Association of Structural Engineers of Serbia
15th CONGRESS
ZLATIBOR, SEPTEMBER 6-8 2018

ASES
ДГКС

ASES INTERNATIONAL CONGRESS PROCEEDINGS

IN COOPERATION

SUPPORTED BY

Република Србија
Министарство
просвете, науке и
технолошког развоја

SPONSORS

PERI
ŠIRBEGOVIĆ INŽENJERING
PUT INŽENJERING
ADING sastojak svake građevine
peikko group
ENERGOPROJEKT
MARTINI GRADNJA D.O.O.
CRH
ARMONT

CIP - Каталогизација у публикацији
Библиотека Матице српске, Нови Сад

624+69(082)

ASSOCIATION of Structural Engineers of Serbia. Congress (15 ; 2018 ; Zlatibor)
ASES International congress proceedings [Elektronski izvor] / Association of Structural Engineers of Serbia, 15th Congress, September 6-8th, 2018, Zlatibor ; [editors Đorđe Lađinović, Zlatko Marković, Boško Stevanović]. - Beograd : Društvo građevinskih konstruktera Srbije, 2018 (Novi Sad : Grafički centar - GRID, Fakultet tehničkih nauka Univerziteta). - 1 elektronski optički disk (CD-ROM) : tekst, slika ; 12 cm

Sistemski zahtevi: Nisu navedeni. - Nasl. sa naslovnog ekrana. - Tiraž 250. - Radovi na srp. i engl. jeziku. - Bibliografija.

ISBN 978-86-6022-070-9

a) Грађевинарство - Зборници
COBISS.SR-ID 325104647

Izdavač: **Društvo građevinskih konstruktera Srbije**
Beograd, Bulevar kralja Aleksandra 73/I

Urednici: prof. dr **Đorđe Lađinović**
prof. dr **Zlatko Marković**
prof. dr **Boško Stevanović**

Tehnički urednik: doc. dr **Jelena Dobrić**

Tehnička priprema: asist. **Nina Gluhović**
asist. **Marija Todorović**

Gafički dizajn: asist. **Tijana Stevanović**

Dizajn korica: asist. **Tijana Stevanović**

Štampa: **Grafički centar – GRID**
Fakultet tehničkih nauka Univerziteta u Novom Sadu

Tiraž: **250 primeraka**

Beograd, septembar 2018.

ДГКС
Друштво грађевинских
конструктера Србије
ASES
Association of Structural
Engineers of Serbia
S-76
15. KONGRES
6-8. 9. 2018. Zlatibor

Nikola Ivanović[1], Jelena Dobrić[2], Milan Spremić[3], Nina Gluhović[4], Jovana Milovanović[5]

NELINEARNA ANALIZA BOČNO-TORZIONOG IZVIJANJA U PROFILA OD ALUMINIJUMSKE LEGURE

Rezime:

Ovaj rad prikazuje komparativnu analizu granične nosvosti konstruktivnih elemenata od aluminijumske legure na bočno-torziono izvijanje implementrajući uticaje početnih geometrijskih imperfekcija. Rezultati nelinearne numeričke analize, koja je sprovedena u programu ABAQUS, se porede sa rezultatima dobijenim primenom metode za proračun elemenata na BTI date u EN 1999-1-1. Svrha analize je procena tačnosti pomenute računske metode, definisane od strane standarda, na primeru realnih Al elemenata U poprečnog preseka sa inicijalnim imperfekcijama kod kojih opterećenje ne deluje u centru smicanja.

Ključne reči: aluminijum, kritični moment, U profil, bočno-torziono izvijanje, imperfekcije

NON-LINEAR LATERAL BUCKLING ANALYSIS OF ALUMINIUM ALLOY CHANNEL BEAMS

Summary:

This research paper presents the comparative analysis of lateral-torsional buckling resistance of structural aluminium alloy members, accounting for the effects of initial geometrical imperfections. The results of non-linear numerical analysis, conducted using ABAQUS software, are compared with the results obtained by utilizing the procedure for calculation of LTB resistance suggested by EN 1999-1-1. The purpose of the analysis is the assessment of accuracy of the above-mentioned code-prescribed design method using the real Al channel-section members with initial imperfections on which the load does not act in the shear centre.

Key words: aluminium, critical moment, channel, lateral-torsional buckling, imperfections

---

[1] *MSc, Structural Engineer, DNEC d.o.o., Belgrade*
[2] *PhD, MSc, BSc, assistant professor, University of Belgrade, Faculty of Civil Engineering*
[3] *PhD, MSc, BSc, assistant professor, University of Belgrade, Faculty of Civil Engineering*
[4] *MSc, PhD student, University of Belgrade, Faculty of Civil Engineering*
[5] *MSc, Structural Engineer, Jaroslav Černi Institute for the Development of Water Resources, Belgrade*

1. INTRODUCTION

This paper has focus on comparison of lateral-torsional buckling resistances of channel-section aluminium alloy beams with initial geometrical imperfections, obtained by finite-element analysis and by applying the procedure for sections mono-symmetric with respect to their major axis, described in EN 1999-1-1 [4]. The Finite Element (FE) modelling for the linear lateral-torsional buckling analysis, which incorporates the information about the cross-sectional properties, material properties, FE mesh, member lengths and structural systems, as well as the position of the line load, is thoroughly explained in the research paper of Ivanović et al. [5]. In the aforementioned paper, authors stated the following conclusions:

- Equation for calculation of critical moment of beams with cross-sections symmetrical about the major principal axis, with load applied perpendicular to the major axis in the direction of the shear centre, is available in EN 1999-1-1, annex I [4]. However, this equation is found to be the same as the equation proposed by EN 1993-1-1 [2] for estimation of critical moment of beams with standard doubly-symmetric cross-sections, with load applied in the symmetry plane.
- The deviations between the FEM analysis values of the critical moment of the channel-section beams and the corresponding values calculated according to EN 1999-1-1 [4] range from 1% to 1.5%. The values of the critical moment obtained in accordance with EN 1999-1-1 [4] did not appear to be safe-sided compared with the FEM results.
- FEM analysis critical moment values are observed to be decreasing with the increase of horizontal eccentricity of the load; therefore, by extrapolating this discovery in the opposite direction, it is rationally concluded that the lateral-torsional buckling capacity would increase if the load is applied in the vertical plane passing through the shear centre. Considering relatively small deviations from the values calculated using the Code [4], it is assumed that the equation for critical moment could provide results with a certain safety factor relative to the FEM analysis values if the equation is modified in a manner which would include the effect of horizontal eccentricity of the load.

The question remains whether the design approach prescribed by EN 1999-1-1 [4] will be more rigorous than the FEM procedure, if non-linear features of the material and initial crookedness and twist are introduced in the analysis. This research was conducted with the objective to shed light on this issue and to bring the problem of lateral-torsional buckling of channel-section beams closer to understanding. It should be mentioned beforehand that the procedure adopted by EN 1999-1-1 [4], which addresses the calculation of lateral-torsional buckling capacity of real beams, is somewhat similar to that displayed in EN 1993-1-1 [2]. It is assumed that the reader is familiar with this procedure, thus no significant effort will be made in order to clarify it.

2. NON-LINEAR LATERAL-TORSIONAL BUCKLING ANALYSIS OF CHANNEL BEAMS WITH INITIAL IMPERFECTIONS

In the preceding research paper [5], Linear Buckling Analysis (LBA) is described, from which the values of elastic critical moments of the analysed channel-section aluminium alloy beams are obtained. LBA implies that the material is linearly elastic and that the examined members are ideally straight, in which case the value of the bending moment in the governing section is increasing without lateral displacements until it reaches its critical value. The point on the Moment - Lateral Displacement diagram in which it occurs is called the bifurcation point, which, by definition, is a point where more than one equilibrium states coexist. This point is represented by M_{cr} in Fig. 1 and the determination of this point is one of the key problems of the theory of elastic stability. If member imperfections are introduced, the value of maximum moment will descend below the value of elastic critical moment. The maximum value of bending moment, in this scenario, represents lateral-torsional buckling capacity of a beam with initial imperfections (see Fig. 1).

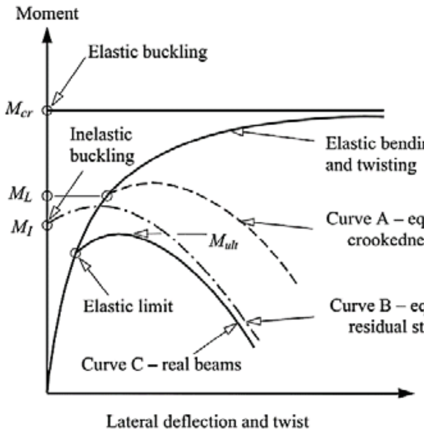

Figure 1 – Behaviour of beams with imperfections [7]

All that was said in the preceding research paper [5] regarding the FE models in terms of boundary conditions, geometry and FE mesh still holds. For the sake of clarity, cross-section with position of the loads, shear centre (S) and centroid (T), and the FE model are shown in Fig. 2. Load q1 from Fig. 2 represents the line load in the direction of the centroid, while load q2 represents the line load applied in the middle of the top flange.

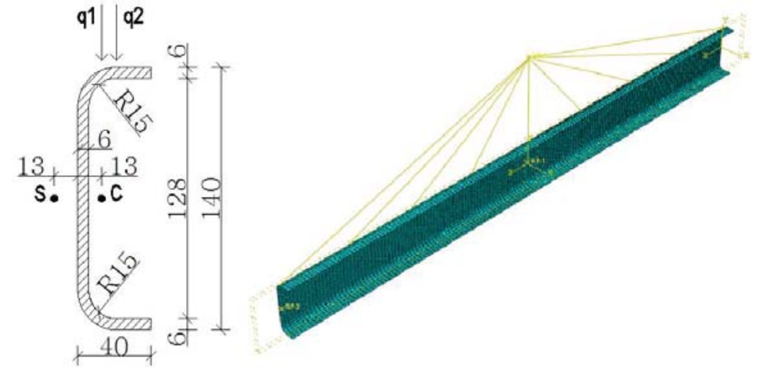

Figure 2 – Cross-section with load positions and FE model

The difference lies in the type of the analysis, which will be described here more precisely. The method of analysis which was used is the Explicit Dynamic Method (EDM). The first option to be considered for the analysis of the members prone to exhibit large deformations caused by the loss of stability should be the Modified Riks Method [1]. However, this method is not known to be effective for the entire spectra of possible models, which was the case for this analysis, as well. However, one method, which has been proven to be satisfactory for the broader spectra of cases, including non-linear lateral buckling, is the EDM. When opting for this method, it should be borne in mind that it is essentially a dynamic method used to simulate static load and that the results are greatly dependent on, among other parameters, the time of the load application.

For the numerical analysis of elastic critical moment the entire stress-strain relationship is of little significance, acknowledging the fact that LBA demands only the value of modulus of elasticity. On the other hand, when analysing the bending capacity of a member as a whole, the stress-strain relationship is crucial, owing to the pronounced non-linearity of aluminium alloy EN-AW 6061-T6/T651, for it conditions the behaviour of the member subjected to stresses beyond the proportionality limit of the material, set as 120 MPa in the analysis [5]. In Annex E, EN 1999-1-1 [4], various types of stress-strain curve models are proposed: bilinear model with and without strengthening branch, trilinear model with and without strengthening branch, along with the continuous models (see Fig. 3).

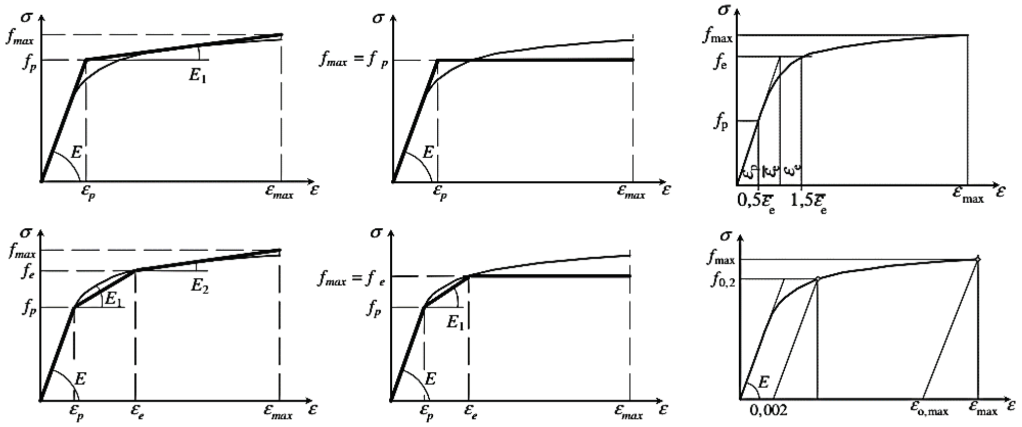

Figure 3 – Models of stress-strain curves suggested by EN 1999-1-1 [4]

Stress-strain model which was used is the continuous model in the form of $\sigma=\sigma(\varepsilon)$, as described in E2.2.1, EN 1999-1-1 [4]. True stress-strain curve is then obtained as recommended in clause C.2, EN 1993-1-4 [3], in order to account for the "necking" phenomenon. After this transformation the stress-strain curve assumes the shape as shown in Fig. 4. From Fig. 4 it can be observed that the material demonstrates significant strengthening, which would not have been so pronounced hadn't the true values of the stresses and strains been employed. In ABAQUS software [1], the elastic behaviour of the material is governed by Hook's law; above the proportionality limit, which was set at 120 MPa, the plastic deformations commence. The non-linear material properties are taken into account by subtracting the value of elastic deformation, which is equal to the ratio of the stress and the modulus of elasticity, from the true values of strains.

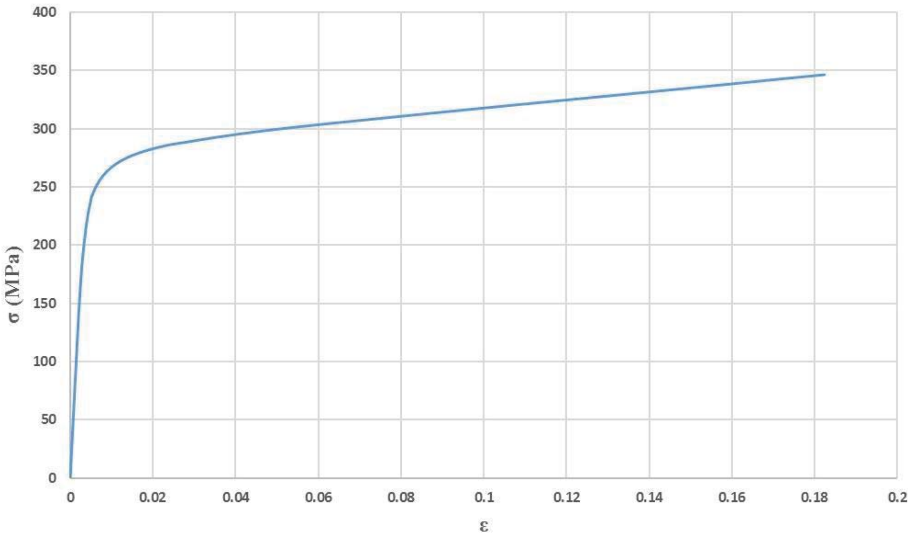

Figure 4 – True stress-strain curve

Stress-strain curve with true stress values from 120 MPa onward and with true values of irreversible strains, which for the stress value of 120 MPa have the value of 0, is imported in the software in a part where the inelastic material behaviour is to be defined.

Parameters which define the dynamic analysis have to be chosen carefully. One of the parameters which demonstrate great influence on the results is the time period. For the purpose of the analysis of the models described in the paper [5], the initial time period of 100 sec was adopted as the time in which the assigned load is applied gradually.

The method employed in obtaining the results demands definition of the partition in the mid-span, by which the governing cross-section is isolated. The function used for this purpose is Integrated Output with variable SOM, which defines the plot of total bending moment against time.

The imperfections were imported as the deformed shape of the models used for linear buckling analysis. The described members are relatively sensitive to imperfections, so that the first mode of lateral-torsional buckling is sufficient to trigger the reduction of the bending capacity below the value of the elastic critical moment. The first lateral-torsional buckling mode is comprised of two deformation components which are unfavourable from the stability standpoint: twist and lateral deflection. Taking into account the fact that the analysed members are formed by the process of extrusion, residual stresses, as well as the variations of material parameters along the cross-section can be neglected, as opposed to the case when the members are welded [6]. The amplitude value of initial imperfection, comprising both the twist and the bow imperfection, is chosen to be equal to the maximum manufacturing imperfection of L/750, in accordance with EN 1090-2, where L is the span length. The visual representation of initial geometric imperfection is shown in Fig. 5.

Figure 5 – Model without and with initial geometric imperfection

The applied load slightly exceeds the critical load obtained from LBA and it is equal to the critical load multiplied by 1.5.

The results obtained from EDA are presented as the plots of mid-span bending moment against the rotation of the support sections (see Fig. 6 – Fig. 8).

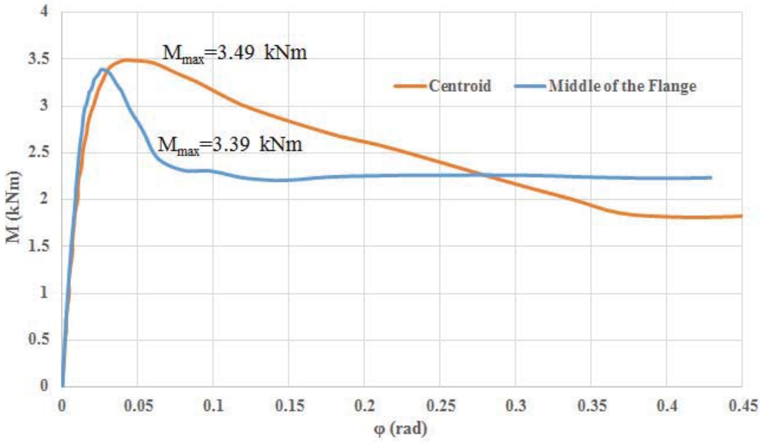

Figure 6 – Bending capacity of 1.5 m beams

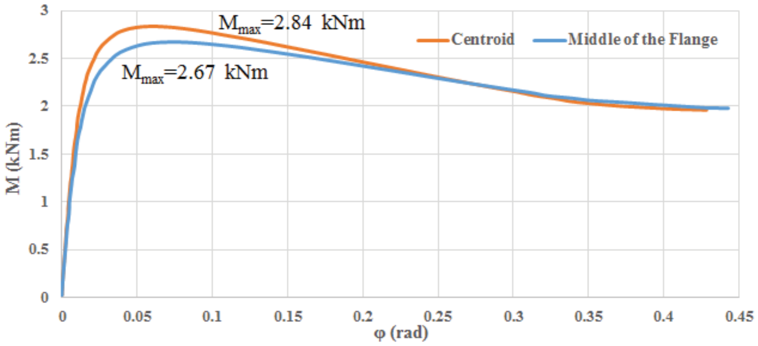

Figure 7 – Bending capacity of 2 m beams

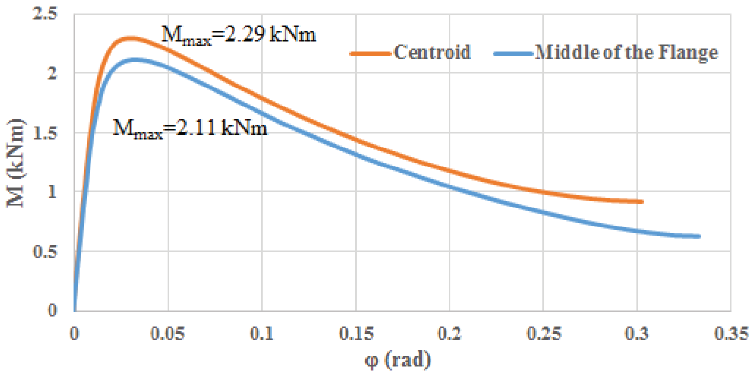

Figure 8 – Bending capacity of 2.5 m beams

In Fig. 9 the example model which has undergone inelastic deformations at the moment of failure is shown for the purpose of visual description. Von Mises stresses, expressed in MPa, are shown in the legend in the upper left corner.

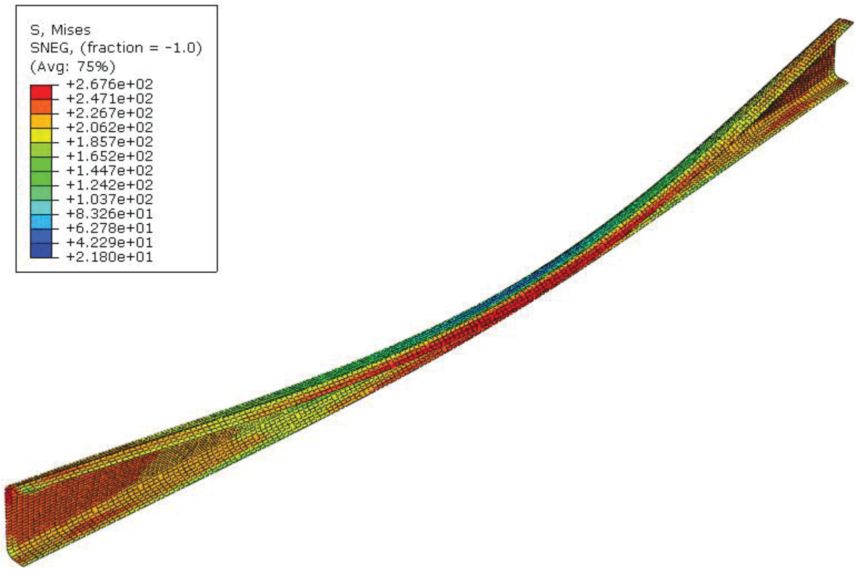

Figure 9 – Example model at the moment of failure

3. RESULTS AND DISCUSSION

The values of bending resistances of the analysed beams obtained from non-linear analysis in ABAQUS are compared in Table 1 with the values obtained using the design approach from Eurocode 9. The comparison includes different beams' spans and variation of load position related to cross-section geometry.

Table 1 – Comparison of bending resistances of the analysed beams

Span		1.5 m		2 m		2.5 m	
Load position		Centroid	Middle of the flange	Centroid	Middle of the flange	Centroid	Middle of the flange
Bending resistances (kNm)	*FEM*	3.49	3.39	2.84	2.67	2.29	2.11
	EC9	3.36	3.36	2.57	2.57	2.1	2.1
Deviation (%)		3.7	0.9	9.5	3.8	8.3	0.5

It can be concluded from the results presented above that the deviations of the values of bending resistances are generally larger than it is the case with the values of critical moments. Graphic comparison of these results is shown in Fig. 10 – Fig. 11.

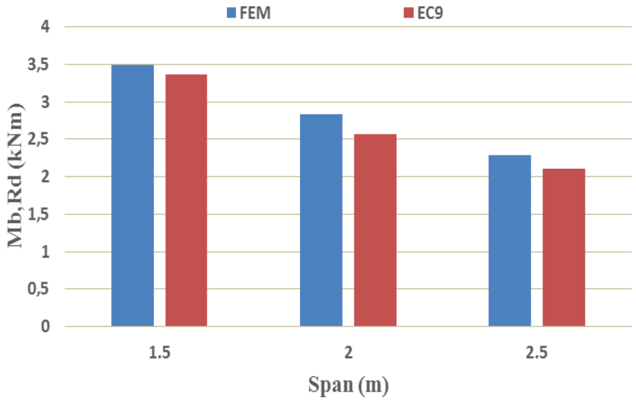

Figure 10 – Bending resistances for load applied in the centroid plane

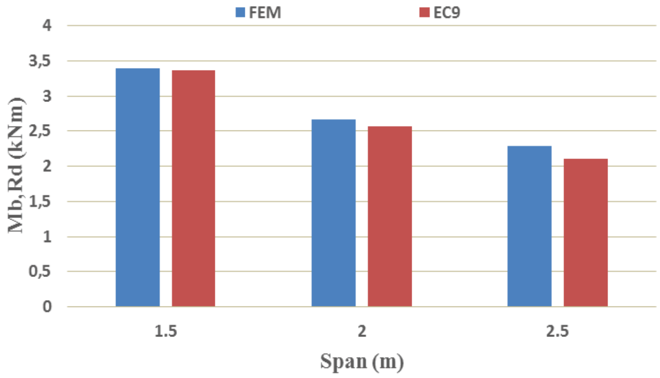

Figure 11 – Bending resistances for load applied in the middle of the flange

It can be noticed from Table 1, as well as from Fig. 10 and Fig. 11, that the values of bending resistances obtained in accordance with Eurocode 9 [4] appear to be safe-sided with respect to the values obtained by FEM analysis. This was not the case with the values of the critical moments, but the fact that the strengthening of the material is not included in the Eurocode design approach against lateral buckling should be appreciated, for it may contribute to this increase of bending capacity.

4. CONCLUSIONS

This paper focuses on the comparative analysis of lateral-torsional buckling capacity of structural channel-section aluminium alloy beams with initial imperfections. The results obtained using FEM analysis and design propositions provided in EN 1999-1-1 are compared and the following conclusions can be accentuated:

- Initial geometric imperfections are introduced in the FEM analysis as the first mode of lateral-torsional buckling scaled so that the amplitude value in the mid-span is equal to L/750, where L is the span length.
- The values of maximum bending moments in the mid-spans obtained by the means of FEM analysis deviate from the values calculated using the design recommendations available in EN 1999-1-1 by 0.5% to 9.5%
- The values of bending capacity of members with initial imperfections obtained using EN 1999-1-1 are shown to be on the safe side, as compared to the values obtained using FEM analysis. Therefore, with respect to the results from this and the preceding research paper [5] it is concluded here that overall lateral buckling resistance of examined aluminium alloy beams with initial imperfections is not overestimated by the Code [4].

REFERENCES

[1] Abaqus 6.12 Analysis User's Manual

[2] EN 1993-1-1:2005, Eurocode 3 – Design of steel structures – Part 1-1: General rules and rules for buildings

[3] EN 1993-1-4:2006, Eurocode 3 – Design of steel structures – Part 1-4: General rules – Supplementary rules for stainless steels

[4] EN 1999-1-1:2007, Eurocode 9 – Design of aluminium structures – Part 1-1: General structural rules

[5] Ivanović N., Marković Z., Dobrić J., Spremić M., Gluhović N.: *Numerical lateral buckling analysis of aluminium alloy beams with channel sections*, Symposium of Association of Structural Engineers of Serbia, 2016., Zlatibor, Serbia.

[6] Mazzolani, F., 2003, Aluminium structural design

[7] Trahair, N. S., Bradford, M. A., Nethercot, D. A., Gardner, L., 2007, The behaviour and design of steel structures to EC3