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Isidora Jakovljević[1], Saša Kovačević[2], Nenad Marković[3]

ELASTIČNA KRITIČNA SILA PLOČA I LIMENIH NOSAČA POD DEJSTVOM LOKALIZOVANOG OPTEREĆENJA

Rezime:

Određivanje elastične kritične sile (sile izbočavanja) predstavlja važan elemenat u određivanju granične nosivosti limenih nosača prema Evrokodu za proračun čeličnih konstrukcija. U radu se daje analiza kritične sile izolovane ploče koja odgovara rebru I nosača za različite granične uslove kao i kritične sile samog I nosača. Dati su zaključci o njihovoj vezi. Uporeďene su eksperimentalno određene granične nosivosti I nosača sa vrednostima prema Evrokodu, za modele korišćene u ovoj analizi.

Ključne reči: izbočavanje ploča, izbočavanje I nosača, lokalizovano opterećenje

BUCKLING OF PLATES AND PLATE GIRDERS SUBJECTED TO PATCH LOAD

Summary:

The determination of the elastic critical load (buckling load) is an important element of the assessment of the ultimate load of plate girders. An analysis of the critical load of a plate corresponding to the web of an I-girder for different boundary conditions and the critical load of the I-girder itself is given in the paper. Conclusions regarding their correspondence are given. Also, experimentally determined ultimate loads of I-girders are compared with values according to Eurocode for the models used in this analysis.

Key words: plate buckling, plate girder buckling, patch loading

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1. INTRODUCTION

The stability problems and ultimate load behavior of steel plate girders have attracted a lot of attention during the last decades. The behavior of the plate girder (welded I-girder) subjected to patch load or partially distributed load on the flange in the plane of a web, without a vertical (transverse) stiffener bellow the load was also intensively investigated. This problem has got the importance with a general trend to avoid vertical stiffeners except at supports and also in the case of moving loads. Except for crane girders loaded by crane wheels, a remarkable realistic load case in which this situation arises is the launching phase of multi-span steel plate girder bridges during construction over temporary or permanent supports.

In the analysis of the behavior of the patch loaded girder the attention is directed towards the distribution of the local direct stresses under the load in the web, the elastic critical load of the web panel and the ultimate load of the girder. For the development of design procedures, the ultimate load is of a great importance. The research workers in many countries have investigated theoretically and experimentally this problem over the last fifty years. Even an increasing number of experimental results and laborious theoretical work has not offered a complete insight into the problem and intensive researches in various segments of this problem are still ongoing worldwide.

In the initial phases of investigations, attempts were made to relate the ultimate load to the elastic critical (buckling) load [1], [2], [3]. Very soon it was found that generally, it was not possible. Also, analytical procedures were not successfully applied, as the problem is very complex, depending on many interrelated parameters. As a corollary, a majority of the suggested solutions were of empirical nature, based on experimental researches. The development of computer programs based on the finite element (FE) method and increased capabilities of computers enabled the development of "numerical experiments" that are nowadays widely applied.

With the development of Eurocodes for the design of civil engineering structures, a new approach was applied to harmonize solutions of the patch load problem with other stability problems. The main points in the determination of the ultimate patch load are elastic critical load, yield load, and resistance function. The determination of the elastic critical (buckling) load got again the importance, as it is needed for the determination of the slenderness parameter.

The solution applied in Eurocode for steel structures [4] uses simplified expressions for the buckling coefficients. Together with some other simplifications and uncertainties, it gives very often rather conservative values for the ultimate load.

The investigation of patch load is carried out at the Faculty of Civil Engineering University of Belgrade within an international cooperation, initially in the eighties, with the University of Cardiff (with T. M. Roberts) and with the Czech Academy of Sciences in Prague (with M. Skaloud), and in last period with the Faculty of Civil Engineering University of Montenegro in Podgorica (with D. Lucic and his collaborators).

The results obtained in the experimental research [5] compared with the procedures applied in Eurocode had shown [6] significantly conservative results given by Eurocode. An attempt is made in this paper to give some improvements to that procedure.

Elastic critical load for an isolated web plate for different boundary conditions is compared with an elastic critical load of an I-girder. Moreover, the experimentally determined ultimate load is compared with the procedure as currently given in Eurocode, as well as with the suggested correction for the determination of the elastic critical load (that includes taking into account the distribution length of an applied patch load).

Nowadays, numerical analysis techniques are widely used in research involving structural steel and in analyses and designs of steel structures and elements. The FE method based numerical analysis is the most popular computational tool in this field and it has been successfully applied in many papers regarding the critical load of plate girders under different loading conditions. The commercial multi-purpose FE analysis software Abaqus was used for the numerical analysis [7]. In order to get a better insight into the problem, geometry and loading of the girders were considered according to those applied in the accompanying experimental research [5].

2. ELASTIC CRITICAL LOAD

Introductory it was stated that the purpose of this paper is the comparison of the elastic critical loads for an isolated web plate, varying boundary conditions, with critical loads of an I-girder. The two plates schematically presented in Figure 1, labeled as *Model 1* with an aspect ratio a/h_w=1 and *Model 2* with an aspect ratio a/h_w=2 (a=1000 mm), were investigated. The web plate thickness was t_w=4 mm. The length of an applied uniform load s_s was varied. The degree of freedom 2 is only constrained in the vertical edges while the degree of freedom 3 is constrained in all edges. Three different cases were analyzed: (a) simply supported plate with no additional constraints, (b) plate simply supported on the vertical and clamped on the horizontal edges (degree of freedom 4 is constrained), (c) plate clamped along all edges (degree of freedom 5 is constrained in the vertical edges and the degree of freedom 4 in the horizontal edges). For the sake of brevity, through the rest of the paper, these cases will be marked as SS, CS, and CC, respectively. In the same vein, the two models of an I–girder according to the previously described Model 1 and Model 2 are displayed in Figure 2. The flange width and thickness was set to 120 mm and 8 mm, respectively. The boundary conditions are according to the experimental procedure described in [5]. The considered material for all cases is homogenous with an elastic modulus of E=210 GPa and Poisson's ratio of v=0.3.

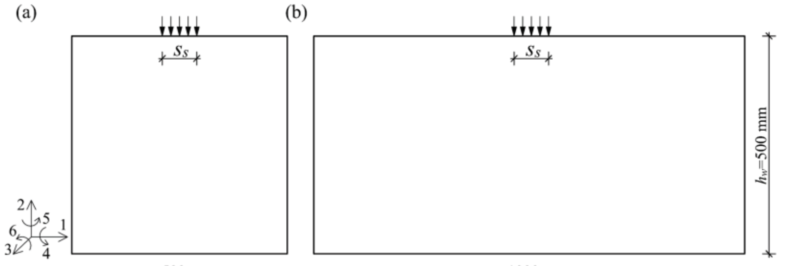

Figure 1 – Plate under patch load: (a) Model 1; (b) Model 2

For the FE analysis, a general-purpose four-node quadrilateral shell element with reduced integration and six degrees of freedom per node S4R from the Abaqus element library was used. According to the mesh convergence study performed only on the isolated web plate of Model l, the adopted finite element size is 5 mm for all numerical runs. The variation of the buckling coefficient due to change of finite element size in the case of Model 1 for a simply supported plate is graphically presented in Figure 3(a). The results of the calculated elastic critical load are presented using the non-dimensional buckling coefficient k_f and tabulated in Table 1 and 2 for Model 1 and Model 2, respectively. Conclusively, it may be stated that they show a good agreement with the corresponding values found in the literature [8–14]. The buckling coefficient depends on boundary conditions, loading type and an aspect ratio a/h_w. On the other hand, it is not influenced by plate thickness or material properties. However, for a different plate thickness applied on a simply supported plate of Model 1, a slight variation in the results could be noticed, as charted in Figure 3(b).

Another sensitivity of the buckling coefficient is pictured in Figure 4. A simply supported plate model is analyzed when the degree of freedom 2 was constrained along the vertical edges, *i.e.* the edge support is set as previously described, and point supports are applied in the lower corners of the plate. In the case of Model 2, this change of boundary conditions does not affect the value of k_f. On the contrary, in the case of Model 1 and a longer uniform load, the difference in the results comes up to 25 %. Expectedly, the boundary conditions do not play a decisive role for higher aspect ratios since the applied load is localized and for an aspect ratio $a/h_w \geq 2$ their influence is negligible.

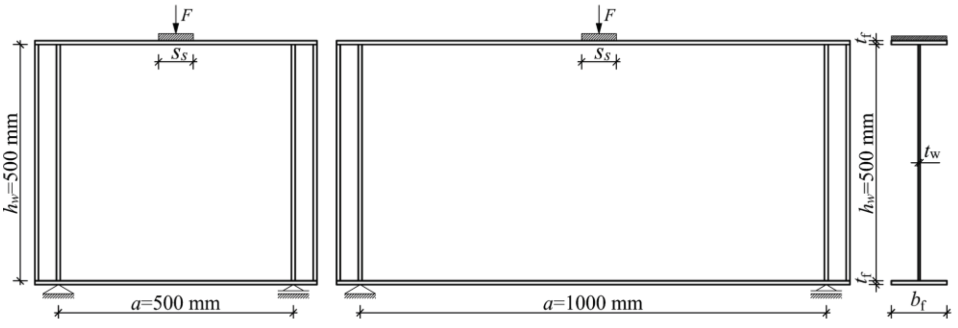

Figure 2 – Model 1 and Model 2 for I–girders

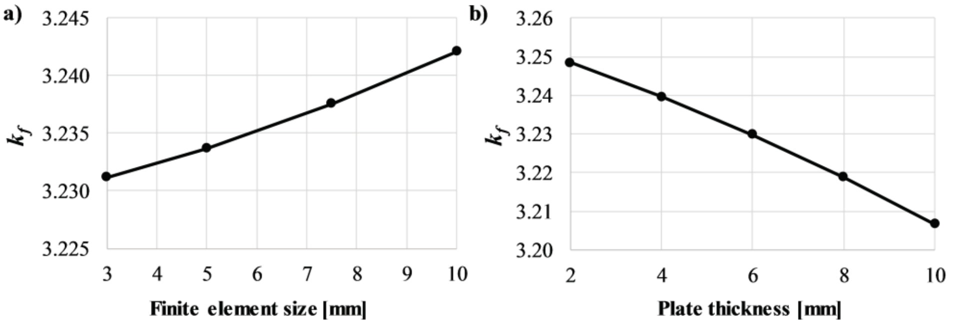

Figure 3 – Variation of the buckling coefficient due to change of: (a) FE size; (b) plate thickness

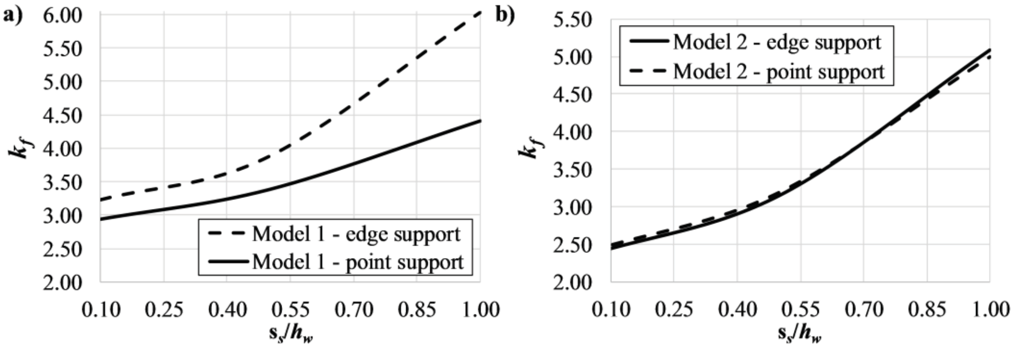

Figure 4 – Buckling coefficient for the plate supported on the vertical edges and for the plate with point supports in the corners

3. RESULTS AND DISCUSSION

The aim of this chapter is to highlight the obtained numerical results of the buckling coefficient considering an isolated web plate and I–girder. The results for all described numerical models are listed and discussed thoroughly. Furthermore, a new model for the determination of the buckling coefficients, as a function of patch load length, is presented. The purpose of the proposed model is to improve the ultimate load calculated using the procedure given in Eurocode. At the end of this section, detailed conclusions are given along with recommendations for further investigations.

Table 1 and Table 2 shows a comparison summary of the numerical results and the buckling coefficients obtained by Eurocode while a graphical representation is shown in Figure 5. One can instantaneously see that the SS plate for both models and for all lengths of an applied patch load gives extremely low values of the critical loads. Introducing the clamped constraint on the horizontal edges (CS plate) the k_f values are improved but still too far below the k_f of the I–girder, especially for Model 1.

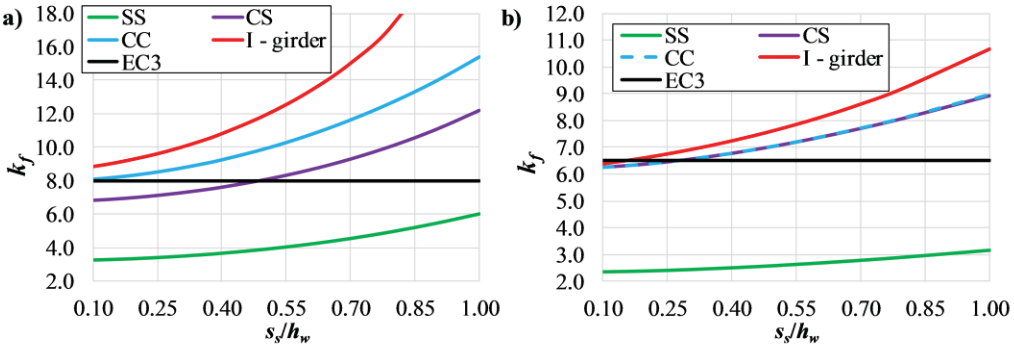

Figure 5 – Comparison of the buckling coefficient for: (a) Model 1; (b) Model 2

On the other hand, it can be noticed that the buckling coefficients for the I–girder are well-captured with the CC plate for shifted values of s_s. Consequently, it may be possible to obtain the buckling coefficient of plate girders by considering a clamped web plate. It is interesting to speculate on whether the values for the CC plate can be related to the values of an I-girder for different geometries and how they are connected.

It is noteworthy to observe another interesting point regarding the clamped boundary conditions for large values of s_s. It is clear from Figure 5 that the rigid transversal stiffeners (see Figure 2) influence the buckling coefficient more than clamped constraint whereas for small values of s_s their influence is not present since the load is localized. Additionally, the clamped constrain on the vertical edges (the difference between the CC and CS plate) reveals the fact that constrained rotations along these edges do not have an influence on the buckling coefficient for an aspect ratio $a/h_w \geq 2$, as stated before. Conversely, they influence the buckling coefficients with a uniform factor for an aspect ratio $a/h_w=1$ (Model 1) for all values of s_s. A further parametric study should be made in order to make an airtight conclusion about this boundary condition and its influence on the buckling coefficient.

Table 1 – Buckling coefficients for different boundary conditions for Model 1

s_s [mm]	***s/a***	***s/h_w***	***k_f***				
			SS	***CS***	***CC***	***I–girder***	***EC3***
0	***0.00***	***0.00***	***3.20***	***6.78***	***8.00***	***8.61***	***8.00***
25	***0.05***	***0.05***	***3.21***	***6.80***	***8.02***	***8.69***	***8.00***
50	***0.10***	***0.10***	***3.23***	***6.85***	***8.10***	***8.85***	***8.00***
75	***0.15***	***0.15***	***3.27***	***6.92***	***8.21***	***9.06***	***8.00***
100	***0.20***	***0.20***	***3.32***	***7.02***	***8.35***	***9.32***	***8.00***
125	***0.25***	***0.25***	***3.38***	***7.13***	***8.53***	***9.61***	***8.00***
150	***0.30***	***0.30***	***3.46***	***7.27***	***8.74***	***9.96***	***8.00***
175	***0.35***	***0.35***	***3.54***	***7.44***	***8.98***	***10.36***	***8.00***
200	***0.40***	***0.40***	***3.64***	***7.62***	***9.26***	***10.81***	***8.00***
250	***0.50***	***0.50***	***3.88***	***8.07***	***9.91***	***11.90***	***8.00***
300	***0.60***	***0.60***	***4.18***	***8.62***	***10.70***	***13.27***	***8.00***
350	***0.70***	***0.70***	***4.54***	***9.29***	***11.64***	***15.01***	***8.00***
400	***0.80***	***0.80***	***4.97***	***10.10***	***12.73***	***17.37***	***8.00***
450	***0.90***	***0.90***	***5.47***	***11.05***	***13.98***	***21.60***	***8.00***
500	***1.00***	***1.00***	***6.04***	***12.17***	***15.41***	***31.95***	***8.00***

As a further comparison between the I-girder and CC plate, the first buckling mode for Model for these two cases is juxtaposed. It can be seen in Figure 6 that the buckled shape for Model 1 for an isolated plate subjected to patch load length s_s=210 mm and the buckled shape of an I- girder subjected to patch load length s_s=100 mm correspond to each other, both numerically and graphically, due to the fact that the flange plate enables spreading the loading length into the web. Therefore, based on these results for the CC plate and I–girder, we can conclude that the buckling coefficients of an I-girder under patch loading can be determined considering an isolated web plate. For instance, for this particular case of Model 1 of the web plate and I–girder the values of buckling coefficient are shifted by patch load length of 115 mm for the cases $s_s/h_w \leq 0.5$ and approximately equal, *i.e.* $k_{f,girder}(s_s)=k_{f,plate}(s_s+115$ mm). Supposedly, this approximation is influenced by aspect ratio and for Model 2 the shifted length is notably less (approximately around 60 mm). It must be emphasized that the shifted lengths are not equally distributed for all patch load lengths and in order to draw a better conclusion about their influence, further analyses considering smaller patch load lengths rates should be performed.

Table 2 – Buckling coefficients for different boundary conditions for Model 2

s_s [mm]	s/a	s/h_w	k_f				
			SS	CS	CC	I–girder	EC3
50	0.05	0.10	2.37	6.24	6.25	6.38	6.50
100	0.10	0.20	2.40	6.36	6.37	6.61	6.50
150	0.15	0.30	2.45	6.54	6.54	6.90	6.50
200	0.20	0.40	2.51	6.77	6.77	7.23	6.50
250	0.25	0.50	2.59	7.04	7.04	7.63	6.50
300	0.30	0.60	2.68	7.35	7.35	8.08	6.50
350	0.35	0.70	2.78	7.70	7.71	8.60	6.50
400	0.40	0.80	2.90	8.08	8.09	9.20	6.50
500	0.50	1.00	3.15	8.93	8.96	10.65	6.50
600	0.60	1.20	3.44	9.86	9.94	12.56	6.50
700	0.70	1.40	3.78	10.85	11.03	15.17	6.50
800	0.80	1.60	4.16	11.92	12.24	19.00	6.50
900	0.90	1.80	4.60	13.11	13.56	25.20	6.50
1000	1.00	2.00	5.08	14.47	15.00	47.26	6.50

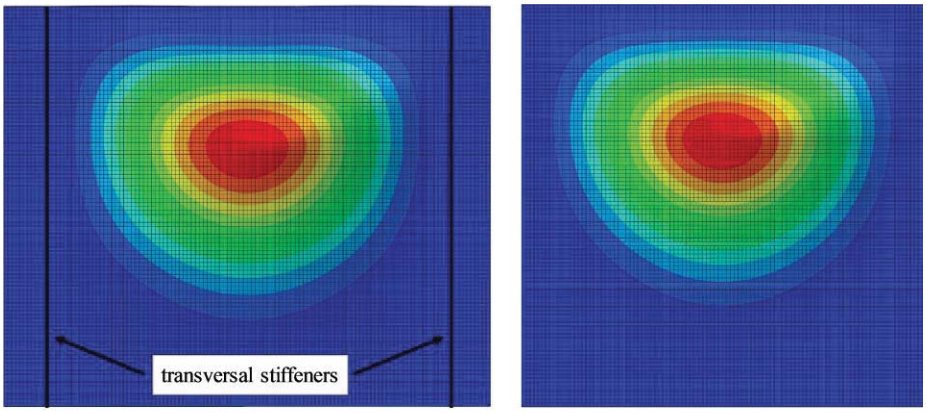

Figure 6 – First buckled shape for Models 1: (a) I–girder; (b) isolated web plate

According to our analysis of the buckling coefficients for the I–girder, a close fit can be established for the k_f values using high order polynomials. However, keeping in mind the simplicity of the design procedure in Eurocode a first or second order polynomial is desirable. For example, the following general form gives a good approximation for the k_f values of the I–girder:

$$k_f = \alpha s_s^2 + \beta s_s + \gamma$$

where:

α, β and γ are unknown constants. For these investigated cases of the I–girders, the constants are $\alpha=4.7\cdot10^{-5}$, $\beta=2\cdot10^{-3}$, $\gamma=8.7$, $\alpha=1.8\cdot10^{-5}$, $\beta=0$ and $\gamma=6.5$ for Model 1 and Model 2, respectively. A graphical comparison of the numerically obtained buckling coefficient of the and I–girder and coefficients approximated by the proposed expressions is shown in Figure 7. One should bear in mind that these values for the constants are determined only for the analyzed geometries. A detailed parametric study is necessary in order to obtain a general expression as a function of geometric parameters, *i.e.* thickness of the web and flange, flange width, etc.

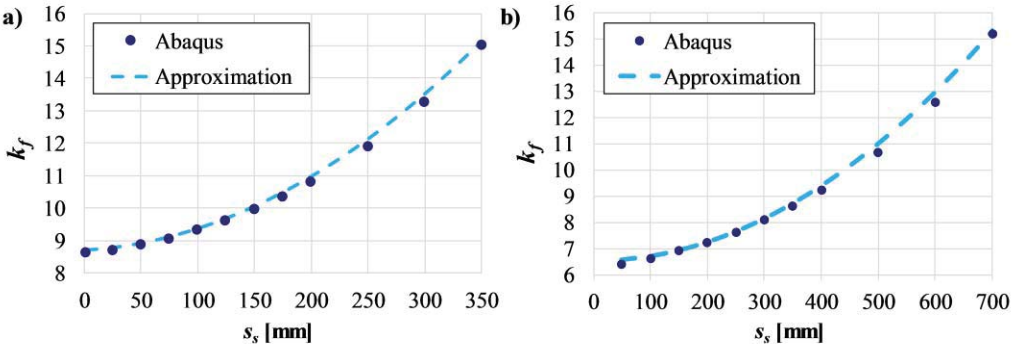

Figure 7 – Abaqus vs. proposed expressions for the k_f values for: (a) Model 1; (b) Model 2

Values of the experimentally obtained ultimate loads and corresponding ultimate loads according to the procedure applied in Eurocode with k_f given in Eurocode (not dependent on the loading length s_s) are pictorially compared in Figure 8. Noticeably, the ultimate loads calculated by Eurocode are considerably lower than the experimentally obtained ultimate loads and the difference is more pronounced for higher patch load lengths.

In order to improve the results from Eurocode, we propose a new model for the determination of k_f. The model includes the calculation of the buckling coefficient as a function of patch load length while the procedure for obtaining the ultimate load is the same as in Eurocode. The new values obtained by this improved model are also presented in Figure 8. One can immediately see that better values are produced by the proposed model. A salient feature of the suggested model is a simple correction regarding only the buckling coefficient while the ultimate loads are improved, especially for longer patch load lengths. However, in order to elucidate the influence of the length of patch load on buckling coefficient and to make a straightforward connection with an isolated web plate considering different geometries of an I–girder, further analyses are required.

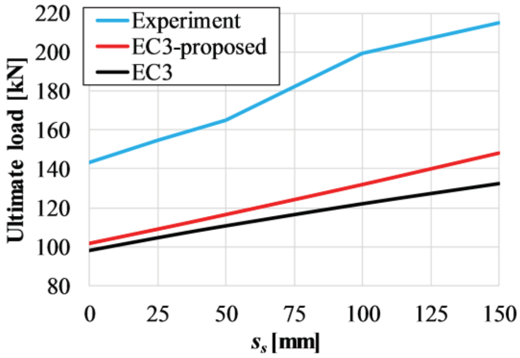

Figure 8 – Comparison of the ultimate load by different approaches

In a nutshell, the presented model enabled a fruitful background for parametric analyses and producing an erroneous number of numerical tests that can be used, in order to better determine the ultimate loads of plate girders using the buckling coefficients. Furthermore, since the ultimate loads are still too far below the experimentally obtained ultimate loads, the present model can be also exploited in order to improve different elements currently present in the design procedure in Eurocode, *i.e.* the effective loaded length.

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