which is defined by the value of maximum mean effective stress $\bar{p}'_0$ (Figure 2). The bounding surface can be called the surface of normal consolidation:

$$\frac{\bar{p}'}{\bar{p}'_0} = \frac{M^2}{M^2 + \eta^2} \tag{2}$$

gde je η – trenutni naponski odnos, a M – nagib linije kritičnog stanja (CSL) u naponskoj ravni.

where η is the current stress ratio and M is the slope of the critical state line (CSL) in the stress plane.



Slika 2. Koncept granične površi
Figure 2. Bounding surface concept

Tačka *A (p',q)* koja predstavlja trenutno naponsko stanje nalazi se na unutrašnjoj površi tečenja (*loading surface*), čiju veličinu definiše vrednost srednjeg efektivnog napona p'_0 :

Point *A (p',q)* that represents current stress state is located on the inner yield surface, the size of which is defined by the value of the mean effective stress p'_0 :

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2} \tag{3}$$

Pretpostavka na kojoj se zasniva HASP model jeste da se plastične deformacije razvijaju od početka opterećivanja i da se tačka *A* uvek nalazi na površi tečenja. Tački *A* odgovara konjugovana tačka $\bar{A}(\bar{p}',\bar{q})$ na graničnoj površi, tako da je ispunjeno:

The assumption on which the HASP model is based is that plastic strains develop from the beginning of loading and point *A* is always located on the yield surface. Conjugate point $\bar{A}(\bar{p}',\bar{q})$ on the bounding surface corresponds to point *A*, so the following is fulfilled:

$$\eta = \frac{q}{p'} = \frac{\bar{q}}{\bar{p}'} \qquad (4)$$

Važi asocijativni zakon tečenja, odnosno to da je vektor priraštaja plastičnih deformacija uvek upravan na površ tečenja. Granična površ ima sve karakteristike MCC površi: za naponski odnos ispod linije kritičnog stanja smanjuje se zapremina i površ se širi, dok se za naponski odnos iznad linije kritičnog stanja povećava zapremina i površ se skuplja. S druge strane, površ tečenja se širi (ojačanje) do dostizanja vršne čvrstoće pri naponskom odnosu $\eta=M_f$, a zatim se skuplja (omekšanje) do dostizanja kritičnog stanja $\eta=M$.

Associated flow rule applies, i.e. plastic strain increment vector is always normal to the yield surface. Bounding surface possesses all the characteristics of the MCC surface. For stress ratio below the critical state line, the volume decreases and the surface expands, while for stress ratio above the critical state line, the volume increases and the surface shrinks. On the other hand, yield surface expands (hardening) until peak strength is reached at stress ratio $\eta=M_f$, after which it shrinks (softening) until critical state is reached $\eta=M$.

3.1 Zakon ojačanja HASP modela

Zakon ojačanja MCC modela zavisi samo od zapreminske plastične deformacije. Generalni zahtev za prekonsolidovana tla je prelaz iz kompresije u ekspanziju pre dostizanja vršne čvrstoće. Zakon ojačanja – koji je u funkciji samo zapreminske plastične deformacije – ne omogućava adekvatno opisivanje dilatancije i ojačanja kod prekonsolidovanih glina. Da bi površ tečenja nastavila da se širi i za vrednosti naponskog odnosa $M<\eta<M_f$, potrebno je koristiti kombinovani zakon ojačanja i formulisati ga u funkciji i plastične smičuće deformacije [28, 50]:

3.1 The hardening rule of the HASP model

The hardening rule of the HASP model depends only on plastic volumetric strains. General requirement for overconsolidated soil is transition from contractive to dilatant behaviour before the peak strength is reached. The hardening rule that is only the function of volumetric plastic strain does not allow adequate description of dilatancy and hardening for overconsolidated clays. In order for the yield surface to continue expanding also for stress ratio values $M<\eta<M_f$, it is necessary to use the combined hardening and express the hardening rule as a function of plastic shear strain also [28, 50]:

$$dp_0' = \frac{v}{\lambda - \kappa} p_0' \left(d\varepsilon_v^p + \xi d\varepsilon_q^p \right) \qquad (5)$$

gde je ξ parametar koji treba definisati, a p_0' parametar ojačanja MCC modela. Parametri λ i κ predstavljaju nagibe linije izotropne konsolidacije i linije bubrenja u *v-lnp'* dijagramu. Kombinovani zakon ojačanja utiče na putanju napona koja prelazi liniju kritičnog stanja i dostiže se vršna čvrstoća u dreniranim uslovima. U nedreniranim uslovima, kombinovano ojačanje omogućava predviđanje putanje efektivnih napona „S” oblika, što je karakteristično za prekonsolidovane gline. Ako definišemo dilatanciju kao odnos priraštaja zapreminske i smičuće komponente plastične deformacije:

where ξ is the parameter to be defined, and p_0' is hardening parameter of the MCC model. Parameters λ and κ are slopes of isotropic consolidation line and swelling lines in *v-lnp'* plane. The combined hardening rule influences the stress path that crosses the critical state line and the peak strength is reached in drained conditions. In undrained conditions, the combined hardening is key to achieve "S" shaped effective stress path, which is typical for overconsolidated clays. If we define dilatancy via the ratio of increment of volumetric and shear component of plastic strain:

$$d = \frac{d\varepsilon_v^p}{d\varepsilon_q^p} \qquad (6)$$

a trenutni stepen prekonsolidacije u toku procesa deformisanja kao:

and the current overconsolidation ratio during the deformation process as:

$$R = \frac{\bar{p}'}{p'} = \frac{\bar{q}}{q} = \frac{\bar{p}_0'}{p_0'} \qquad (7)$$

izraz za zakon ojačanja postaje:

the expression for the hardening rule becomes:

$$dp_0' = \frac{v}{\lambda - \kappa} p_0' d\varepsilon_v^p \left(1 + \frac{\xi}{d} \right) R = \frac{v}{\lambda - \kappa} p_0' d\varepsilon_v^p \omega \qquad (8)$$

gde je ω koeficijent ojačanja (*hardening coefficient*):

where ω is the hardening coefficient:

$$\omega = \left(1 + \frac{\xi}{d} \right) R \qquad (9)$$

Kompletne konstitutivne relacije HASP modela mogu se sada predstaviti kao:

Complete constitutive relations of the HASP model can be presented as:

$$\begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{K} + \dfrac{\lambda - \kappa}{vp'} \dfrac{1}{\omega} \dfrac{M^2 - \eta^2}{M^2 + \eta^2} & \dfrac{\lambda - \kappa}{vp'} \dfrac{1}{\omega} \dfrac{2\eta}{M^2 + \eta^2} \\ \dfrac{\lambda - \kappa}{vp'} \dfrac{1}{\omega} \dfrac{2\eta}{M^2 + \eta^2} & \dfrac{1}{3G} + \dfrac{\lambda - \kappa}{vp'} \dfrac{1}{\omega} \dfrac{4\eta^2}{\left(M^2 + \eta^2\right)\left(M^2 - \eta^2\right)} \end{bmatrix} \begin{Bmatrix} dp' \\ dq \end{Bmatrix} \tag{10}$$

Koeficijent ojačanja ω direktno utiče i na veličinu plastičnih deformacija, tako da se adekvatnom formulacijom koeficijenta ojačanja mogu značajno redukovati plastične deformacije prekonsolidovane gline u početnoj fazi opterećivanja, kada MCC model predviđa samo elastične deformacije. Na taj način je moguće pretpostaviti da tlo od samog početka opterećivanja trpi i plastične deformacije, koje su tada veoma male. Kako se u procesu deformisanja polako smanjuje i stepen prekonsolidacije tla, tako se i koeficijent ω smanjuje ($\omega \rightarrow 1$) i plastične deformacije postaju dominantne. Pri dostizanju vršne čvrstoće (prelaz iz ojačanja u omekšanje), uočava se maksimalni gradijent promene zapremine – maksimalna dilatancija i iz izraza (8) sledi da je ω=0. Tada važi relacija $\xi = -d_{max}$, što znači da parametar ξ predstavlja maksimalnu vrednost dilatancije pri vršnoj čvrstoći u dreniranim uslovima [29].

U izrazu za koeficijent ojačanja (9) odnos ξ/d je definisan preko parametra stanja. Parametar stanja za trenutnu naponsku tačku (Slika 3) može se izraziti kao:

The hardening coefficient ω directly affects the value of the plastic strains, and thus, with the adequate formulation of the hardening coefficient, it is possible to significantly reduce the plastic strains of overconsolidated clay in the initial load phase, when the MCC model predicts only elastic strains. It is then possible to assume that soil deforms plastically from the very beginning of loading. As the overconsolidation ratio of soil decreases in the deformation process, the hardening coefficient ω also decreases ($\omega \rightarrow 1$) and plastic strains become dominant. When reaching the peak strength (transition from hardening to softening) the maximum volume change gradient is observed – maximum dilatancy and from expression (8) it can be concluded that ω=0. Then the relation $\xi = -d_{max}$ applies, which means that parameter ξ is the maximum dilatancy value at peak strength in drained conditions [29].

In the expression for hardening coefficient (9) the ration ξ/d is defined via the state parameter. State parameter for the current stress point (Figure 3) can be expressed as:

$$\Psi = v + \lambda ln p' - \Gamma \tag{11}$$

gde je Γ – parametar koji definiše položaj CSL u kompresionoj p'-v ravni. Parametar stanja je negativan za jako prekonsolidovane gline, dok je za lako prekonsolidovane i normalno konsolidovane gline – pozitivan. Parametar stanja za imaginarnu naponsku tačku iznosi:

where Γ is the parameter that defines the position of CSL in compression p'-v plain. State parameter is negative for highly overconsolidated clays, while for lightly overconsolidated and normally overconsolidated clays it is positive. State parameter for conjugate stress point is:

$$\bar{\Psi} = \left(\lambda - \kappa\right) ln\left(\frac{2M^2}{M^2 + \eta^2}\right) \tag{12}$$

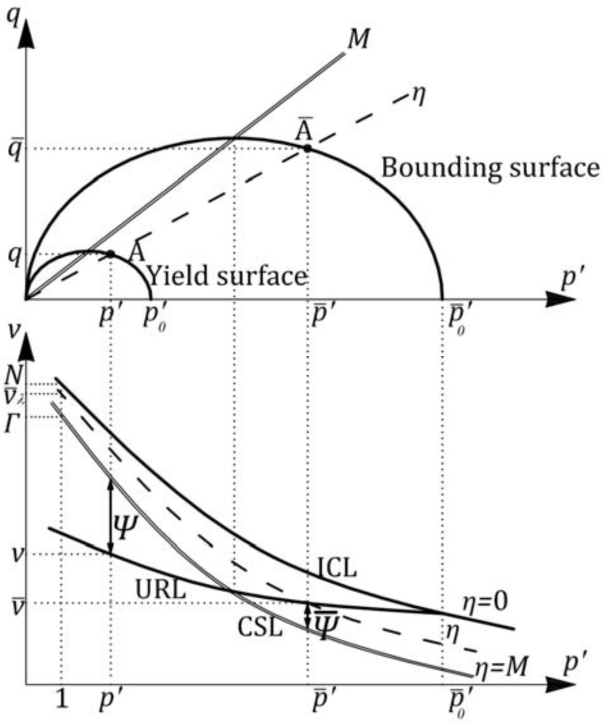
Stepen prekonsolidacije (7) može se takođe izraziti kao funkcija parametara stanja:

The overconsolidation ratio (7) can also be expressed as a function of state parameter:

$$R = \frac{\bar{p}'}{p'} = \frac{\bar{q}}{q} = exp\left(\frac{\bar{\Psi} - \Psi}{\lambda - \kappa}\right) \tag{13}$$

Na osnovu velikog broja triaksijalnih opita na pesku i prekonsolidovanoj glini, Parry [34] je pokazao da je dilatancija pri vršnoj čvrstoći u dreniranim uslovima proporcionalna stepenu prekonsolidacije, a Been & Jefferies [4] su pokazali da je parametar stanja linearno proporcionalan dilatanciji. U skladu s navedenim i imajući u vidu vezu između parametra stanja i stepena prekonsolidacije (13), pretpostavljeno je da je maksimalna vrednost dilatancije u direktnoj zavisnosti od $\bar{\Psi} - \Psi$. Takođe, može se pokazati da se dilatancija menja na sličan način kao parametar stanja za imaginarnu tačku $\bar{\Psi}$. Na osnovu navedenog, sledi da se odnos ξ/d može izraziti preko parametra stanja kao:

On the basis of a large number of triaxial tests on sand and overconsolidated clays, Parry [34] showed that the dilatancy at peak strength in drained conditions is in proportion to the overconsolidation ratio. Also, Been & Jefferies [4] showed that the state parameter is in linear proportion to the dilatancy. In accordance with the aforementioned and taking into account the relationship between the state parameter and overconsolidation ratio (13), it is assumed that the maximum value of dilatancy is directly dependent on $\bar{\Psi} - \Psi$. On the other hand, it can be shown that the dilatancy changes in a similar manner as the state parameter for conjugate stress point $\bar{\Psi}$. Based on the above, it can be concluded that the ratio ξ/d can be expressed via the state parameter:



Slika 3. Parametri stanja za trenutnu i imaginarnu naponsku tačku
Figure 3. State parameters for current and conjugate stress points

$$\frac{\xi}{d}=\frac{\bar{\Psi}-\Psi}{\bar{\Psi}} \tag{14}$$

pa je izraz za koeficijent ojačanja:

and the expression for the hardening coefficient becomes:

$$\omega=\left(1+\frac{\bar{\Psi}-\Psi}{\bar{\Psi}}\right)R \tag{15}$$

Deo izraza (15) u zagradi određuje znak koeficijenta ojačanja i zajedno sa stepenom prekonsolidacije određuje magnitudu koeficijenta ojačanja, a samim tim i veličinu plastičnih deformacija u skladu sa izrazom (10). Za normalno konsolidovane gline važi da je $\Psi=\bar{\Psi}$ i koeficijent ojačanja je $\omega=1$. HASP model tada automatski prelazi u MCC model. Za opis kompletne konstitutivne veze potrebno je pet materijalnih parametara (*M, λ, κ, Γ, μ* - Poisson-ov koeficijent), kao i kod MCC modela i mogu se odrediti iz konvencionalnog triaksijalnog opita, opita direktnog smicanja i edometarskog opita. HASP model, uvođenjem parametra stanja kao unutrašnje promenljive, prevazišao je nedostatke MCC modela, zadržavajući isti set ulaznih parametara, što predstavlja prednost u inženjerskoj implementaciji u poređenju s drugim modelima za prekonsolidovane gline.

The part of the expression (15) in parenthesis controls the sign of the hardening coefficient and with the overconsolidated ratio determines the magnitude of the hardening coefficient and hence affects the magnitude of plastic strains according to expression (10). For normally consolidated clays, the HASP model automatically transforms into the MCC model since $\Psi=\bar{\Psi}$ and the hardening coefficient is $\omega=1$. For the description of stress-strain relations, five material parameters (*M, λ, κ, Γ, μ* - Poisson's coefficient) are needed, just like with the MCC model, and all parameters can be determined from the conventional triaxial test, direct shear test and oedometer test. By introducing the state parameter as an internal variable, the HASP model overcomes many deficiencies of the MCC model, while keeping the same set of input parameters, which is an advantage in engineering implementation compared to other constitutive models for overconsolidated clays.

4 VALIDACIJA HASP MODELA

Validacija HASP modela sprovedena je poređenjem rezultata simulacije laboratorijskih opita sa publikovanim eksperimentalnim rezultatima s različitim putanjama totalnih napona. Da bi se potvrdila efikasnost HASP modela, urađeno je i poređenje s predviđanjem MCC modela. U postupku validacije, izabrane su gline s različitim stepenima prekonsolidacije, za koje u literaturi postoje dobro dokumentovana ispitivanja u triaksijalnom aparatu i za koje su već određeni parametri konstitutivnog MCC modela (Tabela 1). Navedeni parametri predstavljaju ujedno i parametre HASP modela.

4 VALIDATION OF THE HASP MODEL

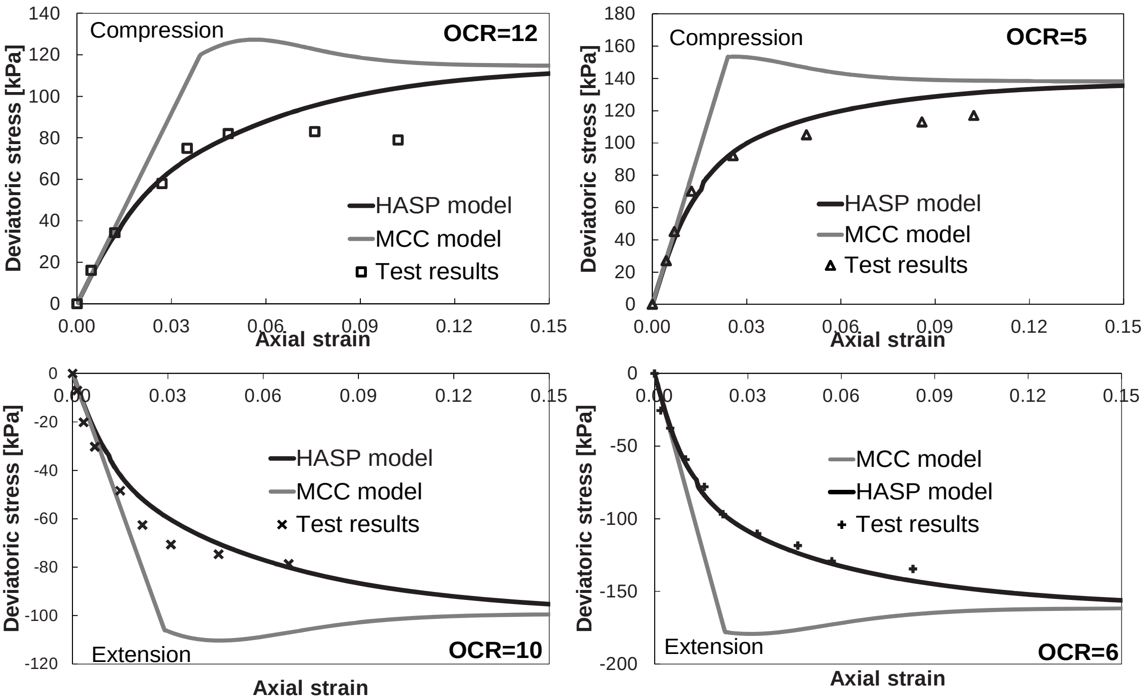
The HASP model validation is performed by comparing the results of simulation of laboratory tests with published experimental results with different total stress paths. In order to confirm the HASP model efficiency, comparison was also made with the prediction of the MCC model. Clays with different overconsolidation ratios were selected, for which in literature there are well-documented triaxial test results and for which parameters of the MCC model have already been determined (Table 1). These parameters are at the same time the parameters of the HASP model.

Tabela 1. Parametri MCC i HASP modela
Table 1. Parameters of MCC and HASP model

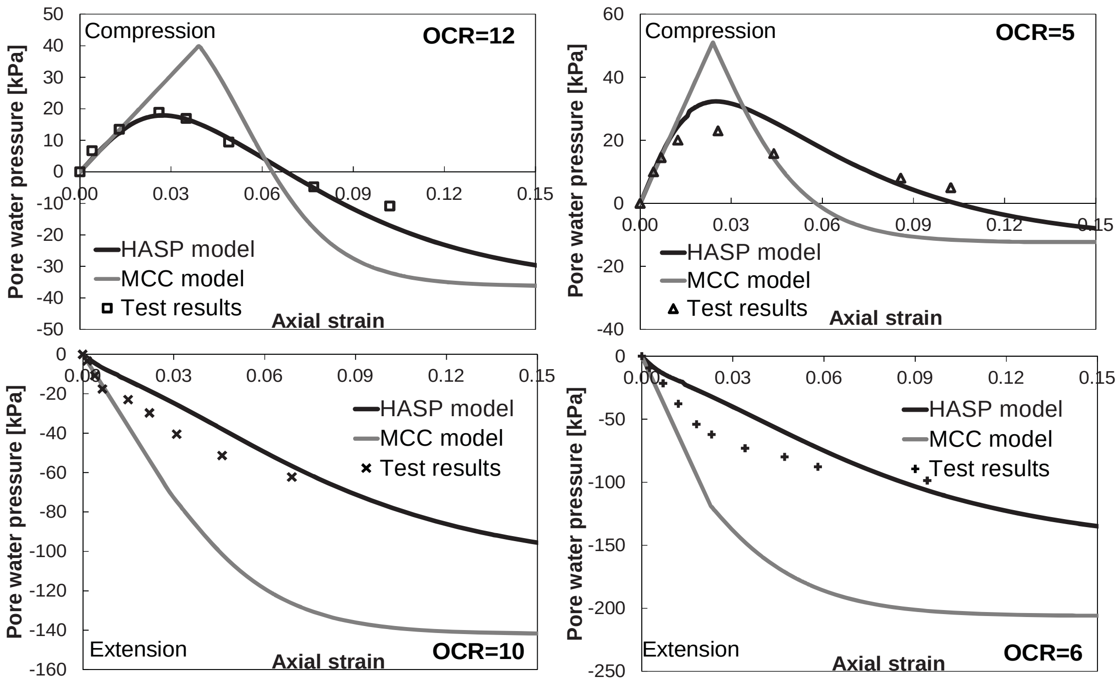
.	λ	κ	M_c	M_e	Γ	μ
Cardiff glina [3] – CU opiti *Cardiff clay* [3] – *CU tests*	0.140	0.050	1.05	0.85	2.63	0.2
Kaolin glina [5] – CD opiti *Kaolin clay* [5] – CD tests	0.230	0.030	0.81	/	3.44	0.2

Prikazani su rezultati dva nedrenirana opita triaksijalne kompresije na prerađenim uzorcima Cardiff gline [3] sa stepenima prekonsolidacije 5 i 12, kao i rezultati dva nedrenirana opita triaksijalne ekstenzije sa stepenima prekonsolidacije 6 i 10 (CU opiti).

The results shown are from two undrained triaxial compression tests on remolded samples of Cardiff clay [3] with overconsolidation ratios 5 and 12, as well as results of two undrained triaxial extension tests with overconsolidation ratios 6 and 10 (CU tests).



Slika 4. CU opiti, Cardiff glina – naponsko-deformacijske relacije
Figure 4. CU tests, Cardiff clay – stress-strain relations



Slika 5. CU opiti, Cardiff glina – promena pornog pritiska
Figure 5. CU tests, Cardiff clay – pore water pressure

Naponsko-deformacijske relacije (Slika 4) i promene pornog pritiska (Slika 5), dobijene HASP modelom pokazuju veoma dobro slaganje sa eksperimentalnim rezultatima, za sve stepene prekonsolidacije pri triaksijalnoj kompresiji i ekstenziji. Može se uočiti da MCC model ne opisuje adekvatno ponašanje prekonsolidovane gline u nedreniranim uslovima. Vrednosti devijatora napona i pornog pritiska znatno su precenjene i odstupanja su veća što je veći stepen prekonsolidacije.

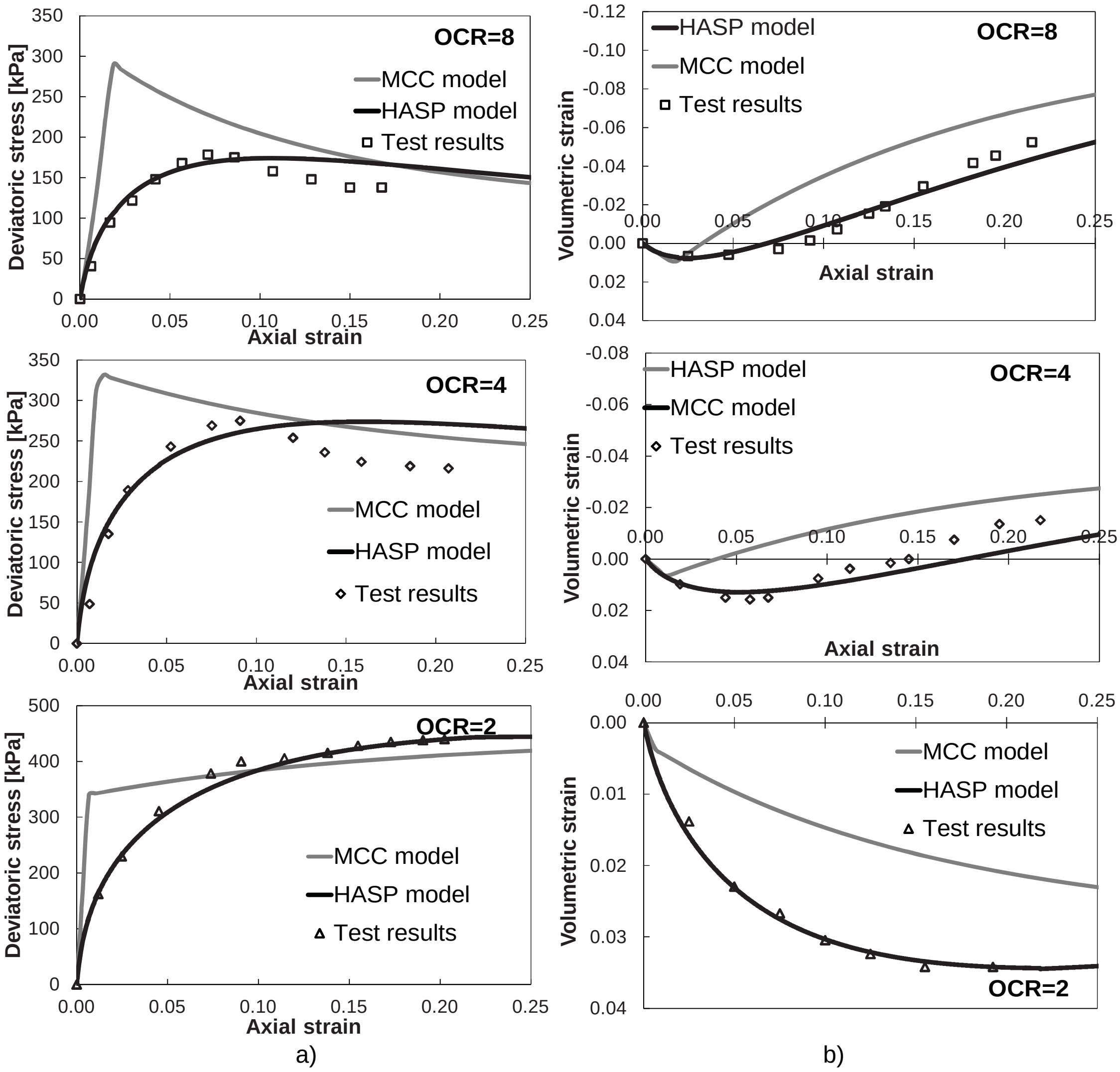
Na Slici 6 su prikazani rezultati dreniranih opita triaksijalne kompresije (CD opiti) na kaolinskoj glini [5] sa stepenima prekonsolidacije 8, 4 i 2.

Ponašanje prekonsolidovanih glina tokom ojačanja veoma je dobro opisano HASP modelom. Za uzorke sa stepenima prekonsolidacije 8 i 4, HASP model predviđa pad čvrstoće – omekšanje pri deformacijama većim od oko 10% (Slika 6a). Za jako prekonsolidovane uzorke (OCR=8, 4), nakon početne kompresije uzoraka, dolazi do ekspanzije i povećanja zapremine (Slika 6b), što je u skladu sa eksperimentalnim rezultatima i uočava se odlično predviđanje promene zapreminskih deformacija s promenom smičućih deformacija. Nedostaci MCC modela, pri opisu mehaničkog ponašanja prekonsolidovanih glina, mogu se uočiti i u dreniranim uslovima. Vršna čvrstoća je precenjena i do dva puta. Detaljan prikaz validacije HASP modela na nekoliko prekonsolidovanih glina s različitim stepenima prekonsolidacije dat je u radu [18].

Stress-strain relations (Figure 4) and changes in pore water pressure (Figure 5) obtained using the HASP model correspond well to the experimental results, for all overconsolidation ratios at triaxial compression and extension. It can be seen that the MCC model fails to adequately describe the behaviour of overconsolidated clays in undrained conditions. Values of deviatoric stresses and pore water pressure are significantly overestimated and deviations are bigger with greater overconsolidation ratio.

Figure 6 shows the results of drained triaxial compression tests (CD tests) on kaolin clay [5] with overconsolidation ratios 8, 4 and 2.

The behaviour of overconsolidated clays during hardening is very well described with the HASP model. For samples with overconsolidation ratios 8 and 4, the HASP model predicts a drop in strength – softening, at strains greater than about 10% (Figure 6a). For highly overconsolidated samples (OCR=8, 4), after the initial compression of the samples, there is an increase in volume (Figure 6b) which is in accordance with experimental results, and excellent prediction of the change in volumetric strains is observed. Deficiencies of the MCC model in describing mechanical behaviour of overconsolidated clays can also be seen in drained conditions. The peak strength is overestimated up to twice the real value. Detailed overview of the validation of the HASP model on several overconsolidated clays with different overconsolidation ratios is shown in the paper [18].



Slika 6. CD opiti, kaolinska glina a) naponsko-deformacijske relacije; b) zapreminske deformacije
Figure 6. CD tests, kaolin clay a) stress-strain relations b) volumetric strains

5 IMPLEMENTACIJA HASP MODELA

Praktična primena složenih elasto-plastičnih konstitutivnih modela u proračunu geotehničkih konstrukcija zahteva korišćenje numeričkih metoda kao što je metoda konačnih elemenata (MKE). Da bi se jedan takav model implementirao u MKE, neophodno je izvršiti numeričku integraciju konstitutivnih relacija, tj. izvršiti integraciju napona za dati inkrement deformacije. Postupak numeričke integracije mora biti stabilan i dovoljno tačan, jer od tačnosti postupka integracije zavisi tačnost rešenja razmatranog graničnog problema

Postoje eksplicitne i implicitne metode za numeričku. integraciju. U slučaju eksplicitnih metoda integracije, do priraštaja napona dolazimo koristeći poznato naponsko stanje na početku inkrementa, u konfiguraciji *t*. U literaturi se mogu naći brojne eksplicitne metode integracije [27, 33, 31, 43, 39, 40, 44]. Razvoj implicitnih metoda počinje sa Wilkins-om [49]. U implicitnim metodama integracije, do priraštaja napona dolazimo koristeći poznate veličine na kraju inkrementa, u

5 IMPLEMENTATION OF THE HASP MODEL

Practical implementation of complex elastic-plastic constitutive models in geotechnical analysis requires the use of numerical methods such as the Finite Element Method (FEM). In order for constitutive model to be implemented in the FEM, it is necessary to perform numerical integration of the constitutive relations, i.e. to perform integration of stresses for the given strain increment. The procedure of numerical integration must be stable and sufficiently accurate, because accuracy of the solution of the considered boundary value problem depends on the accuracy of the integration procedure.

There are explicit and implicit methods for numerical integration. With explicit methods of integration, stress increment is determined by using known stress state at the beginning of the increment, in configuration *t*. In the literature, there are numerous explicit methods of integration [27, 33, 31, 43, 39, 40, 44]. Development of implicit methods begins with Wilkins [49]. In implicit methods of integration, stress increment is determined konfiguraciji $t+\Delta t$. Pocedura se generalno sastoji od dva koraka: proračuna elastičnog rešenja za dati inkrement (elastično predviđanje) i povratka na površ tečenja (plastični korektor). Ovaj pristup kasnije su koristili i razvijali brojni autori i tako je nastala klasa procedura integracije koja se naziva povratno preslikavanje [30, 41, 32, 6, 7, 42, 14]. Implicitnu šemu integracije – nazvanu Metoda vodećeg parametra (Governing Parameter Method) GPM – razvijali su Kojić i Bathe [19-21]. Predstavlja generalizaciju *radial return* metode koju je predstavio Wilkins [49]. Osnovni princip jeste da se sve nepoznate veličine izraze u funkciji jednog parametra (vodeći parametar) i problem se svodi na rešavanje jedne nelinearne jednačine po nepoznatom vodećem parametru. Za HASP model je korišćena GPM metoda, gde je kao vodeći parametar korišćen srednji normalni efektivni napon p' [47] kao veličina s jasnim fizičkim značenjem i s definisanim intervalom mogućih vrednosti. HASP model je implementiran u Abaqus/Standard [1], koristeći korisnički potprogram UMAT i numeričku proceduru za integraciju napona GPM.

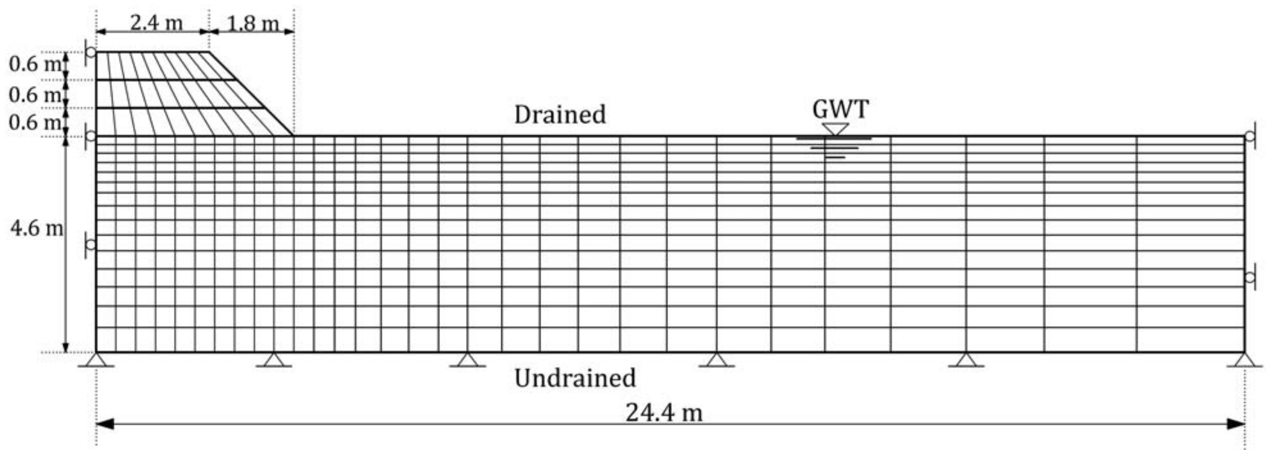
5.1 Konsolidacija sloja gline

Kao primer implementacije HASP modela, urađena je analiza konsolidacionog sleganja tla usled fazne izgradnje nasipa na površini terena (primer u knjizi *Applied Soil Mechanics with Abaqus Applications* [13]). Model se sastoji od sloja gline, debljine 4.6 m, koji leži na nepropusnoj i nestišljivoj podlozi. Nivo podzemne vode se nalazi na površini terena, kao što je prikazano na Slici 7. Nasip se gradi u tri jednaka sloja debljine 0.6 m. Ukupna visina nasipa iznosi 1.8 m. Konstrukcija nasipa se izvodi po fazama/slojevima, a izgradnja jednog sloja traje dva dana, dok izgradnja čitavog nasipa traje šest dana. U modelu, konsolidacija gline nakon izgradnje nasipa traje još 200 dana.

by using known variables at the end of the increment, in configuration $t+\Delta t$. The procedure generally consists of two steps: estimate of the elastic solution for the given increment (elastic prediction) and return to the yield surface (plastic corrector). This approach was later used and further developed by numerous authors and so the class of integration procedures was created, called return mapping [30, 41, 32, 6, 7, 42, 14]. The implicit integration scheme that is called the Governing Parameter Method (GPM) was developed by Kojić and Bathe [19-21]. It is a generalization of the *radial return* method which was introduced by Wilkins [49]. The basic principle is that all unknown variables are expressed in the function of one parameter (the governing parameter) and the problem is reduced to the solving of one non-linear equation with respect to the governing parameter. For the HASP model, the mean effective stress p' [47] was selected as the governing parameter as a value with clear physical meaning and with defined interval of possible values. The HASP model is implemented in Abaqus/Standard [1] using the user subroutine UMAT and GPM as numerical procedure for stress integration.

5.1 Consolidation of clay layer

As an example of implementation of the HASP model, analysis of the soil consolidation as the result of phased construction of the embankment on the clay surface was performed (example in book *Applied Soil Mechanics with Abaqus Applications* [13]). The FEM model consists of a layer of clay, 4.6 m thick, which lies on impermeable and incompressible base. The ground water table is on the clay surface, as shown in Figure 7. The embankment is built in three equal layers, 0.6 m thick. Total height of the embankment is 1.8 m. The structure of the embankment is made by phases/layers and construction of one layer takes two days, while the construction of the entire embankment takes six days. The consolidation of clay after construction of the embankment takes another 200 days.



Slika 7. Model s mrežom konačnih elemenata za numeričku analizu
Figure 7. Model with the finite element mesh for numerical analysis

Ispitana je mogućnosti HASP modela da tokom simulacije navedenog procesa predvidi promenu pornog natpritiska, kao i veličine vremenskog sleganja nasipa i sloja gline.

Materijali

Nasip se gradi od prašinastog peska i modeliran je linearno-elastičnim modelom. Parametri linearno-elastičnog modela prikazani su u Tabeli 2. Sloj visokoplastične gline ispod nasipa modeliran je HASP modelom (Tabela 3).

The possibility of the HASP model to predict the change of pore water pressure, as well as the value of the consolidation settlement of the embankment and clay layer was performed.

Materials

The embankment is built of silty sand and is modelled using the linear-elastic model. Parameters of the linear-elastic model are shown in Table 2. The layer of highly overconsolidated clay below the embankment is modelled using the HASP model (Table 3).

Tabela 2. Parametri nasipa
Table 2. Parameters of the embankment

Linearno-elastični model *Linear-elastic model*		Karakteristike materijala nasipa *Characteristics of embankment material*		
E [MPa]	*μ*	*γ*[kN/m^3]	*k* [m/s]	e_0
5	0.3	18.85	0.001	0.889

Tabela 3. Parametri HASP modela
Table 3. Parameters of the HASP model

λ	*κ*	*M*	*Γ*	*μ*
0.174	0.026	1.5	3.87	0.28

Analiza je rađena s različitim inicijalnim uslovima, odnosno različitim početnim stepenom prekonsolidacije sloja gline (prikazanim u Tabeli 4) i sprovedena je u pet proračunskih koraka, za svaki stepen prekonsolidacije. U prvom proračunskom koraku, nasip je uklonjen iz mreže konačnih elemenata. U sledeća tri koraka je simulirana izgradnja nasipa u tri sloja, pri čemu je svaki naredni sloj dodat na već deformisani prethodni. Peti korak je konsolidacija gline i nasipa u trajanju od 200 dana.

The analysis was performed with different initial conditions, i.e. different initial overconsolidation ratios of clay layer (Table 4) in five calculation steps. In the first calculation step, the embankment is removed from the finite element mesh. The next three steps consist of simulation of the construction of the embankment in three layers, where each subsequent layer was added to the already deformed previous one. The fifth step is consolidation of clay and the embankment for a period of 200 days.

Tabela 4. Inicijalni uslovi
Table 4. Initial conditions

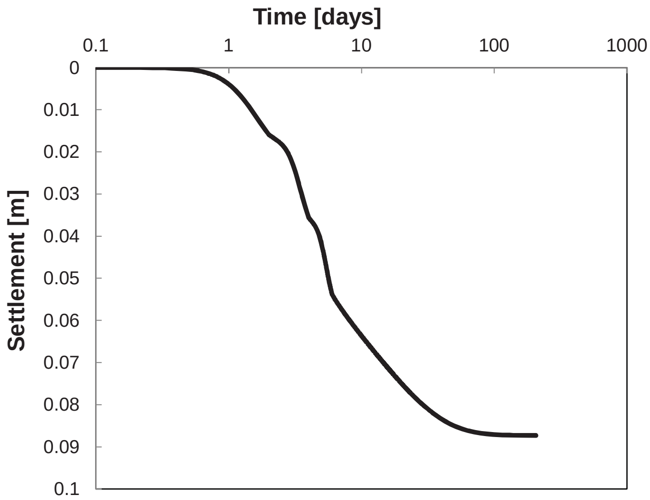
e_0	*γ*[kN/m^3]	OCR	k_0
1.1	17.75	2	0.75
1.0	18.15	5	0.85
0.9	18.60	8	1.0
0.8	19.10	12	1.3
0.7	19.60	18	1.9

Rezultati

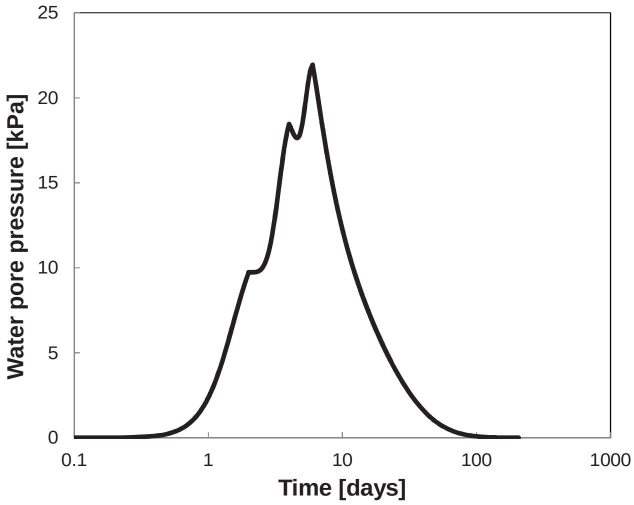
Za analizu pojedinačnih rezultata – kao ilustracija – odabrani su rezultati za stepen prekonsolidacije OCR=5. Na Slici 8 je prikazan vremenski tok sleganja ispod centra nasipa (površina sloja gline) u polulogaritamskoj razmeri. Deformacije se najbrže razvijaju (najveći gradijent) tokom prvih šest dana koliko traje izgradnja nasipa i do tada se desilo oko 50% od ukupnih sleganja. Na slici 9 je prikazana istorija razvoja pornog natpritiska u sredini sloja gline ispod centra nasipa. Porni natpritisak raste tokom izgradnje nasipa (šest dana) i tokom procesa konsolidacije dolazi do njegove potpune disipacije.

Results

For analysis of individual results, the results for the overconsolidation ratio OCR=5 were selected as an illustration. Figure 8 shows the timeline of the settlement under the centre of the embankment (surface of the clay layer) in semi-logarithmic plot. Strains develop most quickly (the highest gradient) during the first 6 days, which is how long the construction of the embankment lasts, and about 50% of the total settlement occurred by that time. Figure 9 shows the history of development of pore water pressure in the middle of the clay layer under the centre of the embankment. The pore water pressure increases during the construction of the embankment (six days) and during the consolidation process its full dissipation occurs.



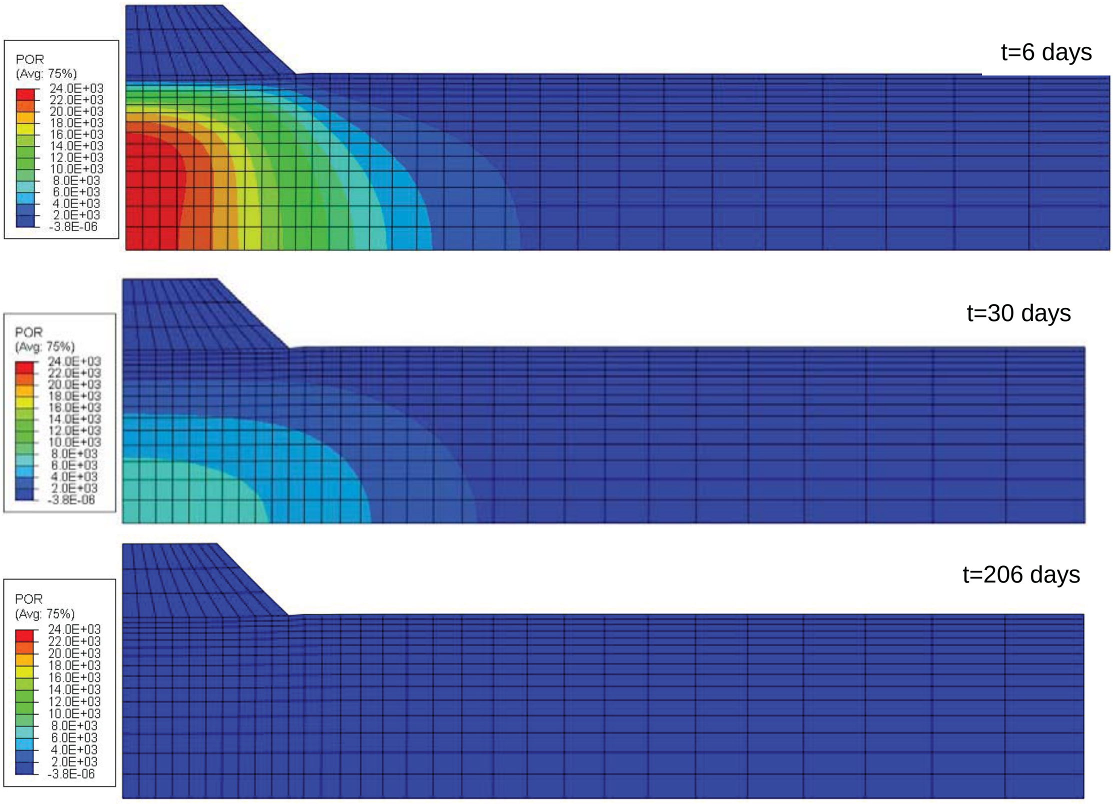
Slika 8. Sleganje gline ispod centra nasipa tokom vremena, OCR=5
Figure 8. Settlement of clay layer under the centre of the embankment over time, OCR=5



Slika 9. Razvoj pornog natpritiska u sredini sloja gline ispod centra nasipa, OCR=5
Figure 9. Development of pore water pressure in the middle of the clay layer under the centre of the embankment, OCR=5

Raspodela pornog natpritiska i disipacija tokom vremena data je na Slici 10. Usled brzog opterećivanja sloja zasićene gline male vodopropusnosti, ispod nasipa se odmah nakon nanošenja opeterećenja razvija porni natpritisak. S obzirom da je omogućeno dreniranje vode samo preko gornje površine, do najbrže disipacije dolazi upravo na gornjoj površini sloja gline.

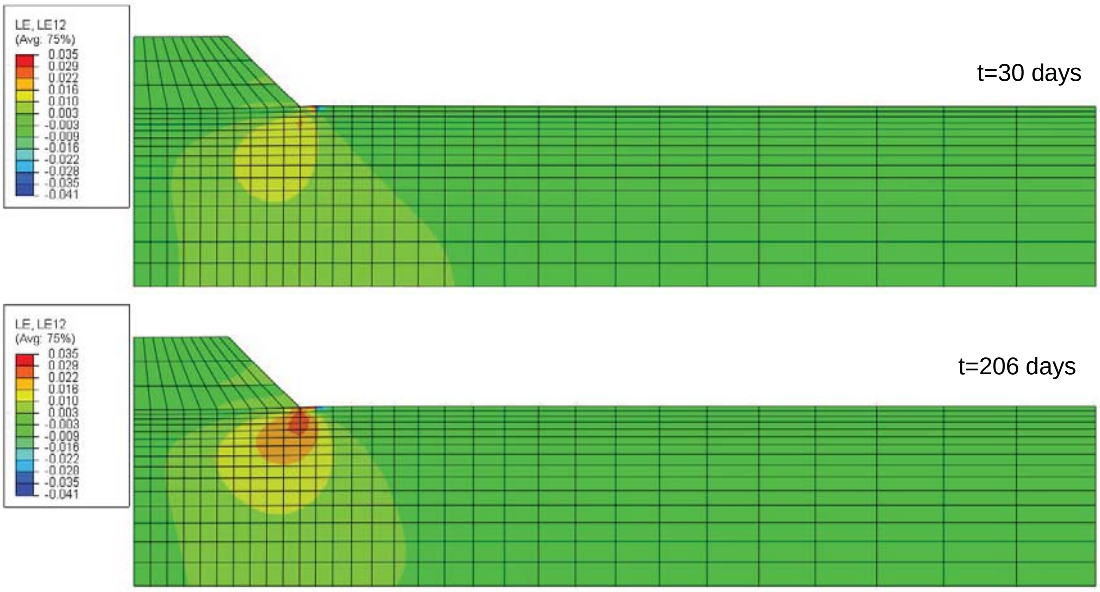
Distribution of the pore water pressure and dissipation over time is shown in Figure 10. As the result of rapid loading of the layer of saturated clay of low water permeability, the pore water pressure develops under the embankment immediately after placing the load. Since water draining is enabled only over the upper surface, the fastest dissipation occurs exactly on the upper surface of the clay layer.



Slika 10. Razvoj pornog natpritiska tokom vremena, OCR=5
Figure 10. Development of pore water pressure over time, OCR=5

Raspodela smičućih deformacija je prikazana na Slici 11, gde se može uočiti da se maksimalne vrednosti smičućih deformacija javljaju u nožici kosine nasipa.

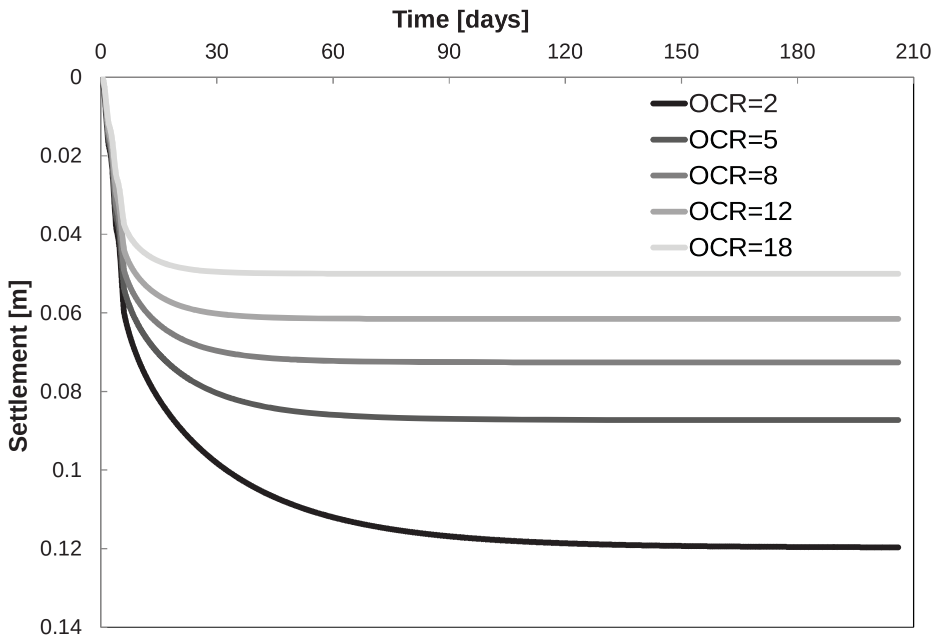
Distribution of shear strains is shown in Figure 11, where it can be observed that the maximum values of shear strains appear in the toe of the slope of the embankment.



Slika 11. Razvoj smičućih deformacija tokom vremena, OCR=5
Figure 11. Development of shear strains over time, OCR=5

Sleganja sloja gline tokom 206 dana ispod centra nasipa, za sve stepene prekonsolidacije, data su na Slici 12. Najveća sleganja, kao što se i očekuje, dobijena su za blago prekonsolidovane gline.

Settlements of clay layer over 206 days under the centre of the embankment for all overconsolidation ratios are shown in Figure 12. The largest settlements were, as expected, obtained for lightly overconsolidated clays.



Slika 12. Sleganje sloja gline za različite stepene prekonsolidacije posle 206 dana
Figure 12. Settlements of clay layer for different overconsolidation ratios after 206 days

5.2 Poređenje s MCC modelom

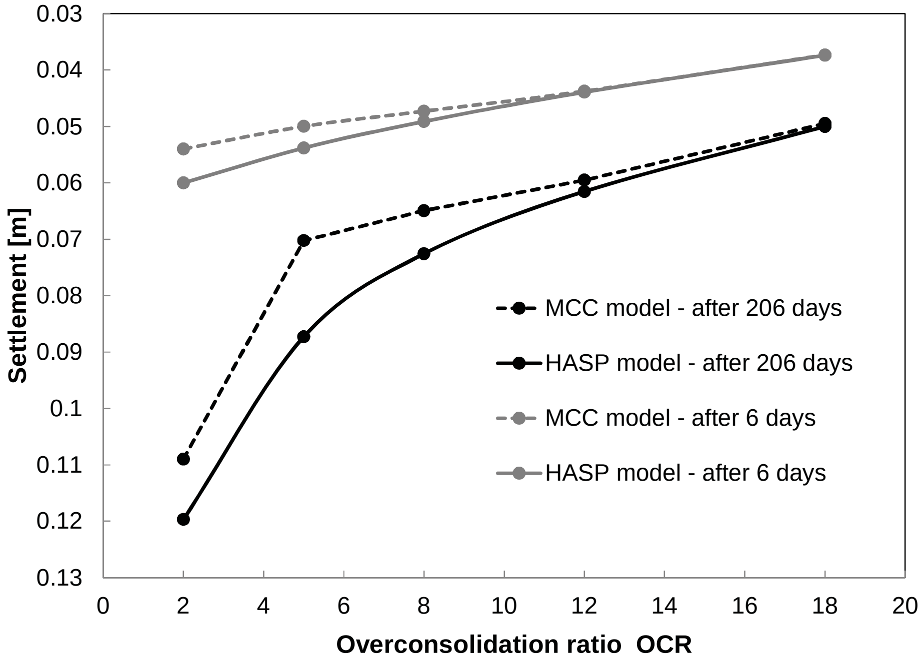
Isti granični problem je analiziran i koristeći MCC model koji već postoji kao standardni materijalni model u Abaqus-u. Predviđa se slična promena pornog pritiska, sleganja i smičućih deformacija tokom vremena, dok je osnovna razlika u veličini zapreminskih i smičućih deformacija. Koristeći HASP model, generalno se dobijaju veće vrednosti deformacija i sleganja u odnosu na MCC model, naročito za manje stepene prekonsolidacije. Takvi rezultati su očekivani, s obzirom na to što HASP model predviđa elasto-plastično ponašanje od samog početka procesa deformisanja, dok MCC model predviđa samo elastično ponašanje unutar inicijalne površi tečenja.

Za veliki stepen prekonsolidacije (u datoj analizi OCR>12), predviđaju se slične vrednosti sleganja za oba modela (Slika 13). HASP model, zahvaljujući velikom koeficijentu ojačanja ω za veliki stepen prekonsolidacije, predviđa male vrednosti plastičnih deformacija i ukupne vrednosti deformacija se ne razlikuju značajno od veličine elastičnih deformacija. Pri manjim vrednostima stepena prekonsolidacije, odstupanja u veličini deformacije značajno su veća. Dok materijal opisan MCC modelom ostaje u elastičnoj zoni za prikazana opterećenja i za manje vrednosti stepena prekonsolidacije, HASP model predviđa veće vrednosti plastičnih deformacija usled manje vrednosti koeficijenta ojačanja ω. U prikazanoj analizi, razlike u sleganjima iznose i do 20–25%.

5.2 Comparison with the MCC model

The same boundary value problem was analyzed by using the MCC model, which already exists as a standard material model in Abaqus. It predicts similar change in pore water pressure, settlements and shear strains over time, while the main difference is in the magnitude of the volumetric and shear strains. By using the HASP model, generally higher values of deformations are obtained compared to those from the MCC model, especially for lower overconsolidation ratios. Such results are expected, since the HASP model predicts elastic-plastic behaviour from the very beginning of the deformation process, while the MCC model predicts only elastic behaviour within the initial yield surface.

For higher overconsolidation ratios (in the given analysis OCR>12), similar values of settlements are predicted for both models (Figure 13). The HASP model, due to the high value of hardening coefficient ω for high overconsolidation ratio, predicts small values of plastic strains and total values of strains are not much different from the values of elastic strains. For lower values of overconsolidation ratio, differences in strain magnitude are more pronounced. While the material described with the MCC model remains in the elastic zone for the given loads and for the lower values of the overconsolidation ratio also, the HASP model predicts higher values of plastic strains as the result of lower values of hardening coefficient ω. In the presented analysis, the differences in the consolidation settlements are up to 20-25%.



Slika 13. Zavisnost veličine sleganja od stepena prekonsolidacije, HASP model i MCC mod
Figure 13. Settlement dependency on the overconsolidation ratio, HASP model and MCC model

6 ZAKLJUČCI

HASP model uspešno prevazilazi mnoge nedostatke MCC modela prilikom opisivanja mehaničkog ponašanja prekonsolidovanih glina, a pri tome je zadržana jednostavnost MCC modela i isti broj parametara. Koristeći kombinovani zakon ojačanja u funkciji plastične zapreminske, plastične smičuće deformacije i parametar stanja, formulisan je koeficijent ojačanja koji kontroliše sve elemente ponašanja prekonsolidovane gline. Koeficijent ojačanja je istovremeno i koeficijent redukcije plastičnih deformacija, čime je omogućeno elasto-plastično ponašanje od samog početka deformisanja.

U dreniranim uslovima, model predviđa postepen prelaz iz kontrakcije u ekspanziju, pre nego što je dostignuta vršna smičuća čvrstoća, kao i postepen prelaz iz ojačanja u omekšanje, bez dodatnog matematičkog opisivanja. U nedreniranim uslovima, model predviđa putanju efektivnih napona "S" oblika, kao i negativan porni pritisak pri lomu za jako prekonsolidovane gline. Što je veća vrednost parametra stanja i veći stepen prekonsolidacije, veća je i vrednost koeficijenta ojačanja, te model predviđa veću krutost tla. Za normalno konsolidovane gline, HASP model automatski prelazi u MCC model, jer je tada koeficijent ojačanja jednak jedinici.

U postupku validacije modela, prikazani rezultati simulacije opita pri različitim putanjama totalnih napona, pokazuju veoma dobro slaganje sa eksperimentalnim rezultatima, za sve stepene prekonsolidacije. U poređenju s predviđanjem MCC modela, značajan napredak postignut je u sledećim elementima: a) HASP model predviđa postepen razvoj plastičnih deformacija od samog početka deformisanja; b) postoji postepen prelaz iz elastične u elasto-plastičnu oblast; c) postoji dobro predviđanje smičućeg napona pri lomu, kao i

6 CONCLUSIONS

The HASP model successfully overcomes many deficiencies of the MCC model when describing the mechanical behaviour of overconsolidated clays, while keeping the simplicity of the MCC model and the same number of parameters. By using the combined hardening rule in the function of plastic volumetric and shear strain and state parameter, the hardening coefficient has been formulated which controls all elements of the mechanical behaviour of overconsolidated clays. The hardening coefficient is at the same time the reduction coefficient for plastic strains, which allows elastic-plastic behaviour from the very beginning of deformation process.

In drained conditions, the model predicts gradual transition from contractive to dilatant behaviour before the peak strength is reached, as well as gradual transition from hardening to softening without additional mathematical description. In undrained conditions, the model predicts effective stress path of "S" shape, as well as negative failure pore pressure for highly overconsolidated clays. The higher the values of state parameter and overconsolidation ratio, higher the value of the hardening coefficient and the model predicts stiffer response. For normally consolidated clays, the HASP model automatically transforms into the MCC model, because the hardening coefficient equals one.

In the model validation process, the presented results of test simulations at different total stress paths are very well aligned with experimental results for all overconsolidation ratios. In comparison with the prediction of the MCC model, a significant progress was achieved in the following elements: a) the HASP model predicts gradual development of plastic strains from the very beginning of the deformation process; b) there is a pornog pritiska za prekonsolidovana tla.

HASP model je implementiran u Abaqus/Standard putem dostupnog korisničkog potprograma UMAT. Za numeričku integraciju konstitutivnih relacija, vrlo uspešno je primenjena Metoda vodećeg parametra.

U razmatranom primeru konsolidacije sloja zasićene prekonsolidovane gline usled fazne izgradnje nasipa, prikazana je sposobnost HASP modela da predvidi vremenski tok promene pornih pritisaka, zapreminskih i smičućih deformacija. Rezultati su poređeni s MCC modelom. Usled brzog opterećivanja sloja zasićene gline male vodopropusnosti, HASP model predviđa pojavu pornog natpritiska, koji raste tokom izgradnje nasipa, te u procesu konsolidacije dolazi do potpune disipacije pornog natpritiska. Deformacije se najbrže razvijaju tokom izgradnje nasipa, a najveća sleganja dobijena su za blago prekonsolidovane gline. U poređenju s MCC modelom, osnovna razlika jeste u veličini zapreminskih i smičućih deformacija. Koristeći HASP model, generalno se dobijaju veće vrednosti deformacija nego u MCC modelu, s obzirom na to što HASP model predviđa elasto-plastično ponašanje od samog početka procesa deformisanja, dok MCC model predviđa samo elastično ponašanje unutar inicijalne površi tečenja.

Na osnovu prikazanih rezultata, može se zaključiti da HASP model ima dobar balans između sofisticiranosti i jednostavnosti, što omogućava njegovu široku praktičnu primenu u rešavanju geotehničkih problema.

gradual transition from elastic into elastic-plastic region; c) there is good prediction of failure shear stress, as well as pore water pressure for overconsolidated soil.

The HASP model is implemented in Abaqus/Standard through the available user subroutine UMAT. For numerical integration of constitutive relations, the Governing Parameter Method was used very successfully.

Through the discussed example of consolidation of saturated overconsolidated clay layer, as the result of phased construction of the embankment, the ability of the HASP model to predict the changes of pore water pressure, volumetric and shear strains was presented. The results were compared with the MCC model. As the result of rapid increase of load on the saturated clay layer with low permeability, the HASP model predicts the increase of pore water pressure during the construction of the embankment and full dissipation of the pore water pressure in the process of consolidation. Strains develop most rapidly during the construction of the embankment and greatest amount of settlement were obtained for slightly overconsolidated clays. In comparison with the MCC model, the main difference is in the magnitude of the volumetric and shear strains. By using the HASP model, higher values of strains are generally obtained against the MCC model, since the HASP model predicts elastic-plastic behaviour from the very beginning of the deformation process, while the MCC model predicts only elastic behaviour within the initial yield surface.

Based on the presented results, it can be concluded that the HASP model has good balance of sophistication and simplicity, which allows its wide practical use in solving various geotechnical problems.

7 LITERATURA REFERENCES

[1] ABAQUS. User manual Hibbit, Karlsson & Sorensen, Inc. 2011.

[2] Al-Tabbaa, A. & Wood, D.M. An experimentally based „bubble” model for clay. Numerical Models in Geomechanics NUMOG III (eds S Pietruszczak and GN Pande), Elsevier Applied Science 1989; 91–99.

[3] Banerjee, P.K. & Stipho, A.S. Elastoplastic model for undrained behavior of heavily overconsolidated clays. Int J Numer Anal Mech Geomech 1979; 3(1): 97–103.

[4] Been, K. & Jefferies, M.G. A state parameter for sands. Géotechnique 1985; 35(2): 99–112.

[5] Biarez, J. & Hicher, P.Y. Elementary Mechanics of Soil Behaviour. Saturated Remoulded Soils. A. A. Balkema, Rotterdam; 1994.

[6] Borja, R.I. & Lee, S.R. Cam clay plasticity, part I: Implicit integration of constitutive relations, Comput. Meth. Appl. Mech. Eng. 1990; Vol. 78, 49–72.

[7] Borja R.I. Cam clay plasticity, part II: Implicit integration of constitutive equations based on nonlinear elastic stress prediction, Comput. Meth. Appl. Mech. Eng. 1991; Vol. 88, 225–240.

[8] Dafalias, Y.F. & Popov, E.P. Plastic internal variables formalism of cyclic plasticity. J. of Applied Mechanics 1976; Vol. 98, 645–651.

[9] Dafalias, Y.F. & Herrmann, L.R. A bounding surface soil plasticity model. Proc. Int. Symp. on Soils under Cyclic and Transient Loading 1980; Vol. 1, 335–345.

[10] Dafalias Y.F., Manzari M.T., Papadimitriou A.G. SANICLAY: simple anisotropic clay plasticity model. International Journal for Numerical and Analytical Methods in Geomechanics 2006; 30: 1231–1257. DOI: 10.1002/nag. 524.

[11] Gajo, A. & Wood, D.M. A new approach to anisotropic, bounding surface plasticity: General formulation and simulations of natural and reconstituted clay behaviour. Int J Numer Analyt Meth Geomech 2001; 25-3: 207–241.

[12] Grammatikopoulou, A., Zdravkovic, L., Potts, D.M. General Formulation of Two Kinematic Hardening Constitutive Models with a Smooth Elastoplastic Transition. Int J Geomech 2006; 6:291–302. doi:10.1061/(ASCE) 1532–3641 (2006) 6:5 (291).

[13] Helwany, S. Applied soil mechanics with ABAQUS applications. John Willey & Sons, Canada, 2007.

[14] Hickman, R.J. & Gutierrez, M. An internally consistent integration method for critical state models. Int J Numer Anal Methods Geomech 2005; 29: 227–48.

[15] Houlsby, G.T., Wroth, C.P., Wood, D.M. Predictions of the results of laboratory tests on a clay using a critical state model. In Proc. int. workshop on constitutive behaviour of soils, Grenoble, Balkema (Rotterdam) 1982; 99–121.
[16] Iwan, W.D. On a class of models for the yield behaviour of continuous and composite systems. Journal of Applied Mechanics 1967, ASME 34; 612–617.
[17] Jefferies, M.G. Nor-Sand: a simple critical state model for sand. Géotechnique 1993; 43(1):91–103.
[18] Jocković, S. & Vukićević, M. Bounding surface model for overconsolidated clays with new state parameter formulation of hardening rule. Comput Geotech 2017; 83:16–29. doi:10.1016/j.compgeo.2016.10.013.
[19] Kojić, M. & Bathe K.J. The effective-stress-function algorithm for thermo-elastoplasticity and creep, Int. J. Num. Meth. Engng. 1987; Vol. 24, pp. 1509–1532.
[20] Kojić, M. The governing parameter method for implicit integration of viscoplastic constitutive relations for isotropic and orthotropic metals, Computational Mechanics 1996; Vol. 19, 49–57.
[21] Kojić, M. & Bathe, K.J. Inelastic Analysis of Solids and Structures, Berlin, Springer-Verlag, 2003.
[22] Krieg, R. D. A practical two-surface plasticity theory. Journal of Applied Mechanics 1975; Vol. 42, No. E3, pp. 641–646.
[23] Li, X.S. & Dafalias, Y.F. Dilatancy for cohesionless soils. Géotechnique 2000; 50:449–60. doi:10.1680/geot.2000.50.4.449.
[24] McDowell, G.R. & Hau, K.W. A simple non-associated three-surface kinematic hardening model. Géotechnique 2003, No. 4, 433−437.
[25] Mita, K.A., Dasari, G.R., Lo, K.W. Performance of a three-dimensional Hvorslev-Modified Cam clay model for overconsolidated clay. ASCE, Int J Geomech 2004; 4(4): 296–309.
[26] Mróz, Z. On the description of anisotropic workhardening. Journal of the Mechanics and Physics of Solids 1967; 15, 163–175.
[27] Nayak, G.C., & Zienkiewicz, O.C. Elastoplastic stress analysis: a generalization for various constitutive relations including strain softening. International Journal for Numerical Methods in Engineering 1972; 5, 113–135.
[28] Nova, R. & Wood, D.M. A constitutive model for sand in triaxial compression. Int J Numer Anal Methods Geomech 1979; 3: 255–278.
[29] Nova, R. Modelling of bonded soils with unstable structure. International Workshop on Modern Trends in Geomechanics – Vienna Springer, 2006.
[30] Ortiz, M., Pinsky, P.M., Taylor, R.L. Operator split methods for the numerical solution of the elasto-plastic dynamic problem, Comp. Meth. Appl. Mech. Engng. 1983; Vol. 39, pp. 137–157.
[31] Ortiz, M. & Popov, E.P. Accuracy and stability of integration algorithms for elastoplastic constitutive relations. International Journal for Numerical Methods in Engineering 1985; 21:1561–1576.
[32] Ortiz, M. & Simo, J.C. An analysis of a new class of integration algorithms for elasto-plastic constitutive relations, Int. J. Num. Meth. Eng. 1986; Vol. 23, 353–366.
[33] Owen, D.R.J. & Hinton, E. Finite Elements in Plasticity: Theory and Practice 1980. Pineridge Press, Swansea, Wales, 594 p.
[34] Parry, R.H.G. Correspondence. Géotechnique 1958; 8:183–186.
[35] Roscoe, K.H., Schofield, A.N., Wroth, C.P. On the yielding of soils. Géotechnique 1958; 8, 22–53.
[36] Roscoe, K.H. & Burland, J.B. On the generalised stress-strain behaviour of 'wet' clay. In: Heyman J, Leckie FA, editors. Engineering plasticity 1968. Cambridge, UK: Cambridge University Press; 535–609.
[37] Schädlich, B. & Schweiger, H.F. Modelling the shear strength of overconsolidated clays with a Hvorslev surface. Geotechnik 2014; 37:47–56. doi: 10.1002/gete. 201300016.
[38] Schofield, A.N. & Wroth, C.P. Models and soil mechanics. Critical state soil mechanics. McGraw-Hill, London, New York, 1968; 20–21.
[39] Sheng, D.C., Sloan, S.W., Gens, A., Smith, D.W. Finite element formulation and algorithms for unsaturated soils. Part 1: Theory. International Journal for Numerical and Analytical Methods in Geomechanics 2003; 27(9):745–765.
[40] Sheng, D.C., Smith, D.W., Sloan, S.W., Gens, A. Finite element formulation and algorithms for unsaturated soils. Part 2: Verification and Applications. International Journal for Numerical and Analytical Methods in Geomechanics 2003; 27(9):767–790.
[41] Simo, J.C. & Taylor, R.L. Consistent tangent operators for rate independent elasto-plasticity, Comp. Meth. Appl. Mech. Engng. 1985; Vol. 48, pp. 101–118.
[42] Simo, J.C. & Hughes, T. Computational inelasticity. Interdisciplinary applied mathematics series. Springer, 1998.
[43] Sloan, S.W. Substepping schemes for the numerical integration of elastoplastic stress-strain relations. Int J Numer Methods Eng 1987; 24: 893–911.
[44] Sołowski, W.T. & Gallipoli, D. Explicit stress integration with error control for the Barcelona Basic Model. Part II: Algorithms efficiency and accuracy. Computers & Geotechnics 2010; 37(1-2):68–81. http://dx.doi.org/10.1016/j.compgeo. 2009.07.00.
[45] Stallebrass, S.E. & Taylor, R.N. The development of a constitutive model for the prediction of ground movements in overconsolidated clay. Géotechnique 1997; 47, 235 - 253.
[46] Tsiampousi, A., Zdravković, L., Potts, D.M. A new Hvorslev surface for critical state type unsaturated and saturated constitutive models. Comput Geotech 2013; 48:156–66. doi:10.1016/j.compgeo.2012.09.010.
[47] Vukićević, M. Governing parameter method for implicit stress integration of Modified Cam Clay model, using the mean stress as the governing parameter. XIV Danube – European Conference on Geotechnical Engineering, DECGE 2010, Bratislava 2010; 274-275.
[48] Whittle, A. & Kavvadas, M. Formulation of MIT-E3 constitutive model for overconsolidated clays. J. Geotech. Eng. 1994; 120 (1), 173–198.

[49] Wilkins, M. L. Calculation of elastic-plastic flow. Methods of Computational Physics. Academic Press, N. Y. 1964; Vol 3.
[50] Yao, Y.P., Hou, W., Zhou, A.N. UH model: three-dimensional unified hardening model for overconsolidated clays. Géotechnique 2009; 59:451–69. doi:10.1680/geot.2007.00029.
[51] Yu, H.S. CASM: A unified state parameter model for clay and sand. International Journal for Numerical and Analytical Methods in Geomechanics 1998; 22, No. 8: 621-653.
[52] Zienkiewicz, O.C. & Naylor, D.J. Finite element studies of soils and porous media. Lecture, finite elements, Oden and de Arantes (eds), UAH Press 1973; 459–493.

REZIME

VALIDACIJA I IMPLEMENTACIJA *HASP* KONSTITUTIVNOG MODELA ZA PREKONSOLIDOVANE GLINE

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Za široku primenu konstitutivnih modela za tlo u savremenoj inženjerskoj praksi postoje dva bitna uslova: a) model treba dovoljno dobro da predviča ponašanje tla pri razlicitim putanjama napona; b) materijalne konstante modela mogu da se odrede iz standardnih opita. Uvažavaju´i oba uslova, formulisan je HASP model za opisivanje mehanickog ponašanja prekonsolidovanih glina, koriste´i teoriju kriticnog stanja i koncept granicne površi. HASP model, na jednostavan nacin, prevazilazi mnoge nedostatke Modifikovanog Cam Clay modela, bez uvočenja dodatnih materijalnih parametara. Formulacijom zakona ojacanja u funkciji parametra stanja i stepena prekonsolidacije, omogu´eno je opisivanje brojnih elemenata mehanickog ponašanja prekonsolidovanih glina. HASP model je implementiran u program Abaqus koriste´i Metodu vode´eg parametra za numericku integraciju konstitutivnih relacija. U radu je prikazana validacija HASP modela – porečenjem s publikovanim rezultatima triaksijalnih opita, kao i mogu´nosti modela da adekvatno predvidi ponašanje prekonsolidovanih glina putem analize granicnog (konturnog) problema metodom konacnih elemenata. Razmatran je problem konsolidacionog sleganja tla usled fazne izgradnje nasipa na površini zasi´ene prekonsolidovane gline, za razlicite stepene prekonsolidacije.

Ključne reči: konstitutivni model, prekonsolidovane gline, parametar stanja

SUMMARY

VALIDATION AND IMPLEMENTATION OF *HASP* CONSTITUTIVE MODEL FOR OVERCONSOLIDATED CLAYS

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There are two important conditions for the wide application of constitutive models for soil in contemporary engineering practice: a) the model should predict sufficiently well the soil behaviour at different stress paths; b) the material constants of the model can be determined from standard laboratory tests. Taking into account both conditions, a HASP model has been formulated to describe the mechanical behaviour of the overconsolidated clays, using the critical state theory and the boundary surface concept. The HASP model in a simple way overcomes many deficiencies of the Modified Cam Clay model, without introducing any additional material parameters. The formulation of the hardening rule in the function of the state parameter and overconsolidation ratio, allows the description of numerous elements of the mechanical behaviour of the overconsolidated clays. The HASP model has been implemented in software Abaqus using the Governing Parameter Method for the numerical integration of constitutive relations. The paper presents validation of the HASP model in comparison with the published results of triaxial tests as well as the possibilities of the model to adequately predict the behaviour of the overconsolidated clays through the analysis of the boundary value problem using the finite element method. The problem of the clay settlements due to phased construction of the embankment on the saturated clay surface was analyzed, assuming different overconsolidation ratios.

Key words: constitutive model, overconsolidated clays, state parameter