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IV INTERNATIONAL SYMPOSIUM FOR STUDENTS OF DOCTORAL STUDIES IN THE FIELDS OF CIVIL ENGINEERING, ARCHITECTURE AND ENVIRONMENTAL PROTECTION

Miroslav Marjanović[1]
Đorđe Vuksanović[2]

TRANSIENT RESPONSE OF CROSS-PLY LAMINATED COMPOSITE PLATES

***Abstract**: This paper describes the transient response of cross-ply laminated composite plates. Possibility of achieving the transient response of cross-ply composites by the use of Reddy's generalized laminated plate theory (**GLPT**) is evaluated in this work. Layerwise linear variation of displacements components, as well as linear kinematic relations and Hook's law are assumed. Navier solution is applied for expansion of generalized displacements in double trigonometric series. The governing partial differential equations are reduced to a set of ordinary differential equations in time. The equations of motion are solved using Newmark's integration schemes. Transient response is calculated with an example of simply supported (0/90) laminate. Different number of layers and variants of stacking sequences are taken into consideration by parametric study. Transient response is investigated using different schemes of dynamic loading (forcing functions). For all loading types, results taking into account the influence of time step are presented. Results are compared with those of other theories existing from the literature. Variety of new results is presented.*

***Key words**: composite plate, transient analysis, layerwise theory, Navier solution*

DINAMIČKI ODGOVOR CROSS-PLY LAMINATNIH KOMPOZITNIH PLOČA

***Rezime**: U ovom radu opisan je dinamički odgovor cross-ply laminatnih kompozitnih ploča. Razmatrana je mogućnost određivanja dinamičkog odgovora cross-ply kompozita korišćenjem Reddy-eve Opšte laminatne teorije ploča (**GLPT**). Pretpostavljena je slojevita linearna varijacija komponenata pomeranja, kao i linearne kinematičke relacije i Hook-ov zakon. Navier-ovo rešenje je primenjeno za razvoj generalisanih pomeranja u dvostruke trigonometrijske redove. Uslovne parcijalne diferencijalne jednačine su redukovane na sistem običnih diferencijalnih jednačina po vremenu. Jednačine kretanja su rešene primenom Newmark-ovih integracionih šema. Dinamički odgovor je sračunat na primeru slobodno oslonjene cross-ply (0/90) laminatne ploče. U parametarskoj analizi su razmatrani uticaji broja slojeva u laminatu, kao i različite varijante šema laminacije. Dinamički odgovor je sračunat za različite šeme dinamičkog opterećenja (forcing functions). Za sve tipove opterećenja prezentovani su rezultati koji uzimaju u obzir uticaj vremenskog inkrementa. Rezultati su upoređeni sa nekim rezultatima po drugim teorijama koji postoje u literaturi. Prikazano je i mnoštvo novih rezultata.*

***Ključne reči**: kompozitna ploča, dinamička analiza, laminatna teorija, Navier-ovo rešenje*

[1] Teaching Assistant – PhD Student, University of Belgrade – Faculty of Civil Engineering, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia, e-mail: mmarjanovic@grf.rs
[2] Full Professor, University of Belgrade – Faculty of Civil Engineering, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia, e-mail: george@grf.bg.ac.rs

1. INTRODUCTION

A wide range of mechanical properties, suitable for different design purposes, can be achieved by the use of laminar composites [1]. Since the fiber direction can be altered arbitrarily, laminated composite plates are attractive in design stage. Development of refined mathematical models of laminar composites is obviously of a great importance. Two main approaches arise in a field of laminated composite plates:

- Theories based on a single equivalent layer (**ESL**) – **CLPT**, **FSDT**, **HSDT** theories etc.,
- Layerwise theories (**LWT**) – the modern approach in the analysis of thick and composite plates.

For the study of laminated composite plates, orthotropic materials are of the greatest importance. They mainly appear in the form of thin plies (*lamina)*. Each ply is composed from fibers, oriented in a certain direction. In engineering practice, it is necessary to form a material that will be able to remain stable under loads from multiple directions. This is achieved by composing multiple laminas in a *laminate*.

2. GENERALIZED LAMINATED PLATE THEORY (GLPT)

Generalized Laminated Plate Theory [2] is a significant step in stress/strain analysis of the laminated composite plates. The plate is analyzed as a multilayered in the true sense of word. GLPT allows independent displacement fields interpolation (as well as of stresses/strains). It is based on the piece-wise linear variation of in-plane displacement components, and constant transverse displacement through the thickness. Considerable computer time cost, which was one of the main disadvantages of 3D-elasticity theory, is significantly reduced in GLPT [3]. Shear deformation due to anisotropic structure is included.

2.1. Basic assumptions and displacement field in GLPT

Material follows Hook’s law, and each ply (of constant thickness) is made of orthotropic material, with fibers oriented in arbitrary directions. Kinematic relations are linear, and inextensibility of normal is imposed. Displacement and stress distributions over thickness coordinate are determined using linear Lagrangian interpolations. Displacement field (u_1, u_2, u_3) in the point (x, y, z, t) of laminate is written as:

$$u_1(x,y,z,t)=u(x,y,t)+U(x,y,z,t) \qquad u_2(x,y,z,t)=v(x,y,t)+V(x,y,z,t)$$

$$u_3(x,y,z,t)=w(x,y,t) \tag{1}$$

In previous expressions, (u, v, w) are the displacement components in three orthogonal directions in the middle plane of the laminate. U and V are functions which vanish in the middle plane of the plate. Following expansions are used to reduce functions U and V to 2D format:

$$U(x,y,z,t)=\sum_{I=1}^{N}U^I(x,y,t)\Phi^I(z) \qquad V(x,y,z,t)=\sum_{I=1}^{N}V^I(x,y,t)\Phi^I(z) \tag{2}$$

U^I and V^I are coefficients to be derived, $\Phi^I(z)$ are layerwise continuous functions of the thickness coordinate (linear, quadratic or cubic Lagrangian interpolations). Linear interpolation is chosen through the thickness coordinate. These functions are presented in detail in [3, 4, 5]. Using assumed displacement field, cross section warping is taken into account. More information about GLPT can be found in [3, 4].

2.2. Kinematic relations of single ply in GLPT

Linear strain – displacement relations are given as follows:

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}+\sum_{I=1}^{N}\frac{\partial U^I}{\partial x}\Phi^I \qquad \varepsilon_{yy}=\frac{\partial v}{\partial y}+\sum_{I=1}^{N}\frac{\partial V^I}{\partial y}\Phi^I \qquad \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\sum_{I=1}^{N}\left(\frac{\partial U^I}{\partial y}+\frac{\partial V^I}{\partial x}\right)\Phi^I$$

$$\gamma_{xz}=\sum_{I=1}^{N}U^I\frac{d\Phi^I}{dz} \qquad \gamma_{yz}=\sum_{I=1}^{N}V^I\frac{d\Phi^I}{dz} \tag{3}$$

Obviously, in-plane deformation components are continuous through the plate thickness, while the transverse strains need not to be. Lamina constitutive equations should be used to derive stress-strain relations of the laminated plate. These relations are linear and they are given in detail in [4, 5, 6].

2.3. Equations of motion

Dynamic version of virtual work principle is:

$$\int_0^T \left(\delta U + \delta V + \delta K\right) = 0\,, \tag{4}$$

δU, δV and δK are virtual strain energy, virtual work of external forces and virtual kinetic energy, respectively. Loading is acting in the middle plane of the plate. There are no tractions on the boundary surface of the plate – homogeneous boundary conditions.

$$\delta V = -\int_\Omega q\delta w dxdy \tag{5}$$

$$\delta U = \int_\Omega \left\{ \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\begin{aligned} &\sigma_{xx}\left(\frac{\partial \delta u}{\partial x} + \sum_{I=1}^{N} \frac{\partial \delta U^I}{\partial x}\Phi^I\right) + \sigma_{yy}\left(\frac{\partial \delta v}{\partial y} + \sum_{I=1}^{N} \frac{\partial \delta V^I}{\partial y}\Phi^I\right) + \\ &+\tau_{xy}\left(\frac{\partial \delta u}{\partial y} + \frac{\partial \delta v}{\partial x} + \sum_{I=1}^{N}\left(\frac{\partial \delta U^I}{\partial y} + \frac{\partial \delta V^I}{\partial x}\right)\Phi^I\right) + \\ &+\tau_{xz}\left(\frac{\partial \delta w}{\partial x} + \sum_{I=1}^{N} U^I \frac{d\delta\Phi^I}{\partial z}\right) + \tau_{yz}\left(\frac{\partial \delta w}{\partial y} + \sum_{I=1}^{N} V^I \frac{d\delta\Phi^I}{\partial z}\right) \end{aligned} \right] dz \right\} dxdy \tag{6}$$

$$\delta K = -\int_\Omega \left\{ \begin{aligned} &I_0\left(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right)dxdy + \sum_{I=1}^{N} I^I\left(\ddot{U}^I\delta u + \ddot{u}\delta U^I + \ddot{V}^I\delta v + \ddot{v}\delta V^I\right) + \\ &+\sum_{I=1}^{N}\sum_{J=1}^{N} I^{IJ}\left(\ddot{U}^I\delta U^J + \ddot{V}^I\delta V^J\right) \end{aligned} \right\} dxdy \tag{7}$$

$I_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 dz$, $I^I = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 \Phi^I dz$ and $I^{IJ} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 \Phi^I \Phi^J dz$ are elements of consistent mass matrix.

We will incorporate following stress resultants:

$$\begin{gathered} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} dz \qquad \begin{Bmatrix} N^I{}_{xx} \\ N^I{}_{yy} \\ N^I{}_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \Phi^I dz \\ \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz \qquad \begin{Bmatrix} Q^I{}_x \\ Q^I{}_y \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \frac{d\Phi^I}{dz} dz \end{gathered} \tag{8}$$

2.4. Laminate constitutive equations and Euler-Lagrange equations

Stress resultants will be incorporated as follows, according to matrix equations [5]:

$$\left\{N^0\right\} = [A]\left\{\varepsilon^0\right\} + \sum_{I=1}^{N}\left[B^I\right]\left\{\varepsilon^I\right\} \quad \text{and} \quad \left\{N^I\right\} = \left[B^I\right]\left\{\varepsilon^0\right\} + \sum_{J=1}^{N}\left[D^{JI}\right]\left\{\varepsilon^I\right\} \tag{9}$$

$$[A] = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left[Q_{pq}^{(k)}\right]dz\,, \qquad \left[B^I\right] = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left[Q_{pq}^{(k)}\right]\Phi^I dz\,, \qquad \left[D^{JI}\right] = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left[Q_{pq}^{(k)}\right]\Phi^I\Phi^J dz$$

Introducing Eq. (9) in (4-7), we derive Euler-Lagrange (equilibrium) equations:

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + I_0\ddot{u} + \sum_{I=1}^{N} I^I \ddot{U}^I = 0 \qquad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} + I_0\ddot{v} + \sum_{I=1}^{N} I^I \ddot{V}^I = 0 \tag{10a-b}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+I_0\ddot{w}+q=0 \tag{10c}$$

$$\frac{\partial N_{xx}^I}{\partial x}+\frac{\partial N_{xy}^I}{\partial y}+Q_x^I+I^I\ddot{u}+\sum_{J=1}^{N}I^{JI}\ddot{U}^J=0 \qquad \frac{\partial N_{xy}^I}{\partial x}+\frac{\partial N_{yy}^I}{\partial y}+Q_y^I+I^I\ddot{v}+\sum_{J=1}^{N}I^{JI}\ddot{V}^J=0 \tag{10d-e}$$

3. NAVIER SOLUTION

By derivation of (10), we have obtained 2N+3 partial differential equations, with 2N+3 unknown displacements components. Navier solution for simply supported rectangular plate under transient loading will be presented. Assumed displacement fields are chosen to satisfy equilibrium conditions on boundary edges. Loading should be expanded in double trigonometric series, too. System of ordinary differential equations in time is then derived and solved. Solution for static loading is given in detail in [5].

Cross-ply is a special type of laminar composites, in which fibers are oriented alternately, with angles of 0^o or 90^o. Some elements of elastic coefficients matrix are identically zero:

$$Q_{13}^{(k)}=Q_{23}^{(k)}=Q_{45}^{(k)}=0 \quad \text{and} \quad A_{13}=A_{23}=A_{45}=B_{13}=B_{23}=\overline{B}_{45}=D_{13}=D_{23}=\overline{D}_{45}=0 \tag{11}$$

3.1. Equilibrium equations, boundary conditions and assumed displacement field

After incorporation of (11) and constitutive equations of lamina from [4], system (10) is compacted. After that, displacement components and loading function are expanded in the following manner:

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}(t)\cdot\cos\alpha x\cdot\sin\beta y \qquad v(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}(t)\cdot\sin\alpha x\cdot\cos\beta y$$

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}(t)\cdot\sin\alpha x\cdot\sin\beta y \tag{12}$$

$$U^I(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}R_{mn}^I(t)\cdot\cos\alpha x\cdot\sin\beta y \qquad V^I(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}S_{mn}^I(t)\cdot\sin\alpha x\cdot\cos\beta y$$

Loading expansion:

$$q(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}(t)\cdot\sin\alpha x\cdot\sin\beta y \tag{13}$$

m and n denote number of members in Fourier series, while $\alpha=\frac{m\pi}{a}$, $\beta=\frac{n\pi}{b}$.

X_{mn}, Y_{mn}, W_{mn}, R^I_{mn}, S^I_{mn} are Fourier coefficients – time functions.

Assumed displacement field satisfies the following boundary conditions for simply supported laminated composite plate:

$$x=0,a: \quad u=w=U^I=N_{xx}=N_{xx}^I=0 \qquad I=1,2,...,N \quad N=n+1$$

$$x=0,b: \quad v=w=V^I=N_{yy}=N_{yy}^I=0 \qquad 0\le t\le T \tag{14}$$

After incorporation of (11) in (10) and re-arranging of expressions, we derive the matrix form of differential equations in time:

$$\begin{bmatrix} k & k^I \\ k^I & k^{JI}\end{bmatrix}\begin{Bmatrix} X_{mn} \\ Y_{mn} \\ W_{mn} \\ R_{mn}^I \\ S_{mn}^I\end{Bmatrix}+\begin{bmatrix} m & m^I \\ m^I & m^{JI}\end{bmatrix}\begin{Bmatrix} \ddot{X}_{mn} \\ \ddot{Y}_{mn} \\ \ddot{W}_{mn} \\ \ddot{R}_{mn}^I \\ \ddot{S}_{mn}^I\end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ -q_{mn} \\ 0 \\ 0\end{Bmatrix}, \qquad [K]\{\Delta\}+[M]\{\ddot{\Delta}\}=\{F\} \tag{15}$$

where:

$$k=\begin{bmatrix} A_{11}\alpha^2+A_{33}\beta^2 & A_{12}\alpha\beta+A_{33}\alpha\beta & 0 \\ A_{12}\alpha\beta+A_{33}\alpha\beta & A_{22}\beta^2+A_{33}\alpha^2 & 0 \\ 0 & 0 & A_{44}\alpha^2+A_{55}\beta^2\end{bmatrix}$$

$$k^I = \begin{bmatrix} B_{11}^I\alpha^2 + B_{33}^I\beta^2 & B_{12}^I\alpha\beta + B_{33}^I\alpha\beta \\ B_{12}^I\alpha\beta + B_{33}^I\alpha\beta & B_{22}^I\beta^2 + B_{33}^I\alpha^2 \\ \overline{B}_{44}^I\alpha & \overline{B}_{55}^I\beta \end{bmatrix} \qquad k^{JI} = \begin{bmatrix} D_{11}^{JI}\alpha^2 + D_{33}^{JI}\beta^2 + \overline{D}_{44}^{JI} & D_{12}^{JI}\alpha\beta + D_{33}^{JI}\alpha\beta \\ D_{12}^{JI}\alpha\beta + D_{33}^{JI}\alpha\beta & D_{22}^{JI}\beta^2 + D_{33}^{JI}\alpha^2 + \overline{D}_{55}^{JI} \end{bmatrix}$$

$$m = \begin{bmatrix} I_0 & & \\ & I_0 & \\ & & I_0 \end{bmatrix}, \; m^I = \begin{bmatrix} I^I & 0 \\ 0 & I^I \\ 0 & 0 \end{bmatrix}, \; m^{JI} = \begin{bmatrix} I^{JI} & \\ & I^{JI} \end{bmatrix}$$

$[K]$ is the global stiffness matrix, $\{\Delta\}$ denotes vector of unknown Fourier coefficients, $[M]$ is consistent mass matrix, and $\{\ddot{\Delta}\}$ is a vector of second derivations of Fourier coefficients. Above matrix equations must be satisfied in all time points $0 \le t \le T$. Global stiffness, as well as consistent mass matrix, remains constant in all time points. In the time point t_n, we have following matrix equation:

$$[K]\{\Delta\}_n + [M]\{\ddot{\Delta}\}_n = \{F\}_n \tag{16}$$

4. TRANSIENT ANALYSIS

Different forcing functions which act on laminated composite plate are analyzed. Other types of dynamic loading can be incorporated easily. Transient response deals with two steps in solution process: assumption of the spatial variation of displacements and then reduction of governing partial differential equations to ordinary differential equations in time, and solving of system of equations in time using analytical or numerical methods.

4.1. Equations of motion and numerical time integration

- Step Loading $F(t) = F_0$
- Sine Loading $F(t) = F_0 \sin\left(\frac{\pi \cdot t}{T}\right)$
- Triangular Loading $F(t) = F_0\left(1 - \frac{t}{T}\right)$
- Blast Loading $F(t) = F_0 \cdot e^{-\alpha t}$

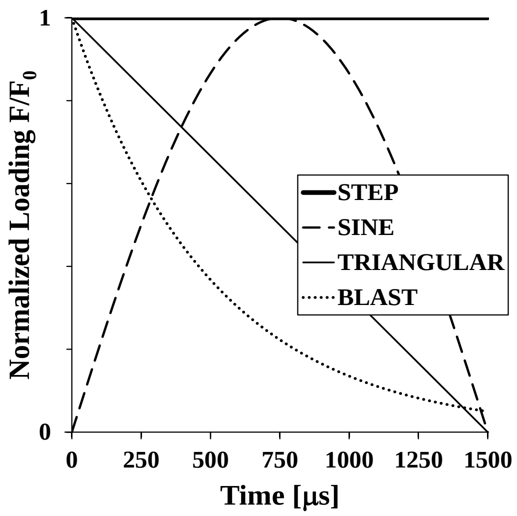

Figure 1 – Different forcing functions

System (15) has to be solved in all time points t_n. Superposed dots denote differentiation with respect to time. In the following discussion we will assume homogenuous initial conditions (initial displacements and their first derivatives – velocities, are zero). Then, the Fourier coefficients X_{mn}, Y_{mn}, W_{mn}, R^I_{mn}, S^I_{mn}, and their first derivatives in time are equal to zero. It is assumed that loading F_0 acts perpendicular to the mid-plane of plate. Load change through time is done according to these loading schemes:

The system of differential equations in time for any fixed m and n can be solved exactly using Laplace transform method or the modal analysis. Alternatively, it can be solved numerically, using the Newmark integration schemes for second-order differential equations. Truncated Taylor series are applied, so the solution is not continuous time function. Terms up to the second derivative are included:

$$\{\Delta\}_{n+1} = \{\Delta\}_n + \Delta t\{\dot{\Delta}\}_n + \frac{1}{2}(\Delta t)^2\{\ddot{\Delta}\}_{n+\gamma} \qquad \{\dot{\Delta}\}_{n+1} = \{\dot{\Delta}\}_n + \Delta t\{\ddot{\Delta}\}_{n+\alpha} \tag{17}$$

$$\{\ddot{\Delta}\}_{n+\alpha} = (1-\alpha)\{\ddot{\Delta}\}_n + \alpha\{\ddot{\Delta}\}_{n+1}$$

$\Delta t = t_{n+1} - t_n$ is the time increment, t_n is the current and t_{n+1} is the next time point in which we seek the solution. Incorporation of first equation into second two gives:

$$\{\dot{\Delta}\}_{n+1} = \{\dot{\Delta}\}_n + a_1\{\ddot{\Delta}\}_n + a_2\{\ddot{\Delta}\}_{n+1} \qquad \{\ddot{\Delta}\}_{n+1} = a_3(\{\Delta\}_{n+1} - \{\Delta\}_n) - a_4\{\dot{\Delta}\}_n - a_5\{\ddot{\Delta}\}_n \tag{18}$$

In above expressions, $a_1 = (1-\alpha)\Delta t$, $a_2 = \alpha\Delta t$, $a_3 = \dfrac{2}{\gamma(\Delta t)^2}$, $a_4 = a_3\Delta t$ and $a_5 = \dfrac{1-\gamma}{\gamma}$.

Parameters α and γ are selected to be 0.5, which correspond to constant-average acceleration method (unconditionally stable) [3]. Stability conditions, as well as more information on this topic, are given in detail in [3, 7]. System of differential equations (17) is solved in the following manner. First, we solve:

$$\left[\hat{K}\right]\{\Delta\}_{n+1} = \left\{\hat{F}\right\} \tag{19}$$

where $\left[\hat{K}\right] = [K]_{n+1} + a_3[M]_{n+1}$ and $\left\{\hat{F}\right\} = \{F\}_{n+1} + [M]_{n+1}\left(a_3\{\Delta\}_n + a_4\{\dot{\Delta}\}_n + a_5\{\ddot{\Delta}\}_n\right)$

From (19) it is obvious that initial conditions $\{\Delta\}_0$, $\{\dot{\Delta}\}_0$ and $\{\ddot{\Delta}\}_0$ are needed for obtaining transient response. $\{\Delta\}_0$ and $\{\dot{\Delta}\}_0$ are known from initial conditions which may or may not be zero. However, acceleration vector $\{\ddot{\Delta}\}_0$ is unknown from initial conditions, and should be calculated from:

$$\{\ddot{\Delta}\}_0 = [M]^{-1}(\{F\} - [K]\{\Delta\}_0) \tag{20}$$

5. NUMERICAL EXAMPLES

Several examples of application of proposed methodology are presented here. In all of the numerical examples, zero initial conditions were assumed. Following lamina properties was used in all calculations:

$h = 1$ cm $\qquad \rho = 8 \times 10^{-6}$ Ns^2/cm^4 $\qquad \nu_{12} = 0.25$

$E_1 = 52.5 \times 10^6$ N/cm^2 $\qquad E_2 = 2.1 \times 10^6$ N/cm^2 $\qquad G_{12} = G_{13} = 0.5E_2$ (21)

Normalized center transverse deflection is presented in all examples: $\bar{w} = w \cdot 100 E_2 h^3 / q a^4$

5.1. Influence of number of elements in Fourier series

In the preliminary calculation, influence of number of members in Fourier series on normalized transverse deflection is analyzed. Composite 2-layer laminate (0/90), with a = b = 25 cm, was examined.

Table 10 –Normalized center transverse deflection at selective times, for different values of $m \times n$

m×n	t=100μs	200	300	400	500	600	700	800	900	1000
1×1	0.4697	1.6270	2.8513	3.4860	3.1907	2.1239	0.8576	0.0708	0.1856	1.1403
3×3	0.4127	1.5855	2.8477	3.4173	3.1681	2.1080	0.7849	0.0667	0.1485	1.0777
5×5	0.4185	1.5864	2.8520	3.4202	3.1701	2.1130	0.7853	0.0727	0.1487	1.0823

Table 1 clearly shows that the number of members in double trigonometric series does not affect severely the transient response of laminated composite plate. According to this, in all following calculations it is assumed that m = n = 1.

5.2. Influence of time increment

Influence of time increment was investigated with 2 composite plates with characteristics (21): 2-layer (0/90) and 8-layer plate $(0/90)_4$. Different time steps were used: Δt = 25, 50, 75, 100, 125 and 150 μs. It is obvious that larger time step increases the period of oscillation, and reduces the amplitude. Note that the maximum transient deflections of both plates are about 2 times that of the static deflection.

2-layer: $\bar{w}_{\max,dynamic} = 3.5009$ $\qquad \bar{w}_{\max,static} = 1.7519$ $\qquad \bar{w}_{\max,d} / \bar{w}_{\max,s} = 1.998$

8-layer: $\bar{w}_{\max,dynamic} = 1.5824$ $\qquad \bar{w}_{\max,static} = 0.7912$ $\qquad \bar{w}_{\max,d} / \bar{w}_{\max,s} = 2.000$

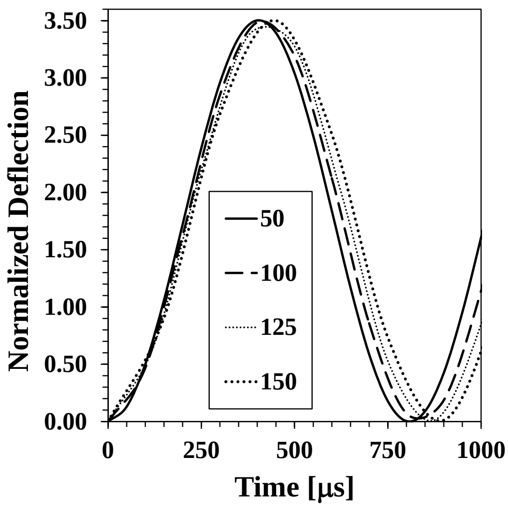

Figure 2 - Simply supported 2-layer cross-ply (0/90) laminate subjected to uniformly distributed step loading

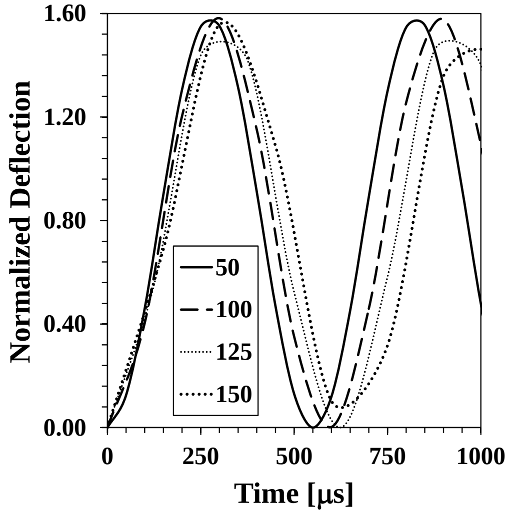

Figure 3 - Simply supported 8-layer cross-ply $(0/90)_4$ laminate subjected to uniformly distributed step loading

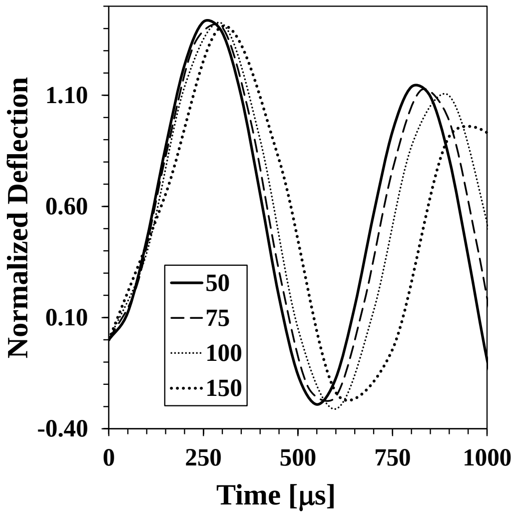

Figure 4 - Simply supported 8-layer cross-ply (0/90) laminate subjected to uniformly distributed triangular loading

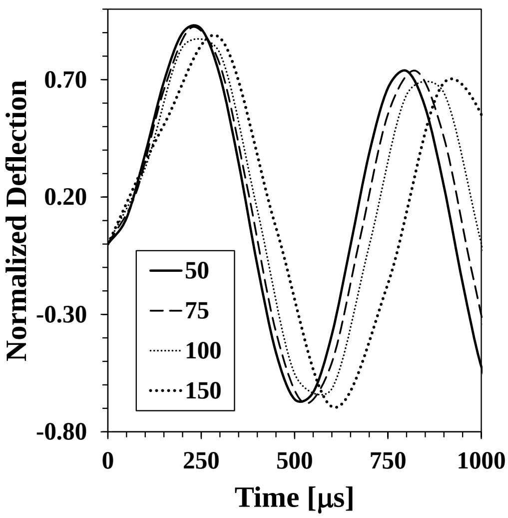

Figure 5 - Simply supported 8-layer cross-ply (0/90) laminate subjected to uniformly distributed blast loading

5.3. Influence of lamination scheme

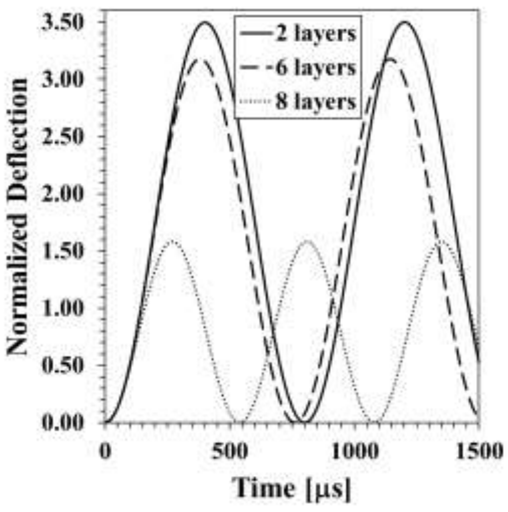

Figure 6 - Simply supported cross-ply laminates subjected to uniformly distributed step loading ($\Delta t = 25\mu s$)

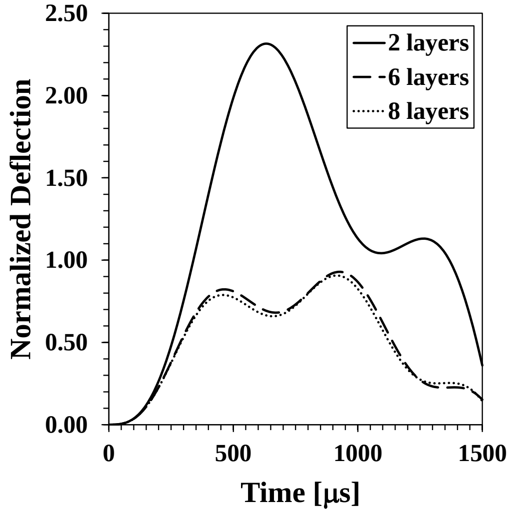

Figure 7 - Simply supported cross-ply laminates subjected to uniformly distributed sine loading ($\Delta t = 25\mu s$)

5.4. Response of plate under different schemes of dynamic loading

The influence of the dynamic loading type is investigated using a simply supported 2-layer (0/90) laminate under uniformly distributed loading. For this purpose, exponential blast loading is chosen as:

$$F(t) = F_0 \cdot e^{-0.002t}$$

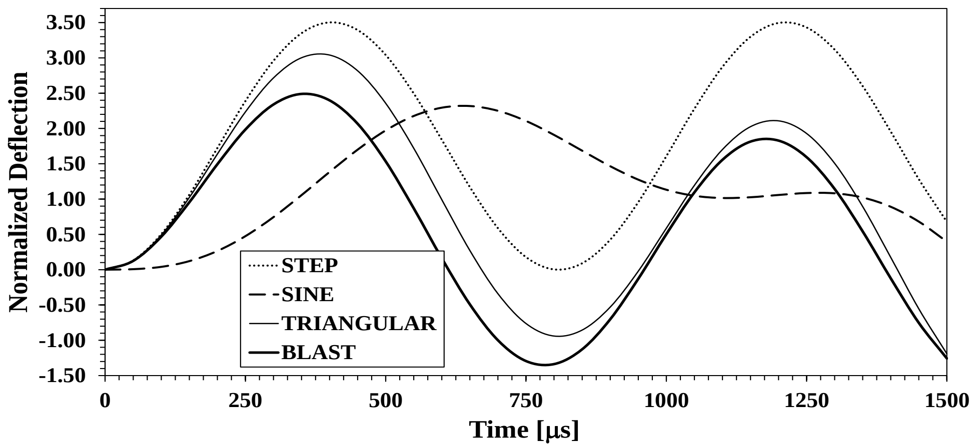

Figure 8 - Simply supported 2-layer cross-ply (0/90) laminate subjected to different schemes of uniformly distributed transient loading (Δt = 50μs, T = 1500μs)

6. CONCLUSIONS

Dynamic version of GLPT is introduced. Using the derived system of differential equations in time, Navier-type solution, as well as Newmark integration scheme, was applied for calculating the transient response, using MATLAB code. It is obviously that the number of elements in double trigonometric series does not affect severely the results of calculation. Using different time steps, influence of time increment on the accuracy of the solution was studied, and it is obvious that larger time step increases the period of oscillation, and reduces the amplitude. Lamination scheme affects the results in a way that reduction in a number of layers leads to a more flexible response of plate – it is increasing the amplitude as well as the period. Using more cross-ply layers in a same plate thickness, we get much stiffer response.

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REFERENCES

[1] Staab GH: *Laminar Composites*, Butterworth – Heinemann, 1999.

[2] Reddy JN, Barbero EJ, Teply JL: *A plate bending element based on a generalized laminate plate theory*, International Journal for Numerical Methods in Engineering, Vol. 28, 1989, pp. 2275-92.

[3] Reddy JN: *Mechanics of Laminated Composite Plates: Theory and Analysis*. CRC Press, 1996.

[4] Ćetković M: *Nonlinear behavior of laminated composite plates*. PhD Thesis (in Serbian). University of Belgrade – Faculty of Civil Engineering, 2010.

[5] Ćetković M: *Analytical solution for multilayer plates using general layerwise plate theory*, Facta Universitatis, Vol. 3, 2005, pp. 121-36.

[6] Ćetković M, Vuksanović Đ: *Bending, free vibrations and buckling of laminated composite and sandwich plates using a layerwise displacement model*, Composite Structures, Vol. 88, 2009, pp. 219–27.

[7] Hinton E: *Numerical Methods and Software for Dynamic Analysis of Plates and Shells*, Pineridge Press, Swansea, 1988, pp. 1-47.