Cellular Automata Approach for 2D Pollution Transport Modelling in Urban Groundwater

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Abstract. Integrated modelling requires many simplifications in order to speed up long time calculations and simulations. Therefore, many non-traditional methods are being widely used. Cellular automata (CA) represents one of these methods. The paper presents the application of the CA approach in modelling of the contaminant transport in unsteady groundwater conditions. It compares the results obtained using the two CA models modified for groundwater problems. Results obtained in this paper show that CA approach can be successfully used for simulations of unsteady groundwater conditions, caused by surface-groundwater interaction, and pollution transport.

Keywords: Integrated modelling · Cellular automata
Contaminant transport, surface-groundwater interaction

1 Introduction

Water quality in urban catchments is one of the crucial problems that need to be addressed to assure high quality of life in cities. Particularly vulnerable to quality deterioration is the groundwater, often used for water supply, especially if the aquifer lies near or beneath the city. This creates a problem with groundwater resources pollution protection. Urban stormwater runoff is being categorized as one of the major pollution sources to receiving waters. Although build-up and wash-off mechanisms of pollution in highly urbanized areas have been subjects of numerous studies (Deletic et al. 1997), interaction between groundwater and sewer systems should be further investigated. Old combined sewer systems (and even inadequately constructed separate ones) may leak underground. Sewer leakage combined with absence of a wastewater treatment system (Belgrade being such an extreme example), puts groundwater resources at a high risk of contamination. To assess various "*what if*" scenarios, particularly when there are water quality monitoring points, it is of outmost importance to develop a usable integrated model.

Due to the complexity of the commonly used physically based models, computation cost occurs as one of the main problems, especially in long term simulations. Combination of several numerical models in order to solve physically based models for interactions in a chain of models can, often, create an unsolvable problem. Hence, simplified models, such as the Cellular Automata (CA) have been a topic for many

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researchers in the last two decades, especially since they can speed up the computation. Additionally, with the development of parallel computing, CA models became highly exploited methods in different areas (Bandini et al. 2001), including the water cycle modelling.

Development of flood inundation models based on CA approach started with Dottori and Todini 2010. They proposed a CA based model for representing diffusion dominated problems such as the flood inundation events. Guidolin et al. 2016 developed weighted cellular automata model for rapid flood inundation analysis. At the same time with development of the CA models for flood inundation and runoff problems (surface water problems), several researchers used this approach to solve problems in groundwater modelling. Ravazzani et al. 2011 used CA for unsteady groundwater state modelling caused by constant pumping rate from a well. At the same time CA approach is used to model pollution (contaminant) and other transport processes, in atmosphere, surface water and groundwater. Modelling of air pollution transport was the subject of researches presented by Guariso and Maniezzo 1992. Palanichamy et al. 2008 developed probabilistic CA model for two dimensional contaminant transport in groundwater in steady state conditions.

This paper presents development of a methodology that can be used to model pollution transport in unsteady groundwater conditions. The primary objective of this research is to analyse and compare the behaviour of the two CA-based models for contaminant transport from linear pollution source in unsteady groundwater conditions caused by infiltration. Analysis shows the CA based algorithm may be utilized for groundwater and diffusion dominant mechanisms of pollution transport modelling.

2 Materials and Methods

Methodology is based on a two-layer (two-stage) modelling (Fig. 1). First layer is used for hydrodynamic modelling of the unsteady groundwater conditions. In this layer, the two CA approaches are possible: the first one is weighted cellular automata, proposed by Guidolin et al. 2016, modified for groundwater conditions, and the second one the cellular automata model based on simple Darcy's law, proposed by Ravazzani et al. 2011. Second layer is used for modelling of the pollution (contaminant) transport using the velocity-field obtained in the first layer (hydrodynamic layer).

2.1 Weighted Cellular Automata for Unsteady Groundwater Flow Modelling – WCAGW

The WCAGW model is developed by modifying the Weighted Cellular Automata (WCA2D) (Guidolin et al. 2016) so it can be used for the simulations in unsteady groundwater conditions. This method uses cellular weights, calculated on the basis of the head gradient. WCAGW differs from the original WCA2D in physical limitations of the maximum intercellular velocity (Eq. 1) and the modification of the cells state update equation (Eq. 2).

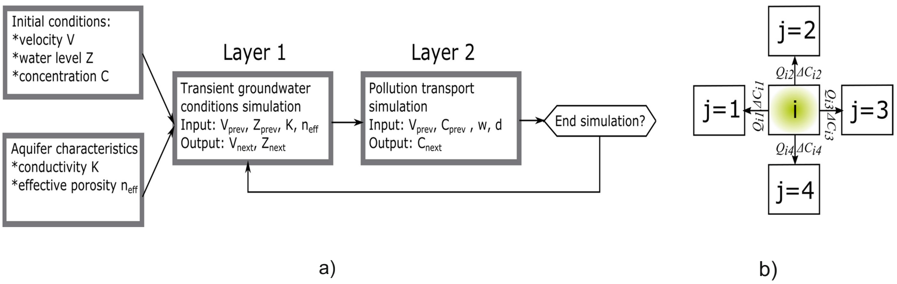


Fig. 1. (a) Schematic view of the CA two-layer model: Layer 1 is computed either by WCAGW or by DCAGW; (b) Cell local numeration

$$v_M = \frac{4 \cdot K_i \cdot H_i \cdot K_M \cdot H_M}{(K_i \cdot H_i + K_M \cdot H_M) \cdot (H_i + H_M) \cdot d_{iM}} (H_i - H_M) \tag{1}$$

$$H_i^{n+1} = H_i^n - \frac{1}{n_{eff}} \cdot \frac{\sum Q_{ij}^n}{A_i} \cdot \Delta t \tag{2}$$

where v_M represent max allowed velocity (M is the index of max weighted cell, according to local numeration Fig. 1b.), K_i represent hydraulic conductivity of the central cell, K_M represent hydraulic conductivity of an adjacent cell with max weight, H_i represent water level in the central cell, H_M represent water level in the max weighted adjacent cell and d_{iM} represent boundary length between two cells. In Eq. (2) n_{eff} represent effective porosity, Q_{ij} represents intercellular discharge and/or volume input/output (well or infiltration), A_i is the area of the central cell basis and Δt is time step.

2.2 Darcy's Law-Based Cellular Automata for Unsteady Groundwater Flow Modelling –DCAGW

Darcy's law-based Cellular Automata for Unsteady Groundwater Modelling (Ravazzani et al. 2011), originally developed for simulation of unsteady groundwater conditions due to the constant pumping rate from a well, was slightly modified so it can be used for simulations of unsteady groundwater conditions caused by homogeneous infiltration over the entire domain (Eq. 3). Cell state update is calculated using Eq. (2).

$$Q_{ij}^n = \frac{2 \cdot K_i \cdot H_i^n \cdot K_j \cdot H_j^n}{K_i \cdot H_i^n + K_j \cdot H_j^n} \left(H_j^n - H_i^n \right) \tag{3}$$

2.3 Cellular Automata for Pollution Transport Modelling – CAPT

Pollution transport model uses velocity field calculated either by WCAGW or DCAGW. Pollution transport is calculated on the basis of the adjacent cells pollution state gradient, advection (convection) factor and diffusion factor (Eq. 4). Updated cell pollution state presents the linear combination of adjacent cells state from the previous time step.

$$C_i^{n+1} = C_i^n + \sum_{j=1}^{4} \left(-2 \frac{v_j^n}{n_{eff}} \cdot \frac{\left(C_j^n - C_i^n \right)}{\Delta x} + 4 \cdot \alpha_{x(y)} \cdot v_j^n \cdot \frac{C_j^n - C_i^n}{\Delta x^2} \right) \cdot \Delta t \qquad (4)$$

Where C_i^{n+1} is pollution concentration in central cell in the next step, C_i^n is pollution concentration in central cell at the present time, $C_j^n (j = 1, 2, 3, 4)$ is pollution concentration at an adjacent cell at the present time, v_j is velocity between cells, Δx spatial resolution, Δt temporal resolution and $\alpha_{x(y)}$ is longitudinal (transversal) dispersivity.

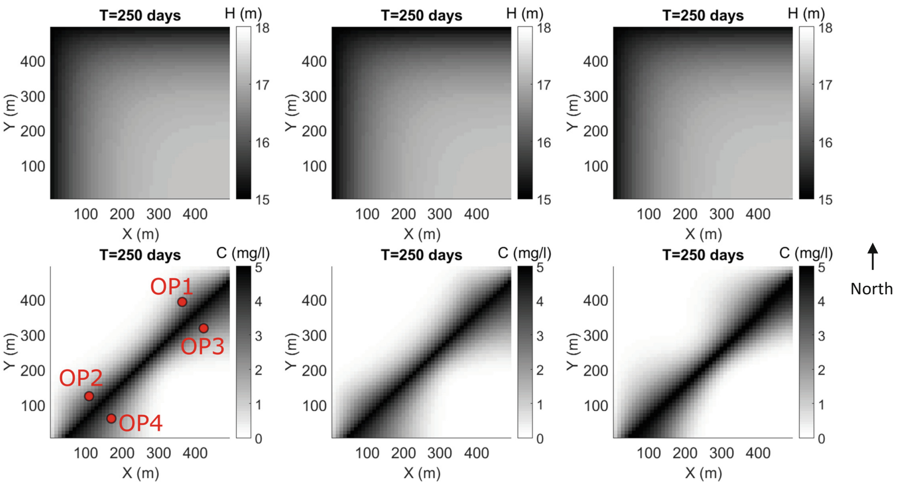
2.4 Test Case

Initial methodology testing is performed on an artificial square domain. Boundary conditions include constant head H = 15 m on the west and the north boundaries and concentration set at 0 *mg/l* and Neumann boundary condition of zero velocity on the east and the south boundaries. The initial condition over the entire domain is a constant head of $H = 15\,\text{m}$ with no flow conditions and concentration of $C = 0\,\text{mg/l}$. The domain is assumed to be homogeneous with a hydraulic conductivity $K = 1.25 * 10^{-5}\,\text{m/s}$ and effective porosity $n_{eff} = 0.26$. The domain is discretized in a *10* x *10* m cellular grid. Pollution is set as a linear source pollution with constant concentration of 5 mg/l during the simulation period. Infiltration is represented by a constant rainfall intensity i = 0.1 mm/h over 250 days with the time step Δt = 6 h, spread homogeneously over the entire domain.

3 Results and Discussion

Initial results of the simulation using coupled DCAGW/CAPT and WCAGW/CAPT models for pollution transport in unsteady groundwater conditions are presented in Figs. 2 and 3, respectively, at one time frame: 250 days after infiltration start and as head and concentration time series at 4 observation points. Figure 2 includes water levels over the entire domain (top graphs) and pollution concentration distribution (bottom graphs) for 3 different models, DCAGW/CAPT, WCAGW/CAPT and MODFLOW, respectively.

DCAGW simulation shows better head matching with MODFLOW results with NRMSD (Normalised Root Mean Square Deviation – RMSE normalized in relation to the difference between max and min value) changing between 0.0547% and 0.0668% for 4 observation points, while the NRMSD is between 0.5247% and 1.6209% when WCAGW model is used. Interestingly, when pollutant concentration are compared, coupled WCAGW/CAPT shows slightly better matching with MODFLOW than

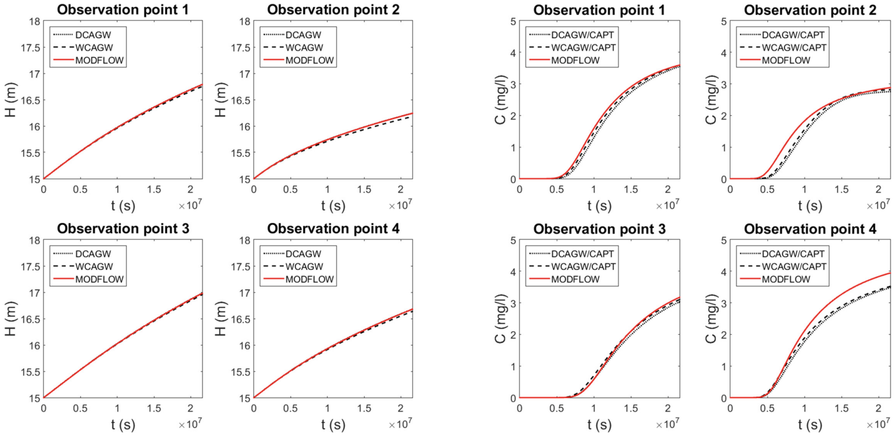


a)

b)

c)

Fig. 2. (a) Head (H) and pollution concentrations (C) over the entire domain using coupled DCAGW/CAPT simulation ($\alpha_x = 1000$ m; $\alpha_y = \alpha_x$) and observation points (OP) marked as red dots; (b) Head (H) and pollution concentrations (C) over the entire domain using coupled WCAGW/CAPT simulation ($\alpha_x = 1000$ m; $\alpha_y = \alpha_x$); (c) Head (H) and pollution concentrations (C) over the entire domain using MODFLOW ($\alpha_x = 1000$ m; $\alpha_y = \alpha_x$).



a)

b)

Fig. 3. (a) Head time series in four observation points marked in Fig. 2a; (b) Concentration time series in four observation points marked in Fig. 2a.

coupled DCAGW/CAPT model. NRMSD for pollutant concentration when WCAGW/CAPT is used are between 0.1681% and 4.3028% for 4 observation points. When DCAGW/CAPT model is used NRMSD is between 1.1843% and 5.6380%.

4 Conclusions

CA approach was used to calculate pollution transport in unsteady groundwater conditions. Two models were tested and compared for simulation of the hydrodynamics effects (WCAGW, DCAGW) and one model was used for pollution transport modelling (CAPT). Analysis shows CA approach usage potential for pollution transport modelling when diffusion is dominant transport mechanism. Future work will include model testing with real scenarios and advection/convection dominant cases and comparison with standard models such as MODFLOW.

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