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ELASTO-PLASTIC ANALYSIS OF STEEL BEAMS

Marina Ćetković, Mira Petronijević

1. Introduction

In the modern structural design the limit state analysis of steel structure is obligatory. It is assumed that structure behave elastic until the maximum stress reaches the yield stress in the critical section. As the bending moment is increased, yielding spreads towards the neutral axis until the section has become the plastic hinge. At that stage the bendig moment is equal to plastic moment of the section. The limit state design is based on plastic hinge theory. This analysis includes calculation of internal forces by linear theory using specific load factors to produce structure against collaps.

This approach is idealisation of the true section behaviour. In fact, the bending moments at sections adjacent to the critical one are sufficient to cause yielding before the plastic hinge can be formed. The result is gradual spreading of plasticity, both by depth of cross section and along the beam element. This causes a gradual bending of beam rather than sharp kink which would result from an idealised plastic hinge.

In this paper effect of spreading of yield through the beam axis and over the depth of the beam is carryed out by finite element method (FEM). The continuous beam is analysed using the elasto-plastic Timoshenko beam theory and layer approach. The influence of the cross section stiffness, number of finite elements, number of layers as well as strain hardening parameter on the development of the plastic zone and on the ultimate limit state are presented.

2. Defining of the model

The main assumptions adopted in the derivation of the governing equations of elasto-plastic Timoshenko beam theory, presented by Hinton and Owen [1],[2], are:

1. stress-strain relationship is non-linear,
2. small deflection theory is valid,
3. strain-deflection relationship is linear,
4. after deformation cross sections remain plane, but not normal to neutral axes,
5. stress normal to the neutral axes is neglibible ($\sigma_z \approx 0$).

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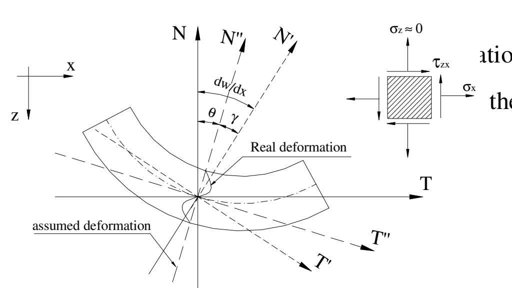


N – normal to the neutral axis before deformation
N' - normal to neutral axis after deformation (Bernoully theory)
N''- normal to neutral axis after deformation (Timoshenko theory)
.... **Fig. 1**

$$\theta(x) = \frac{dw}{dx} - \gamma \tag{1}$$

The axial displacement u(x,z) and the lateral displacement w(x,z) at any point are:

$$u(x,z) = -z * \theta(x), \qquad w(x,z) = w(x) \tag{2}$$

Also, the axial strain ε_x and shear strain γ_{xz} can be written in the following form:

$$\varepsilon_x = \frac{du}{dx} = -z * \frac{d\theta}{dx} \qquad \gamma_{xz} = \frac{du}{dz} + \frac{dw}{dx} = -\theta + \frac{dw}{dx} = \gamma \tag{3}$$

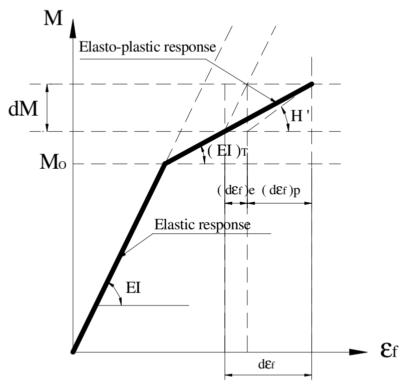


Fig 2.

The moment curvature relationship for Timoshenko beam of elasto-plastic material is shown in Fig. 2.

The beam initially deforms elastically with a flexural rigidity of EI until ultimate bending moment M_0 is reached. By increasing the load further the material is assumed to exhibit linear strain hardening, characterized by the tangential flexural rigidity $(EI)_T$. The incremental increase of bending moment dM is accompanied by a change of curvature $d\varepsilon_f$, which can be separated into elastic and plastic components, so that :

$$d\varepsilon_f = (d\varepsilon_f)_e + (d\varepsilon_f)_p \tag{4}$$

The strain hardening parameter is defined as :

$$\mathbf{H}' = \frac{dM}{(d\varepsilon_f)_p} \tag{5}$$

From equation (2.4) and (2.5) it is possible to write incremental moment curvature relationship in the plastic region as :

$$dM = EI * \left(1 - \frac{EI}{EI + H'}\right) * d\varepsilon_f \tag{6}$$

It can be pointed out that after the yielding the flexural rigidity *EI* have to be replaced by :

$$(EI)_T = EI * \left(1 - \frac{EI}{EI + H'}\right) \tag{7}$$

At the same time the shear force – shear strain relation remains elastic :

$$dT = G\hat{A} * d\gamma_{xz} \qquad (8)$$

where, T denotes shear force, ($\hat{A}=A/\alpha$ is shear cross section.

2.1 Layered beam

In layer approach the beam is divided into chosen number of layers (Fig.3).

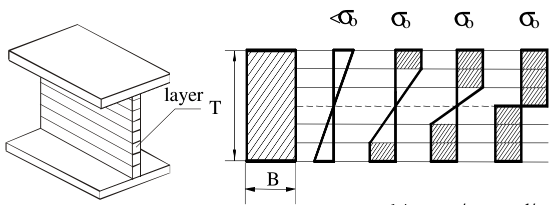


Fig.3

When the stress in the middle of the outer layer reaches the yield value, then this layer becomes plastic, while the rest of the layers remain elastic. As plastification propagates throughout the depth of the beam more layers become plastic, until the eventually plastification of whole cross section. In the layer approach the flexural and shear rigidity are calculated as

$$EI = \sum_i E_i * b_i * z_i^2 * t_i \quad i \quad G\hat{A} = \sum_i G_i * b_i * t_i \qquad (9)$$

where b_i is the layer breadth, t_i is the layer thickness, z_i is z-coordinate at the middle of the layer, E_i and G_i are Young's and shear modulus of the layer material, respectively.

3.Finite element for Timoshenko beam

In this paper Hughes element for linear-elastic Timoshenko beam is used. The stiffness matrix, which is given in [1], consists of the two submatrices: the flexural elment sttifness matrix $\mathbf{K_f}^{(e)}$ and shear element stiffness matrix $\mathbf{K_s}^{(e)}$:

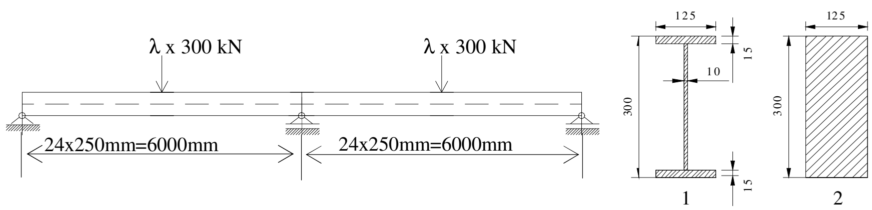
$$\mathbf{K}_f^{(e)} = \left(\frac{GA}{\alpha L}\right)^{(e)} \begin{bmatrix} 1 & \frac{L^{(e)}}{2} & -1 & \frac{L^{(e)}}{2} \\ \frac{L^{(e)}}{2} & \left(\frac{L^{(e)}}{2}\right)^2 & -\frac{L^{(e)}}{2} & \left(\frac{L^{(e)}}{2}\right)^2 \\ -1 & -\frac{L^{(e)}}{2} & 1 & -\frac{L^{(e)}}{2} \\ \frac{L^{(e)}}{2} & \left(\frac{L^{(e)}}{2}\right)^2 & -\frac{L^{(e)}}{2} & \left(\frac{L^{(e)}}{2}\right)^2 \end{bmatrix} \qquad \mathbf{K}_s^{(e)} = \left(\frac{EI}{L}\right)^{(e)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

4.Solution of non-linear eqution

The nonlinear equilibrium equation is solved using the initial stifness metod in order to avoid the singularity of stiffness matrix if H' is equal zero. The incremental procedure is applyed. During each increment of load the problem is linearised and the solution is obtained assuming the sets of successive linear solutions [1].

5.Example

The nonlinear analysis is carried out for continious beam given in Fig. 4. The influence of cross section stifness, number of elements, number of layers and strain hardening parametar H' is analysed.



Material properties : σ_Y=0.25kN/mm^2 E=210kN/mm^2 ν=0.3 H'=0 kN/mm^2

Fig. 4.

The effect of stiffness on the nonliner response of the beam is analysed using two different cross sections with stiffness ratio I_2/I_1=3,6 (Fig. 4). The obtained limit moments are in proportion with beams stiffnesses, while the limit curvature ε_f for the stiffer beam is 10% higher. The moment curvature diagrams for boath beams are given in Fig. 5.

The second analyses is carried out with prametar H' equal 0, 0.04E and 0.06E respectivelly. The strain hardenig parametar increases the plastic moment and curvature of the beam. The nondimensional relation between moment and curvature is given in Fig.6.

H'	M_U/M_o	$\varepsilon_{fu}/\varepsilon_{fo}$
0	1	1
0.04E	1.04	1.08
0.06E	1.09	1.22

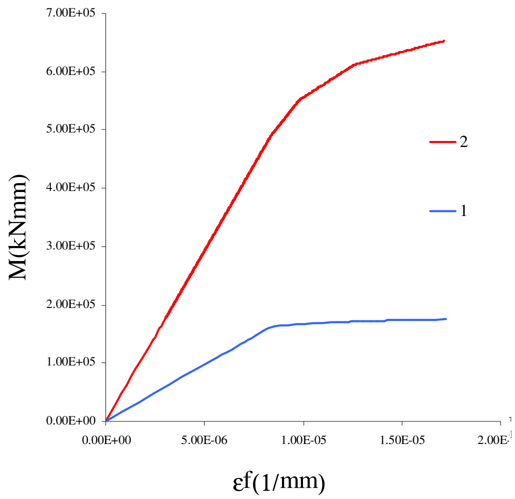


Fig.5

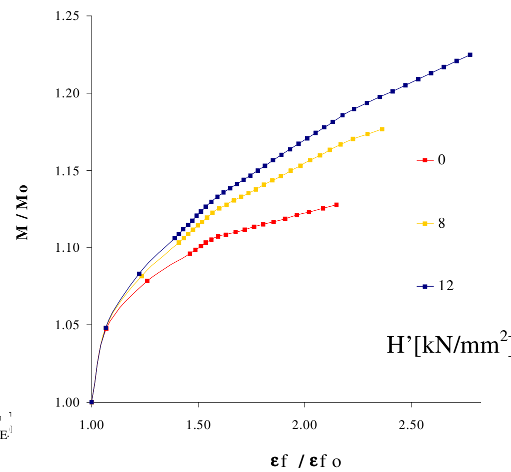


Fig.6

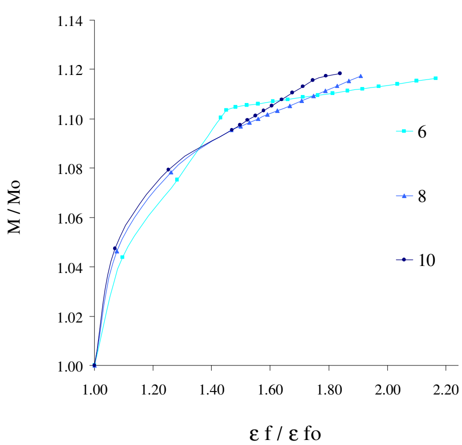


Fig.7

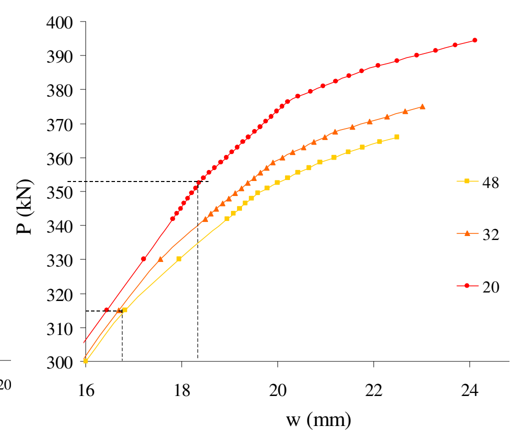


Fig.8

Numb.of layers	P/P6	w/w6
6	1.00	1.00
8	0.986	0.94
10	0.984	0.92

Numb.of Elem.	M/M20	$\varepsilon_f / \varepsilon_{f20}$
20	1	1
32	0.93	0.95
48	0.9	0.93

The effect of number of layers is analysed for 6, 8 and 10 layers. When the number of layers increase ultimate load and ultimate deflection decrease (Fig.7). It means that increasing number of layers decrease the stiffnes of beam.
The effect of number of elements is analysed using 20, 32 and 48 elements. The load deflection diagram in plastic zone is given in Fig.8. It is obvious that if the number of elements increase, the ultimate load decrease, as well as the deflection at limit state.

6. Conclusion

Results of the performed analysis are as follows:
1.Generally we can say that the stiffer beam has greater ultimate capacity and wider plastic zone.
2.Increasing number of elements and layers decrease stiffness of beam, so the ultimate capacity is lower. Also increasing number of elements consderably affect reduction of ultimate strength, while taking great number of layers in account more affect reduction of ultimate deformation.
3.Taking higher strain hardening parameter into account we get the beam of greater ultimate strength. Also, strain hardening parameter have greater influence on limit deformations, than on limit strength.

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Marina Ćetković, Mira Petronijević,
Faculty of Civil Engineering, University of Belgrade