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FREE AND FORCED VIBRATION ANALYSIS OF DELAMINATED COMPOSITE PLATES OF ARBITRARY SHAPE USING TRIANGULAR LAYERED FINITE ELEMENTS

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Abstract:

In the paper, free and forced vibrations of laminated composite plates having complex shape and embedded delaminations, have been considered. Computational model has been derived based on Reddy's Generalized Laminated Plate Theory (GLPT), assuming layerwise linear variation of in-plane displacements and constant transverse displacement through the plate thickness. The assumed kinematics allows for the real cross sectional warping. The delamination openings in three orthogonal directions have been implemented using Heaviside step functions. Linear kinematics and Hooke's constitutive law have been considered. Numerical solution has been obtained using the triangular 6- node layered finite elements, to approximately simulate the arbitrary geometry of the plate. To prevent layer overlapping in the delaminated zone, a novel node-to-node frictionless contact algorithm has been implemented in the computational model.

Delamination propagation has been predicted using a simple and efficient algorithm based on the Virtual Crack Closing Technique (VCCT), requiring the calculation of the virtually closed area in front of the delamination, the delamination openings behind the delamination and the reaction forces along the delamination front. All calculations have been performed using the original MATLAB code. The results have been compared with the existing data from the literature and the results from the commercial software. A variety of new results has been provided to serve as a benchmark for further investigations.

Key words: composite; delamination; GLPT; propagation; contact; forced and free vibrations.

1. Introduction

In the construction of efficient and lightweight engineering structures and their components, in mechanical, aeronautical and civil engineering, as well as in the aircraft, aerospace or automotive industry, laminated composite plates and shells play an important role due to the excellent performance in different loading conditions. During their service life, laminated composite plate structures suffer from the structural defects in the form of delamination (separation or debonding between the adjacent layers), leading to the severe reduction of plate stiffness. This cause the considerable change in plate dynamic properties.

Since delamination is one of the major failure modes of composite laminates, considerable research efforts have been continuously invested to develop the robust and accurate computational models, capable to describe the delamination kinematics, including both stationery and propagating delamination. Modeling of a propagating delamination is of the paramount importance in prediction of the progressive collapse of a structural components.

For the modeling of delamination growth in laminated structures a number of computational strategies exist. The well-known Strain Energy Release Rate (G) is the measure which usually serves as a criterion for delamination growth [1-3]. It is divided in three components G_{I}, G_{II} and G_{III} (depending on the fracture mode) and then compared with the interlaminar fracture toughness, which is a material property obtained experimentally. The Virtual Crack Closing Technique (VCCT) [4-6] is an approximate method which assumes that the strain energy released during the delamination growth is equal to the work required to close the crack to its original length. The application of the Virtual Crack Closing Technique requires the calculation of nodal forces along the delamination front, delamination openings behind the delamination front and the virtually closed area in front of the existing delamination, in the single calculation step of the finite element computation. This requires the additional degrees of freedom to be incorporated in numerical model, to describe the relative displacements between the upper and lower portions of the composite laminate in the delaminated zone of the plate. The previous requirements can be conveniently achieved using the Generalized Layerwise Plate Theory (GLPT) of Reddy [7-8], because the finite element model based on the enhanced GLPT is capable to conveniently describe the independent motion of the adjacent layers during free [9] and forced vibrations [10].

In the paper, the computational model based on the enhanced GLP Theory and FEM has been considered. The tracking of moving delamination front has been performed using the modified algorithm of Xie et al. [11, 12], which is further improved by Riccio et al. [13] and Marjanović et al. [14]. The VCCT approach has been extended in this paper to the unstructured mesh of 6-node layered triangular elements. The computational model for free and forced vibration analysis of laminated composite plates with embedded delamination has been implemented into an original MATLAB [15] code. GiD Pre/Post Processor [16] with an originally coded *problemtype* ”2D_T6” has been used for the generation of numerical models and the post processing.

The model is validated against the results from the commercial software ABAQUS based on the classical FEM and Generalized Differential Quadrature FEM (GDQFEM) based on the First-Order Shear Deformation Theory [17]. The presented model allows for the consideration of laminated composite plates of arbitrary shape. Along with the verification study, a variety of new results is provided as a benchmark for further investigations.

2. Triangular layered finite element based on the GLPT with enhanced kinematics

Due to the layerwise expansion of displacement components through the plate thickness, GLP Theory allows the independent interpolation of in-plane and out-of-plane displacement components, including the possible jump discontinuities at the interfaces between layers. Heaviside step function is introduced to simulate the jumps at the delaminated interfaces. Piece-wise linear variation of the in-plane displacement components and the constant transverse displacement through the thickness are imposed, which provides the plane stress description of every material layer. Cross-sectional warping is taken into account. In the paper, the authors consider a laminated composite plate composed of n orthotropic laminae. The assumptions of the extended GLPT are provided in [8, 14].

The FE model based on the GLPT [8-10, 14] consists of the middle plane, N numerical layers through the plate thickness (except the middle plane), and finally an additional numerical layer in which the delamination occurs. The layered finite elements require only C0 continuity of the generalized displacements along element boundaries, because only translational displacement components are adopted as the nodal degrees of freedom. It is important to highlight that the out of plane coordinate has been eliminated in the calculation after the explicit integration of the displacement field in out of plane (z) direction. This allows the formulation of the family of the two-dimensional layered (plate) finite elements. In the paper, quadratic layered triangular elements (6-node) have been considered. Reduced integration is adopted to avoid locking.

After the standard finite element calculation, the displacement field obtained in the finite element analysis is used for the computation of three modes of the Strain Energy Release Rate along the crack front using the Virtual Crack Closure Technique. The algorithm for tracking a moving delamination front presented here is based on the assumptions from Refs. [14, 18].

The post-processing algorithm presented in this paper for the detection of the delamination front, calculation of G_{i} components, and the prediction of the delamination growth is applied in all nodes of the finite element model after each calculation step. The procedure is repeated for all delamination zones in the plate. To start, the algorithm detects all nodes in which the essential condition $U^I=V^I=W^I=0$ is satisfied (where $U^I=V^I=W^I$ are the relative displacements between the adjacent layers in three orthogonal directions [14, 18]). This is performed by checking of the vicinity of each a priori imposed delaminated node. The detected nodes are intact nodes defining the undamaged area of the plate. Nodes dividing the undamaged from the delaminated plate area are the nodes which define the node-to-node delamination front. The delaminated zone is encapsulated by the polygonal line connecting the nodes along the delamination front [14, 18].

For the embedded delaminations of arbitrary shape (which generally occur in structural applications), the orientation of the normal vector to describe the propagation direction is also arbitrary. The delamination front in the considered node on the front is defined by the two direction vectors, and the propagation vector is calculated as a unit vector along the symmetry line defined by the previously calculated direction vectors.

The virtually closed area has been finally defined using the 6 control points for every node N along the delamination front (see [14, 18] for the detail). After the six control points have been determined, the virtually closed area is calculated using the MATLAB function. Note that an overlap of the virtually closed areas corresponding to the adjacent nodes on the delamination front is a priori prevented using the explained procedure.

3. Modeling of laminated composite plate in ABAQUS CAE

A flat plate of arbitrary shape with an elliptic hole is considered in this example (see Figure 1a). The laminate is composed of three layers: two Graphite-Epoxy faces (E_1=137.9GPa, E_2=8.96GPa, G_{12}=G$_{13}$=7.1GPa, G_{23}=6.21GPa, ν_{12}=0.3, ρ=1450kg/m^3) and a core of Glass–Epoxy (E_1=53.78GPa, E_2=13.93GPa, G_{12}=G13=8.96GPa, G_{23}=3.45GPa, ν_{12}=0.25, ρ=1900kg/m3), in the following stacking sequence: (30/65/45). The sheets have a constant thickness h_s=3cm and the core thickness is equal to h_c=4cm. All external edges are clamped and the inner ellipse is free.

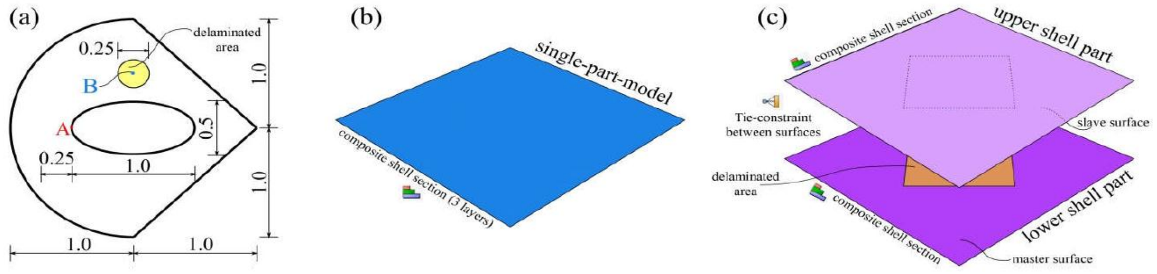

Fig. 1. (a) Plate geometry, (b) model of intact plate in ABAQUS CAE;
(c) model of delaminated plate in ABAQUS CAE

Shell model of the intact plate in ABAQUS CAE has been composed of one shell-like part with composite shell section. Composite shell section consists of three layers made of the orthotropic material with different orientations. Shell model of the delaminated plate has been composed of two different shell-like parts, which are perfectly bonded together using Tie-constraint along the intact area of the plate (see Figure 1c). The model was discretized using STRI65 finite elements (6-node quadratic triangular shell elements, with five DOFs per node). Boundary conditions are prescribed along the clamped exterior edges by restraining all degrees of freedom in nodes.

4. Numerical Example and Discussion

Free vibration analysis of the composite plate has been performed in order to validate the GLPT model against the GDQFEM solution from Ref. [17] and commercial software ABAQUS CAE. The first ten natural frequencies have been calculated for the intact plate, using two different mesh densities (app. element size 0.05m and 0.10m).

Figure 2 illustrates the results obtained using three considered computational models. Obviously, mesh density does not significantly influence the results, both in GLPT and ABAQUS CAE models. Very good agreement has been obtained between GLPT model and the other two. In general, GLPT model is in closer agreement with the GDQFEM-based model, than compared to the ABAQUS CAE model.

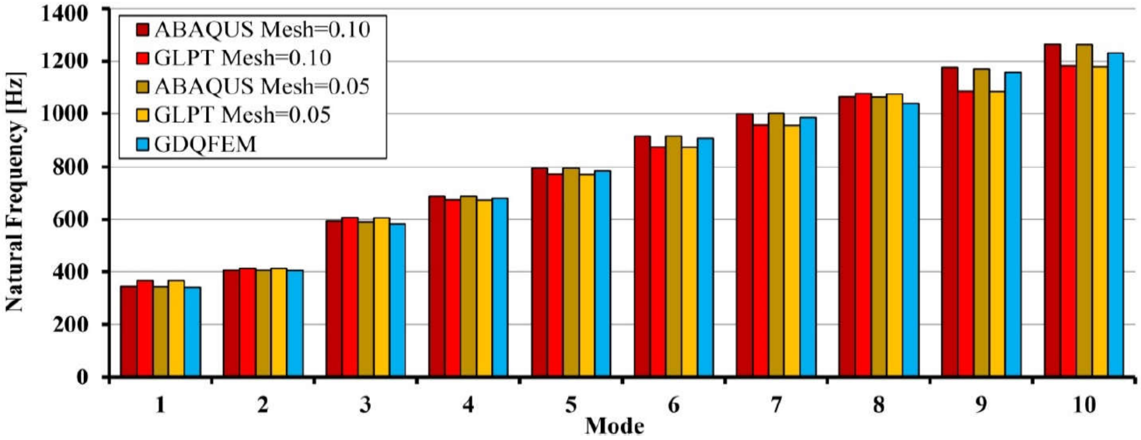

Fig. 2. Natural frequencies of intact composite plate considering different numerical models and different mesh densities

To validate the model for the delaminated plate situation, the circular embedded delamination is inserted as shown in Figure 1a. Two delamination positions through the plate thickness (i.e. between layers 1-2, and layers 2-3) have been considered. The validation is made using GLPT and ABAQUS models with a single mesh size of 0.05m.

The frequency reduction in comparison with the intact models is presented in Figure 3. For all considered cases, the reduction is more pronounced for the delamination between layers 1-2. In ABAQUS CAE, the reduction is 1.4-5.0% depending on the vibration mode, while for GLPT model it is 0.3-10.7%. Both models generally showed the same trend of results, having the higher reduction of the natural frequency in higher modes of vibration.

The transient response of both intact and delaminated composite plate has been calculated in order to check the accuracy of the proposed model for a pulse loading situation. The equally distributed pressure load has been applied over the whole plate area (q=100kPa) in the form of the step pulse lasting for T=7ms. For the solution of the time dependent problem of the GLPT-based finite element model, implicit Newmark's solver [19] with α=β=0.5 (corresponding to the constant average acceleration method) is applied with a constant time step ΔT=0.1ms. In ABAQUS CAE, implicit dynamic solver is applied with the same parameters. Both models were discretized using mesh density of 0.50m.

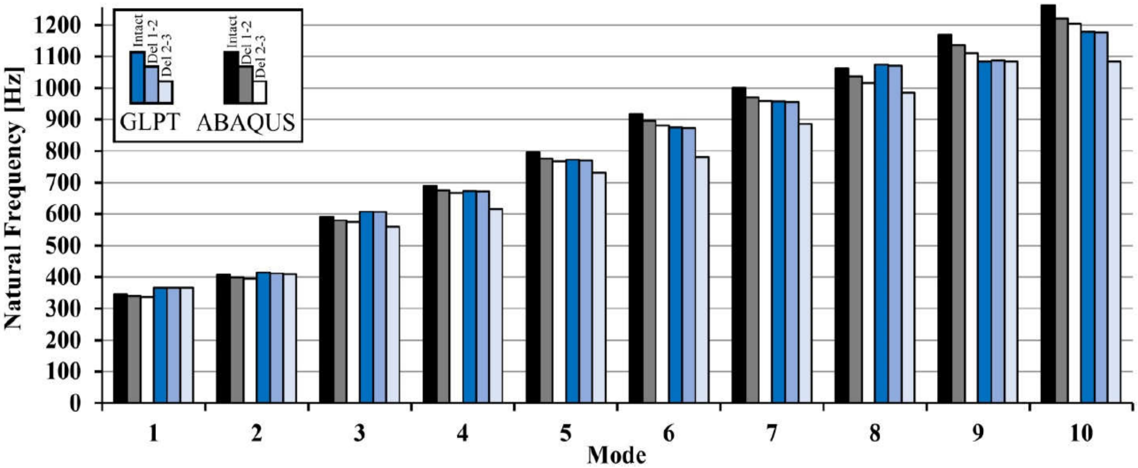

Fig. 3. Natural frequencies of intact/delaminated composite plate considering different numerical models and different delamination positions

The time history of transverse displacement of point A (see Figure 1a) is illustrated in Figure 4. Obviously, the results from the proposed GLPT-based model correspond very well to the ABAQUS CAE solution for the intact plate and both considered mesh densities. Influence of the mesh size is generally negligible, which means that mesh size of 0.10m is already fine enough. The period of vibrations is approximately 2.7ms for both models. Having in mind that the first natural frequency for intact plate is around 360Hz (see Figure 2), fundamental period is really expected to be close to 2.7ms. Only a marginal difference in amplitudes is detected.

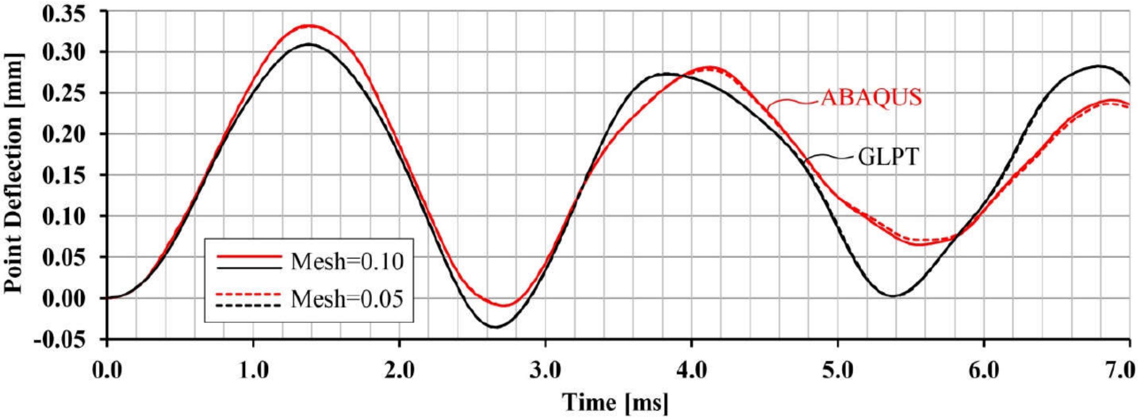

Fig. 4. Time history of transverse deflection of point A considering different numerical models and different mesh densities

After the validation of the model for intact plate, the dynamic analysis of previously described delaminated plate is performed. The time history of point B (see Figure 1a) is plotted for both models and two different delamination positions through the plate thickness (layers 1-2 and 2-3) in Figure 5. During the motion of the plate, two adjacent interfaces in the delaminated zone of the plate oscillate independently (difference between solid and dashed lines in Figure 5). The difference in transverse displacement is so-called Crack Opening Displacement (COD), which temporal evolution is also plotted in Figure 5 (thick solid lines). Obviously, in both models, higher values of COD are obtained if the delamination is located between layers 1-2. The agreement is obtained between the present model and the solution from ABAQUS CAE.

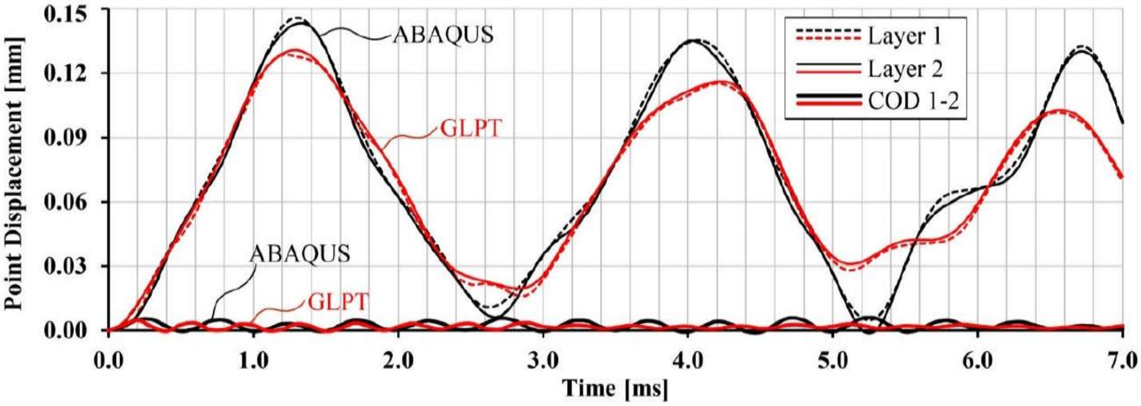

Fig. 5. Time history of transverse deflection of adjacent layers (1-2 or 2-3) and COD in point B, considering different numerical models

5. Conclusions

The computational model based on the enhanced GLPT and FEM is presented for free and forced vibration analysis of laminated composite plates with embedded delamination. The tracking of moving delamination front is performed using the novel algorithm for unstructured mesh of 6-node layered triangular elements. The presented model allows the consideration of arbitrary plate geometry and arbitrary delamination shape.

It is detected that mesh density does not severely influence the results for natural frequencies of intact composite plates. Very good agreement has been obtained between the presented model, ABAQUS CAE shell model and the GDQFEM-based model. In addition, frequency reduction due to the presence of delamination in comparison with the intact models is more pronounced for the delamination between layers 1-2. The reduction percent depends on the vibration mode - the higher is in higher modes of vibration. This dependency is, however, more pronounced in the GLPT model than in the standard shell formulation in ABAQUS CAE.

The presented model proved to be capable to predict the transient response of intact plate under pulse dynamic loading, even for the coarse mesh. The results were in good agreement with ABAQUS CAE model, having only minor differences in amplitudes, and an excellent agreement of period of vibrations.

After the validation of the model for intact plate, the dynamic analysis of delaminated plate is performed. The temporal change of transverse displacement of adjacent layers in the delaminated zone of the plate is successfully predicted using the proposed model, for different delamination positions. The temporal evolution of COD for different scenarios is also predicted and proved to be in the excellent agreement with ABAQUS CAE model. Finally, in both models, higher values of COD are obtained if the delamination is located between layers (30/65). Further work includes the tracking of moving delamination front under pulse loading and collapse prediction.

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