THREE DIMENSIONAL FRAME STRUCTURES SUBJECTED TO CYCLIC LOADING

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Summary: In this paper, three dimensional frame structures, subjected to cyclic loading are analyzed. In numerical examples, rectangular and I cross section shapes are used for elements of structures subjected to various loading cases. For definition of material behavior, elastoplastic model of material, based on the Preisach model of hysteresis is used. Some aspects of convergence, as well as numerical performance of this analysis are shown.

Keywords: *frame,cyclic loading, hysteresis*

1. INTRODUCTION

Since steel frames are widely used as structural systems in building industry, optimizing numerical analysis of this problem is of great importance. If the structure is subjected to cyclic loading in plastic domain, convergence of the analysis is hardly achieved and possibility of error increases. One of the method for describing stress-strain diagrams of material behavior is incorporating hysteretic operators in material models. In this paper, Preisach model, of hysteresis [1], which has been implemented in elastoplastic analysis of trusses subjected to cyclic loading [2], is used for the purpose of adequately modeling material mechanical properties in frames. This hysteretic operator [3] is used in various physical problems, where this phenomena occure, since it is probably the most powerful operator [4]. It is shown in[5] that analysis of trusses based on this model of hysteresis can be expanded to frame elements, if the cross section of element is divided on fiber elements, that are modeled as truss elements in [5]. Preisach model of hysteresis can also be used for defining moment-curvature relation as analytical expression in closed form [6], [7] as presented in section 2 of this paper. However, for various cross section types and arbitrary load cases, it is inefficient and difficult to evaluate analytical model for these cases. Hence, fiber element approach can provide, as it is presented in this paper, very effective elastoplastic analysis of three-dimensional frame structures subjected to cyclic loading.
In the second section of this paper, basic expressions and considerations for aplication of Preisach model of hysteresis in structural analysis are shown. In third section, numerical examples are shown and finally conclusions are presented in section 4.

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2. APPLICATION OF HYSTERETIC OPERATORS IN NUMERICAL ANALYSIS

Analytical expression for uniaxial material behavior based on the Preisach hysteretic operator is first presented in [8], [9] :

$$\sigma(t)=\frac{E}{2}\left[\int_{-\varepsilon_0}^{\varepsilon_0} G_{\alpha,\alpha}\varepsilon(t)\,d\alpha-\frac{(E-E_h)}{2}\frac{1}{Y_{max}-Y_{min}}\iint_A G_{\alpha,\beta}\varepsilon(t)\,d\alpha d\beta\right]$$ **(Error! Reference source not found.1)**

Where E is elastic modulus, E_h is hardening modulus, Y_{max} and Y_{min} are limits of initial nonlinear hardening, G□□□□is hysteretic operator and A is corresponding area in Preisach triangle [9]. This stress-strain relation can be extended in such way that different strains in cross section could define corresponding stress diagram in cross section and thus expression for moment can be derived [6]:

$$M(t)=E\int_{-h/2}^{-h/2} y\cdot b(y)\cdot\kappa(t)\cdot y\cdot dy\ -$$

$$\frac{E(E-E_h)}{4}\frac{1}{Y_{max}-Y_{min}}\int_{-h/2}^{-h/2} y\cdot b(y)\cdot\left[\iint_A G_{\alpha,\beta}\kappa(t)\cdot y\cdot d\alpha d\beta\right]\cdot dy \qquad \textbf{(2)}$$

Where b and h are dimensions of cross section and □ is curvature. Just like Preisach triangle represents geometrical interpretation of expression **(Error! Reference source not found.1)Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.**, for expression **(2)**, corresponding geometrical figure for analysis is Preisach prism (*1*(b))

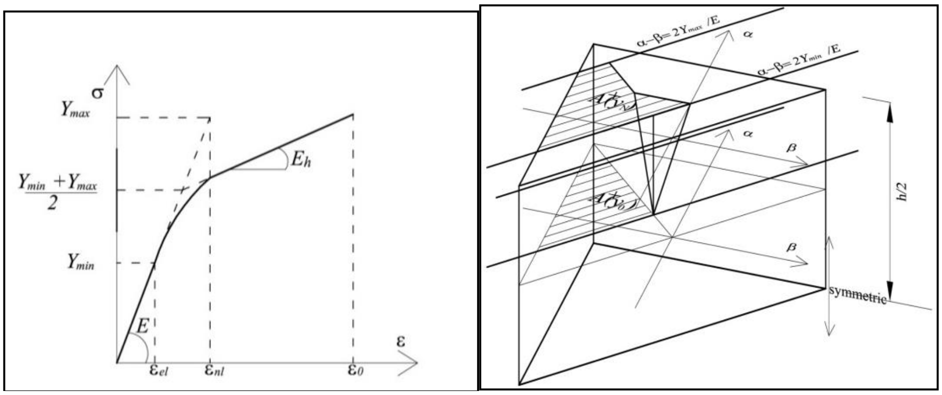

Figure 1.(a) Stress-strain relation for material defined with ***(Error!*** **Reference source not found.1)***; (b) Example of limiting (Preisach) prism in first step of loading in plastic domain (pure bending).*

Numerical implementation of expression **(Error! Reference source not found.1)** was presented in detail in [5] where equation for finite element method for trusses are derived. It was also shown that, for simple 2-dimensional frames with rectangular cross section, fibre-element approach is practical for implementation and that convergence of numerical analysis is easily achieved.
For numerical analysis in section 3 of this paper, stiffness matrix of frame cross sections is derived based on Euler-Bernoulli beam theory.
Defined 2-node frame elemens have possibility to achieve plastic deformation only at nodes (concentrated plasticity method), but effects of distributed platicity are approximated with increased number of elements per frame.

3. NUMERICAL EXAMPLES

In this paper , numerical examples represent analysis of three-dimensional frame structures. In both examples, one storey and one bay frame structure is analyzed under cyclic loading (Figure *2*(a)). Parametric analysis consisted of varying cross section types (rectangular and I cross section), varying number of elements per one frame and varying number of integration point in cross section.
Material properties are also varied in a such way that beside presented material property in Fig.*1* (E=200GPa, Eh=20GPa, Ymax=300MPa, Ymin= 248MPa) corresponding material property with no hardening (nonliner and linear) is also used in same numerical experiments. Analyzing structures and loading scheme are presented in Fig*2*. In numerical examples 1 and 2, one storey frame structure is analyzed under static and cyclic loading in form of horizontal displacement of its top.
This frame represents 3-D variation of El-Zanaty frame experiment. Material properties are presented in Figure *1*. I cross section of frame elements represents W8x31profile, while dimensions of rectangular are b/h=3cm/24cm.

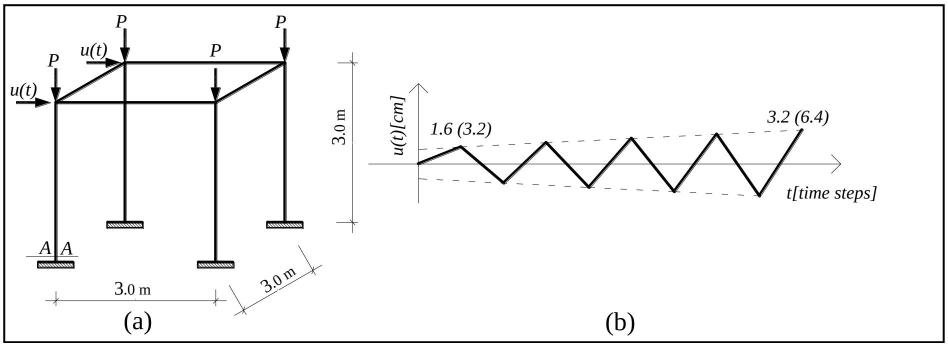

Figure 2.(a) Three dimensional structure in numerical examples; (b) Loading history for displacement function u(t);

Scheme of integration are based on rectangular rule [10] for I cross section and rectangular cross section with three levels of mesh density. The number of integration points is 32,128,512 and 8, 32, 128 for I cross section and rectangular cross section respectively.
Before loading history for displacement u(t) was applied, structure was subjected to gravity load in form of concentrated forces P=450kN (Figure *2*).
Levels of maximal loading displacement u(t) are coresponding for top structure drift of 1.06% (3.2cm) for the first loading case and 2.12%(6.4cm) for the second loading case, as shown in the Figure *2*(b).

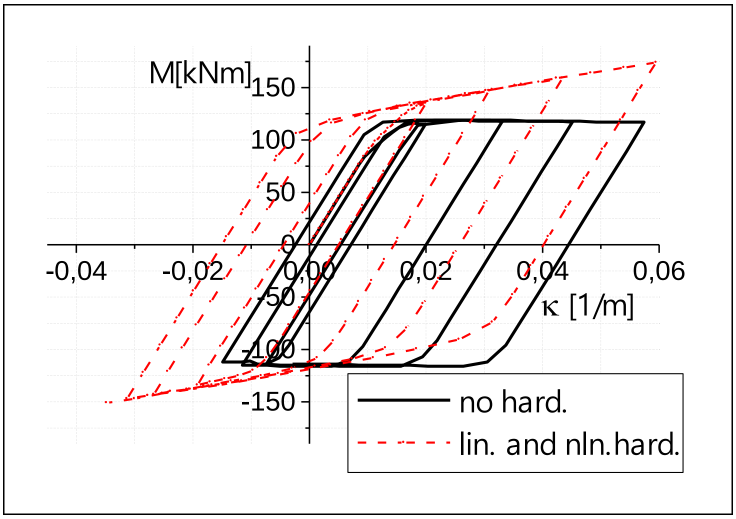

Figure 3.Moment-curvature relation for the case of 2.06% drift of top structure with rectangular cross section (material with and without hardening)

It can be seen from the Fig *3* how hardening affect material behavior in cross section. These curves adequately represent Masing law. It can be seen from the Fig *3* that hardening in material significantly influenced on increase of moment and curvature in both directions. Presented diagrams correspond to section *A-A* from figure *2*(a). That was the section with with highest stress level during loading and its section forces were used in the numerical analyis, as parameters that deteremine convergence of process.
Displayed figures present effects of increased number of elements and integration points (in cross section).
Increased number of integration points lead to converging solution, but function of errors is not always monotone, as it can be seen in the Figure *4*(b).
For the first loading case (drift 1.06%), it can be said that adequate accuracy is obtained with 30 (I cross section) and 10 (rectangular cross section) sub-elements per frame with second level of mesh density. On the other hand, for the higher level of loading (2.12%), it was needed 30 (I cross section) and 30 (rectangular cross section) elements per frame and third level of mesh densinty.
Note that convergent result could be obtained with less frame divisions and integration points if distribution of divisions would be imposed around nodes where plasticity zones

occur. In the displayed figures, errors are estimated in comparing to the exact solution. This solution is obtained when increasing number of frames and integration points in cross section did not affect the result of the numerical analysis.
It can be said that this solution adequately models corresponding effects and results that would have been obtained by distributed plasticity method.

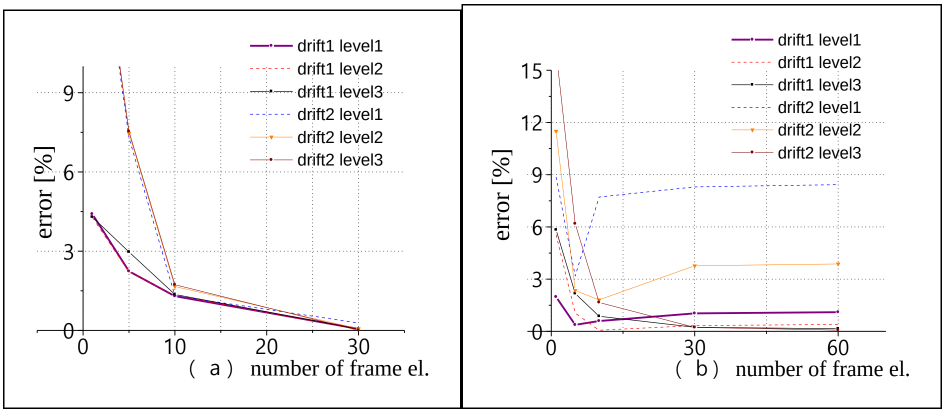

Figure 4.Convergence of results expressed through varying level of load (drift 1 and 2), and level of mesh density of integration points (level 1-3):(a)I cross section;(b)Rectangular cross section

These facts are expected, considering the number of elements (nodes) that enter plastic zone. In the second loading case (drift 2), large zones of structure and significant zone in cross section (Figure555(b)) had plastic deformation, causing higher number of iterations and, consequently, increased CPU time needed for performing calculation. For example, in the second loading case, plastic deformation occurred in about 40% of the most stressed frame element for both cross section types, while only 60% remained in the elastic state (Figure 6).

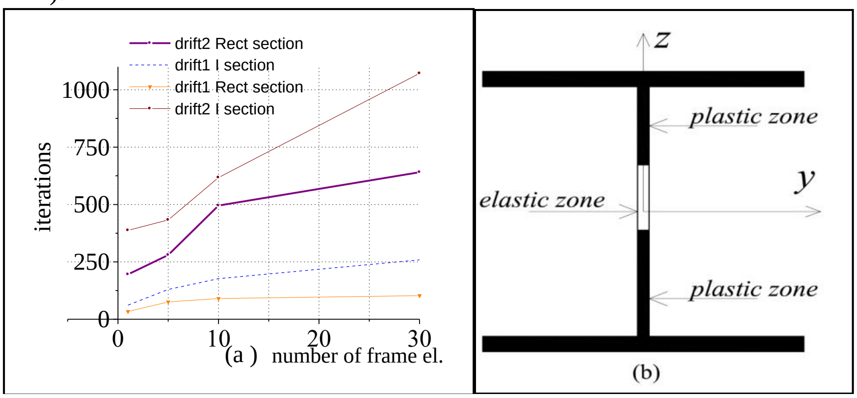

Figure 5.(a)Number of iteration performed during numerical analysis;(b)Plastification in section A-A in second loading case (drift 2)

In the Figure *4*, there are above mentioned dependancies presented. It can be seen that increased number of frame element was essential parameter that influenced process to be convergent. However, number of integration points is also important, since for the coarse mesh of integration points, solution is hardly achieved even for high number of frame divisions.
Number of elements that entered plastic zone (Figure 6) directly caused higher number of iterations (Figure *555*(a)), which needed to be performed in order to redistribution of nodal forces to result in convergent solution. It should be noted that each iteration does not demand equal amount of cpu time within structures that have only 1 element per frame in comparing to structures which elements are divided on large numbar of sub-elements in order to obtain exact solution. Therefore, computational cost for structures that have large number of sub-elements that enter plastic zone is even greater than number of iterations indicate.

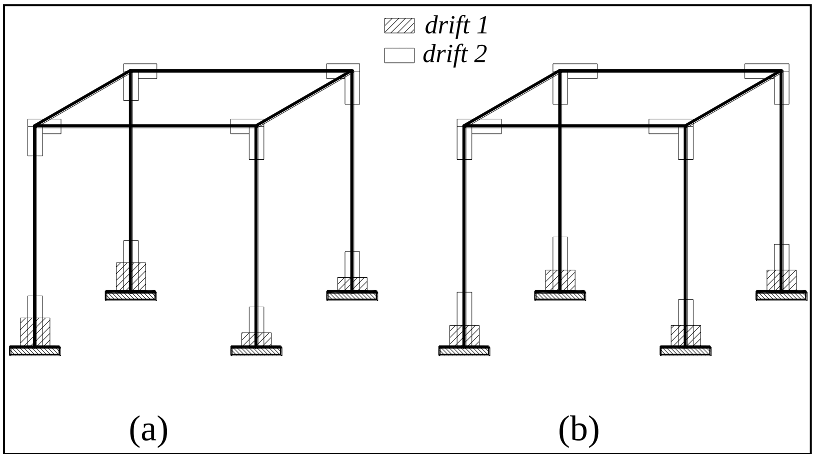

Figure 6.Zones of plastification in structure determined in first (drift 1) and second (drift 2) loading case (a)Structure with I cross sections;(b)Structure with rectangular cross sections

In the corresponding numerical examples where no hardening in material is defined, similar trends relating convergence and numerical efficiency were observed. The only difference is that absence of hardening caused higher computational effort and higher number of iterations that needed to be performed in the analysis. Plastic zones across the frame elements also increased and overall structural behavior with corresponding amount of material nonlinearity became even more questionable.

4. CONCLUSION

In this paper, three-dimensional frame structures are analyzed during cyclic loading. Plastic deformation occurrence were restricted to nodes, hence method of distributed plasticity could only be approximated with division of frame elements. Material behavior is defined by using Preisach model of hysteresis, which has shown its advantages in application in cyclic plasticity. It is shown that convergence and CPU time needed for performing the analysis greatly depend on loading level, and consequently on occurrence and size of zones of plastic deformation in cross section or in number of nodes in frame sub-elements. Convergent solution, for the cases of significant plasticity zones in frame elements, was achieved at the cost of high number of frame divisions and consequently increased CPU time. In those cases number of integration points was less important parameter of convergence. However, large plastic zones in frame are nor practical, nor desired effect in structural analysis. One would expect that plastic deformation remain localized in smaller areas in the cases that are relevant in structural analysi. In such cases, as presented in this paper, it can be said that use of concentrated plasticity method is justified form both standpoints of numerical accuracy and computational cost.

REFERENCES

[1] F. Preisach, "Über die magnetische Nachwirkung," *Z . Phys*, vol. 94, pp. 277–302, 1935.
[2] D. Šumarac and Z. Perović, "Cyclic plasticity of trusses," *Arch. Appl. Mech.*, vol. 85, pp. 1513–1526, 2015.
[3] Mayergoyz. Isaak, *Mathematical Models of Hysteresis and their Applications*. Elsevier, 1991.
[4] A. Visintin, "Mathematical models of hysteresis," in *The Science of Hysteresis*, 2006.
[5] Z. Perovic, "Elastoplastic damage analysis of trusses subjected to cyclic loading," University of Belgrade, Faculty of Civil Engineering, 2016.
[6] D. Sumarac and S. Stosic, "The Preisach Model for the Cyclic Bending of Elasto-plastic Beams," *Eur. J. Mech., A/Solids* , vol. 15, no. 1, pp. 155–172, 1996.
[7] D. Šumarac and Z. Petraškovic, "Hysteretic behavior of rectangular tube (box) sections based on Preisach model," *Arch Appl Mech*, vol. 82, no. 10, pp. 1663–1673, 2012.
[8] A. . Lubarda, D. Sumarac, and D. Krajcinovic, "Hysteretic Response of Ductile Materials Subjected to Cyclic Loads," *Recent Adv. Damage Mech. Plast.*, vol. 123, pp. 145–157, 1992.
[9] A. V. Lubarda, D. Sumarac, and D. Krajcinovic, "Preisach Model and Hysteretic Behavior of Ductile Materials," *Eur. J. Mech., A/Solids*, vol. 12, no. 4, pp. 445–470 , 1993.
[10] S. M. Kostic, "Model Generalisane Plastičnosti kod Nelinearne Analize Prostornih Okvirnih Konstrukcija," Univerzitet U Beogradu, 2013.

ПРОСТОРНЕ РАМОВСКЕ КОНСТРУКЦИЈЕ ПРИ ЦИКЛИЧНОМ ОПТЕРЕЋЕЊУ

Резиме: *У овом раду је приказана анализа тродимензионалних рамовских конструкција при цикличном оптерећењу. У нумеричким примерима, за попречне пресеке елемената рамовских конструкција изложених различитим случајевима* *оптерећења, су коришћени правоугаони и I попречни пресеци. Еластопластично понашање материјала је засновано на Прајзаковом моделу хистерезиса Приказани су неки аспекти конвергенције, као и нумеричке перформансе прорачуна.*

Кључне речи: *рам, циклично оптерећење, хистерезис*