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S-29

NUMERIČKA ANALIZA BOČNO-TORZIONOG IZVIJANJA U PROFILA OD ALUMINIJUMSKE LEGURE

Rezime:

U radu je prikazana analiza bočno-torzionog izvijanja elemenata U poprečnog preseka od aluminijumske legure. Analizirani elementi su statičkog sistema proste grede, opterećeni linijskim opterećenjem koje deluje u nivou gornje flanše, u ravnima koje prolaze kroz težište preseka i kroz sredinu gornje flanše. Numerička analiza je sprovedena korišćenjem programskog paketa ABAQUS. Svrha ove analize je procena prikladnosti izraza za elastičan kritični moment bočno-torzionog izvijanja elemenata konstantnog poprečnog preseka, simetričnog samo u odnosu na jaču osu inercije, datog u EN 1999-1-1.

Ključne reči: kritični moment, U profil, bočno-torziono izvijanje, ABAQUS

NUMERICAL LATERAL BUCKLING ANALYSIS OF ALUMINIUM ALLOY BEAMS WITH CHANNEL SECTIONS

Summary:

This research paper presents the numerical analysis of lateral-torsional buckling of aluminium alloy beams with channel cross-section. Analysed structural elements are simply supported beams, with line load uniformly distributed over the top flange, in the direction towards the centroid of the cross-section and in the middle of the top flange. Numerical analysis is conducted using the finite elements software ABAQUS. The purpose of this analysis is estimation of appropriateness of the equation for elastic critical moment for lateral buckling of beams with uniform cross-sections, symmetrical only about the major axis, according to EN 1999-1-1.

Key words: critical moment, channel section, lateral-torsional buckling, ABAQUS

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1 INTRODUCTION

The lateral-torsional buckling phenomenon of mono-symmetric cross-sections that are symmetrical about the minor axis and of doubly-symmetric cross-sections is well understood. However, civil engineers encounter difficulties when the geometry of the cross-section does not coincide with the geometry for which the expressions are available in the standards. Example of this problem is a channel section beam. If the horizontal eccentricity of the load relative to shear centre is included, then the problem becomes even greater. The research presented here is instigated by the insufficiency of available information for determining the lateral-torsional buckling resistance of a channel section beam that is eccentrically loaded relative to shear centre. The entire paper is only one part of the Master's thesis, which, in addition to the analysis of the critical moment for lateral-torsional buckling, focuses on the ultimate bending resistance of a real channel section beam with initial imperfections.

2 DESIGN AGAINST LATERAL BUCKLING ACCORDING TO EUROPEAN STANDARDS

Lateral-torsional buckling of a structural element with cross-section asymmetrical about the minor principal axis, such as a channel section beam, is not thoroughly covered in the corresponding European standards. In EN 1993-1-1, the expression for calculation of the critical moment of C-section beams is not provided. However, the necessary equation is available in the EN 1999-1-1, in annex I, which addresses the lateral-torsional buckling phenomenon. Important to mention regarding this particular expression is the fact that it is only valid in a situation where the load is applied in the vertical plane which goes through shear centre. Considering the possible load positions relative to shear centre on the channel section structural elements, it can be concluded that this expression is relevant only for theoretically possible scenarios, not for what can be expected in real situations. The expression given in the EN 1999-1-1 has the following form:

$$M_{cr} = \mu_{cr} \frac{\pi \sqrt{\mathrm{E}I_z \mathrm{G}I_t}}{L} \tag{1}$$

where relative non-dimensional critical moment is:

$$\mu_{cr} = \frac{C_1}{k_z} \left[\sqrt{1 + \kappa_{wt}^2 + \left(C_2\, \varsigma_g - C_3\, \varsigma_j\right)^2} - \left(C_2\, \varsigma_g - C_3\, \varsigma_j\right) \right] \tag{2}$$

non-dimensional torsion parameter is:

$$\kappa_{wt} = \frac{\pi}{k_w L} \sqrt{\frac{\mathrm{E}I_w}{\mathrm{G}I_t}} \tag{3}$$

relative non-dimensional coordinate of the point of load application relative to shear centre is:

$$\varsigma_g = \frac{\pi z_g}{k_z L} \sqrt{\frac{\mathrm{E}I_z}{\mathrm{G}I_t}} \tag{4}$$

relative non-dimensional cross-section mono-symmetry parameter is:

$$\varsigma_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{\mathrm{E}I_z}{\mathrm{G}I_t}} \tag{5}$$

Following parameters are included in the previous equations:
E - modulus of elasticity,
L - length of the beam between lateral restraints,
I_w - warping constant,
G - shear modulus,
I_t - torsion constant,
I_z - moment of inertia about the minor principal axis,
k_z and k_w - buckling length factors, which are dependent on the boundary conditions,
C_1, C_2 and C_3 - factors which are dependent on the shape of the bending moment diagram, end restraint conditions, type and position of the load and factor of asymmetry of the cross-section about the major principal axis,
z_g and z_j - load position coordinates relative to shear centre.

It can be demonstrated that, if the equation for critical moment of lateral-torsional buckling is rearranged in a manner that all of the dimensionless parameters and coordinates of the loading position are inserted in expression (2) in their expanded form, it becomes the same as the equation given in EN 1993-1-1:

$$\begin{aligned}
M_{cr} &= \mu_{cr}\frac{\pi\sqrt{\mathrm{E}I_z\mathrm{G}I_t}}{L} \\
&= \frac{\pi\sqrt{\mathrm{E}I_z\mathrm{G}I_t}}{L}\frac{C_1}{k_z}\left[\sqrt{1+\kappa_{wt}{}^2+\left(C_2\varsigma_g-C_3\varsigma_j\right)^2}-\left(C_2\varsigma_g-C_3\varsigma_j\right)\right] \\
&= \frac{\pi\sqrt{\mathrm{E}I_z\mathrm{G}I_t}}{L}\frac{C_1}{k_z}\left[\sqrt{1+\frac{\pi^2\mathrm{E}I_w}{(k_wL)^2\mathrm{G}I_t}+\frac{\pi^2\mathrm{E}I_z}{(k_zL)^2\mathrm{G}I_t}\left(C_2z_g-C_3z_j\right)^2}-\frac{\pi}{k_zL}\sqrt{\frac{\mathrm{E}I_z}{\mathrm{G}I_t}}\left(C_2z_g-C_3z_j\right)\right] \\
&= C_1\frac{\pi\sqrt{\mathrm{E}I_z\mathrm{G}I_t}}{k_zL}\cdot\frac{\pi\sqrt{\mathrm{E}I_z}}{k_zL\sqrt{\mathrm{G}I_t}}\left[\sqrt{\frac{(k_zL)^2\mathrm{G}I_t}{\pi^2\mathrm{E}I_z}+\left(\frac{k_z}{k_w}\right)^2\frac{I_w}{I_t}+\left(C_2z_g-C_3z_j\right)^2}-\left(C_2z_g-C_3z_j\right)\right] \\
&= C_1\frac{\pi^2\mathrm{E}I_z}{(k_zL)^2}\left[\sqrt{\left(\frac{k_z}{k_w}\right)^2\frac{I_w}{I_z}+\frac{(k_zL)^2\mathrm{G}I_t}{\pi^2\mathrm{E}I_z}+\left(C_2z_g-C_3z_j\right)^2}-\left(C_2z_g-C_3z_j\right)\right]
\end{aligned} \tag{6}$$

This is the general formula which refers to the standard mono-symmetric cross-sections, which are symmetric about the minor axis. In this case, the only difference between critical moment of steel and aluminium element lies in Young's modulus of elasticity and in factors C_1, C_2 and C_3. In a situation such as this, when factors k_z and k_w are equal to 1, factors C_1, C_2 and C_3 barely represent a difference, when comparing steel and aluminium simple beam elements with identical cross-sections, geometry and boundary conditions, loaded with uniformly distributed line load. The expression (1) is valid for cross-sections symmetrical about the minor axis; therefore, in order for it to be valid for cross-sections symmetrical only about the major axis, with load perpendicular to the major axis, directed towards the shear centre, the only change that needs to be made is the exclusion of the z_j coordinate. If the same change is made in the expression (6), it becomes the expression for doubly-symmetric I cross-sections. Because of the commonality of these expressions - on the one hand for cross-sections symmetrical only about the major axis, with load applied perpendicular to the major axis in the plane going through the shear centre, which is given in EN 1999-1-1, and, on the other hand, for doubly-symmetric I cross-sections, where load is applied in the symmetry plane, which is given in EN 1993-1-1, the simulation of lateral-torsional buckling using commercial finite element software ABAQUS is conducted. The main purpose is determination of deviations between the values obtained using numerical method and values obtained according to the expressions available in the standard.

3 SIMULATION USING FINITE ELEMENTS SOFTWARE ABAQUS

The basic problem related to the calculation of the lateral-torsional buckling capacity of a channel section member is the eccentricity of the load in horizontal direction relative to shear centre. The position of the shear centre in the C-section is not in the material part of the cross-section; therefore, load application in the point which represents the shear centre is not as straightforward as it may seem to be. Using finite elements software ABAQUS, simply supported beams with channel cross-section are modelled. The basic material used is aluminium alloy EN-AW 6061-T6/T651, with most important physical and mechanical properties shown in Table 1.

Table 1 - Physical and mechanical properties of the material EN-AW 6061-T6/T651

Density (kg/m^3)	Melting point (°C)	Poisson's ratio (-)	Elasticity modulus (N/mm^2)	Yield strength (N/mm^2)	Tensile strength (N/mm^2)	Elongation at failure (%)
2700	582-652	0,3	70000	240	290	≈15

The structural element that is analysed is manufactured using the process of extrusion, which means that the cross-section can be shaped arbitrarily. However, the modelling process required some modifications of the cross-section's geometry (Figure 1).

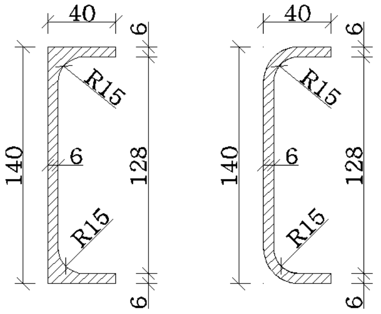

Figure 1 - Real cross-section and cross-section modelled in ABAQUS

As shown in Figure 1, the modelled cross-section has curved edges. Another option was using cross-section with sharp edges, neglecting the rounded parts; however, in a parallel analysis of these two types of cross-section, it was observed that, in the case of the cross-section with sharp edges, the critical moment is rising with the increase of the horizontal eccentricity of the load; that kind of behaviour is not what should be counted on, because it is not safe to assume that the eccentricity is helping to increase the member’s resistance against the lateral buckling. Therefore, the cross-section with curved edges was adopted because of the observed decrease of the critical moment with the increase of the horizontal eccentricity of the load relative to shear centre. The position of the shear centre is, referring to Figure 1, 12.36 mm left of the edge of the web. Parametric numerical analysis was carried out: the span lengths of the beams and position of the line load are varied. Three span lengths are analysed: 1.5 m, 2 m and 2.5 m, with the line load positioned on the top flange, directed towards the centroid and in the middle of the top flange. The full stress-strain relationship is not of particular interest in this analysis, because of the fact that the lateral-torsional buckling of the analysed structural elements is taking place in the elastic area far below the material proportionality limit, which represents the point in the stress-strain diagram in which the non-linearity effect is starting to gain influence. For material EN-AW 6061-T6/T651, the proportionality limit is in between 120 and 140 MPa, which significantly exceeds the stresses that can be achieved in the linear lateral-torsional buckling analysis of the elements described. The finite elements used for this analysis in ABAQUS software are S4R, doubly-curved four-node shell elements. The average size of the finite element is 6 mm in both directions, which remains constant throughout the analysis. Geometry was defined by sketching the cross-section, extrusion to the desired span length and by assignment of the thickness to the shell elements. The thickness of the web and flanges is 6 mm and it remains constant in all of the elements analysed. The most significant parameters for definition of the material are Poisson’s ratio and Young’s modulus of elasticity; for the numerical analysis purposes, it is sufficiently correct to assume the value of Poisson’s ratio to be 0.3 and the value of Young’s modulus of elasticity to be 70000 GPa. Assigning the material and defining the element’s geometry is, generally, simple, when the topic is stability analysis of the described elements. Slightly greater challenge is defining the line load which is not applied on the free edge of the element, but which is positioned on the outside surface, in the specified position. In the finite elements software ABAQUS it is possible to apply the line load directly on the model consisted of shell elements, but only if it is applied on the free edge. In order for the load to be positioned on the outside surface – in this case on the top flange – the following steps are necessary:

1) Forming the partition so that it becomes possible to isolate a single line on which the load will be placed,
2) Creating the reference point which is positioned above the element, in the direction of the centre of the newly formed line,
3) Applying the concentrated force in the reference point,
4) Connecting the reference point and the line formed by partitioning of the element’s face, using continuum distributing coupling constraints.

In this way, the concentrated force is distributed uniformly throughout the line. For purposes of the analysis, the values of the concentrated forces are equivalent to the uniformly distributed line load with magnitude of 1 kN/m, multiplied by the span length.

Besides modelling of the line load on the shell's surface, defining the material properties and forming the finite elements mesh, the next crucial step, which influences the results in a great manner, is the definition of the support conditions. The essential factor which requires attention while modelling the support conditions for the purposes of lateral-torsional buckling analysis is the proper simulation of the cross-section's rotation about the major and minor principal axis. Acknowledging the fact that the analysed element is a simply supported beam, the support conditions should permit the rotation of the cross-section about the major and minor principal axis, as well as warping of the cross-section. Rotation about the longitudinal axis should be restrained, which corresponds to the assumption of the fork support conditions. At one end, the translation in the direction of the longitudinal axis is restrained, while on the other end it is allowed. On both ends the translation in the plane of the cross-section is restrained. One of the methods for defining the support conditions which will satisfy the above-mentioned is the application of the kinematic coupling constraints. The reference point, with purpose of kinematic connection of the support section's nodes and assignment of the unique support conditions for the entire cross-section, is defined in the centroid of the cross-section. In this case, the support conditions at one end keep the rotation about the longitudinal axis, as well as the translation in the direction of the longitudinal axis and the translation in the plane of the cross-section - restrained, while, at the other end, the same conditions remain, with the exception of the translation in the longitudinal axis direction, which is not restrained. Support conditions defined in this manner allow the requested rotations about the major and minor axis of inertia; however, deviations observed with respect to the value of the critical moment obtained from expression (1) are up to 20%, which is not negligible, considering the fact that the analysed elements are slender elements with relatively small spans. In other words, the critical moment obtained from the finite elements analysis is much greater than the one calculated using expression (1). For the purpose of explanation of these influences, the equivalent steel FE models with I cross-sections are formed. Two span lengths are analysed: 2.5 m and 3 m. The geometry is selected so that all of the cross-section's parts can be classified as class 1, 2 or 3 (Figure 2), according to the European standards, so that global instability would manifest itself as the first buckling mode, without the local buckling effects.

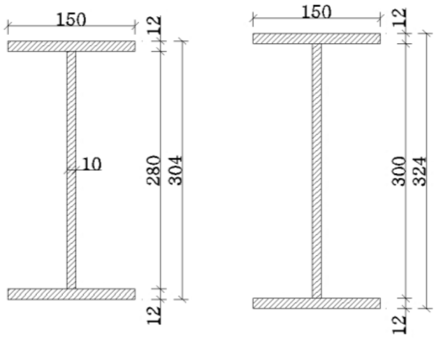

Figure 2 - Geometry of I cross-sections for 2.5m and 3m spans, respectively

In both cases, deviations observed with respect to expression (6) reach the value of 54% for both models. It means that, with supports defined as it was previously explained, the critical moment is more than two times greater than the critical moment expected, based on expression (6). In both cases, critical moments obtained from expression (6) are 100.67 kNm and 79.3 kNm, respectively. The values obtained from the finite elements analysis are 219.44 kNm and 169.52 kNm, respectively. Since this type of support conditions did not appear to be sufficiently flexible, new support conditions are designed. The reference point in the centroid of the cross-section is kinematically connected only with three nodes closest to the centroid and the translation in the direction of the longitudinal axis is restrained in that point (Figure 3). It is possible that, in previously described support conditions, the constraints defined in such manner are restraining the longitudinal displacements of the nodes farther away from the centroid, which results in decreased rotational capacity of the cross-section. In all of the nodes of the support cross-section the translation in the plane of the cross-section and the rotation about the longitudinal axis of the element are restrained.

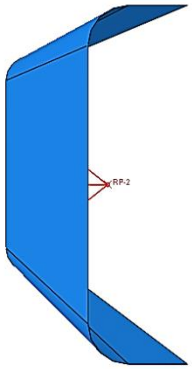

Figure 3 - Nodes connected to the centroid of the C-section beam

The support conditions, defined as described above, are tested in the analysis of I cross-section elements. The results obtained from this analysis are different from those obtained from expression (6) by no more than 2%, which is a satisfying precision. Shown in Figure 4 are the first buckling modes and the corresponding eigenvalues of the analysed I-section beams. The values of critical moments are 98.55 kNm and 78.41 kNm, respectively.

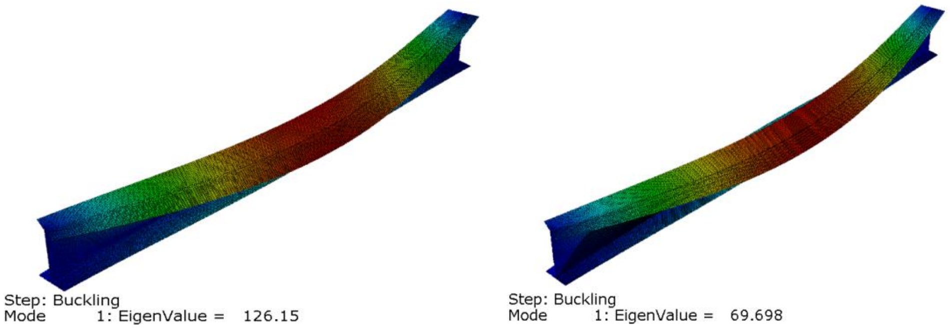

Figure 4 - First buckling modes and eigenvalues of the I-section beams

4 RESULTS AND DISCUSSION

After the conducted buckling analysis of the channel cross-section simple beam elements with spans of 1.5 m, 2 m and 2.5 m, with load position on the top flange level, directed towards the centroid and in the middle of the top flange, appropriate buckling modes and eigenvalues are obtained (Figure 5a-5c). On the left side, the results are presented for the situation when the load is applied on the top flange, in the plane that goes through the centroid of the cross-section and on the right side - when the load is applied in the middle of the top flange.

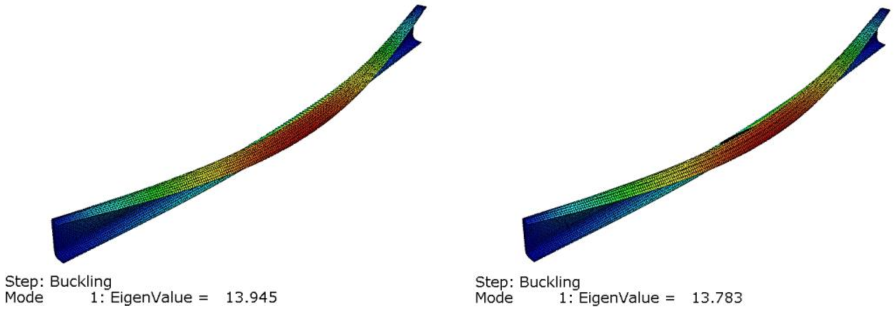

Figure 5a - First buckling modes and eigenvalues of channel beams with span length of 1.5 m

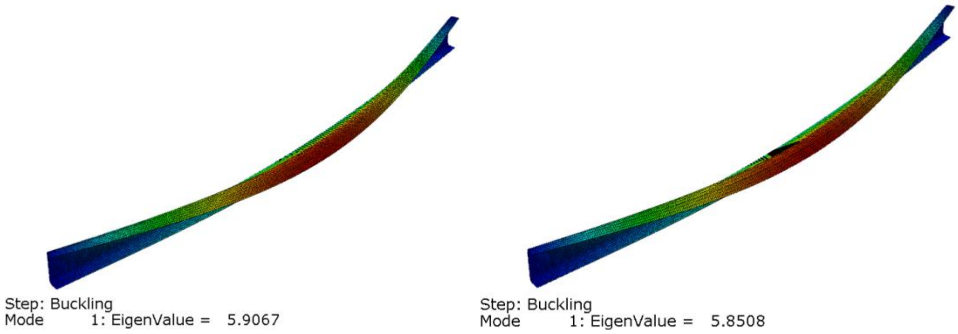

Figure 5b - First buckling modes and eigenvalues of channel beams with span length of 2 m

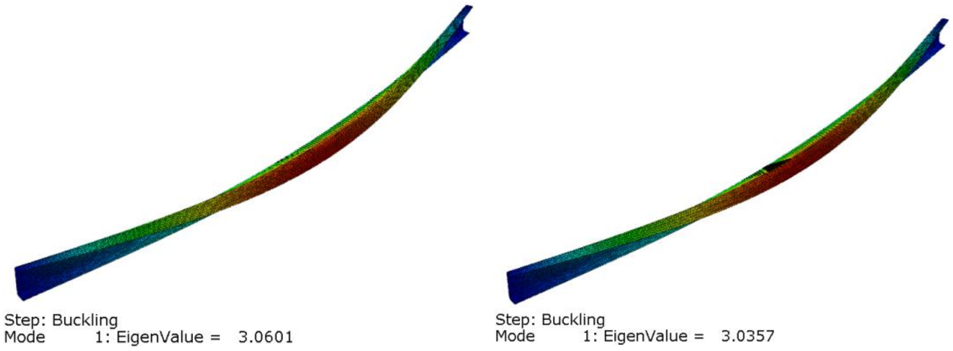

Figure 5c - First buckling modes and eigenvalues of channel beams with span length of 2.5 m

The value of the critical moment of lateral-torsional buckling can be calculated using a well-known formula for maximum bending moment in the middle of the span of a simply supported beam:

$$M_{cr} = qL^2/8 \tag{7}$$

where q is the line load which can be calculated by multiplying the applied line load with magnitude of 1 kN/m by the factor represented by the eigenvalue obtained from the buckling analysis. Values of the critical moments calculated using expression (7) were then compared to the values obtained from expression (1), with the remark that geometrical properties used in expression (1) correspond to the cross-section with rounded edges. In the interest of better transparency of the comparison of the results obtained using numerical method and the values calculated using expression (1), the results are shown in the Table 2.

Table 2 - Comparison of the critical moments

Span length	1,5 m		2 m		2,5 m	
Load position	Centroid	Middle of the flange	Centroid	Middle of the flange	Centroid	Middle of the flange
Abaqus (kNm)	3,922	3,876	2,953	2,925	2,391	2,372
EC9 (kNm)	3,926	3,926	2,966	2,966	2,408	2,408
Deviation (%)	0,1	1,27	0,44	1,38	0,71	1,5

As shown in Table 2, for analysed spans and load positions, deviations of the values obtained using finite elements software ABAQUS from the values calculated using expression (1) do not exceed the value of 1.5%, which can be considered as a very good precision. Critical moment values shown in Table 2 that are calculated using the European standard for aluminium structures design are only relevant for the situation when the applied load is directed towards the shear centre. The load defined in ABAQUS is horizontally eccentric, which means that, acknowledging the fact that values of the critical moment are decreasing with the increase of the eccentricity, it is logical to assume that the lateral-torsional buckling resistance will be increased if the load is applied in the plane that is going through the shear centre. In that case, it is possible that the values of the critical moment obtained from the ABAQUS software surpass the values calculated using expression (1). That means that, in theoretically possible situations, when the load is directed towards the shear centre, expression (1) could be used with a certain safety factor relative to the results obtained using the software. Considering that the deviations shown in Table 2 are relatively small, it becomes conceivable that, if the formula for the critical moment of lateral-torsional buckling is modified in a way that the horizontal eccentricity effect is included, the values calculated using the formula could have a safety factor relative to the values obtained from the numerical analysis. However, that is what further analysis should determine.

5 CONCLUSIONS

Presented in this research paper is the lateral-torsional buckling analysis of aluminium alloy channel cross-section beams. Material of which the element is comprised is EN-AW 6061-

T6/T651. The greatest difficulty encountered while calculating the critical moment using the equations provided in the standards is the horizontal eccentricity of the load relative to shear centre. European standard for design of aluminium structures EN 1999-1-1 addresses the topic, but not in a way that would remove all of the uncertainties, because of the assumption that the vertical load is directed towards the shear centre, which is primarily theoretically possible, considering the position of the shear centre in the C cross-section. The elements analysed are simple beams with span lengths of 1.5 m, 2 m and 2.5 m, with line load positioned on the top flange, directed towards the centroid, in one case, and applied in the middle of the top flange, in the other. Conclusions which can be derived from the presented analysis are:

- Expression for calculation of the critical moment of beams with cross-sections symmetrical about the major principal axis, with load applied perpendicular to the major axis in the direction of the shear centre, which is available in EN 1999-1-1, annex I, is the same expression given in EN 1993-1-1, for calculation of the critical moment of the beams with standard doubly-symmetric I cross-sections, with the load applied in the symmetry plane.
- The depicted method of modelling using finite elements software ABAQUS provides satisfying results, in terms of buckling analysis, when used for doubly-symmetric I-section beams. For models analysed, the deviations of the FEA values obtained using ABAQUS from the values calculated using the formula for critical moment provided in EN 1993-1-1 are in between 1% and 2%.
- The deviations between the FEA values of the critical moment of channel section beams and the corresponding values calculated according to EN 1999-1-1 are in range from 1% to 1.5%.
- FEA critical moment values are decreasing with the increase of the horizontal eccentricity of the load; therefore, it can be concluded that the lateral-torsional buckling capacity would increase if the load is applied in the plane directed towards the shear centre. Considering relatively small deviations from the values calculated using the standard, it is assumed that the expression for critical moment could provide results with a certain safety factor relative to the FEA values, if the expression is modified so that it includes horizontal eccentricity of the load. However, further analysis is necessary in order for that assumption to be proven.

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