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Editors

Tomislav S. Igić

John T. Katsikadelis

Dragoslav Šumarac

Computer editing

Aleksandra Igić

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VON KARMAN NONLINEAR THEORY OF COMPOSITE PLATES

Vuksanović Đorđe[1], **Ćetković Marina**[1]
[1]Faculty of Civil Engineering, University of Belgrad, Serbia

Abstract: *The von Karman plate theory is used to analyze geometrically nonlinear small strain, large deflection response of composite laminated plates. The 3D elasticity equations are reduced to 2D problem using kinematical assumptions based on layerwise displacement field of Reddy. With the assumed displacement field, nonlinear Green-Lagrange small strain large displacements relations and linear orthotropic material properties for each lamina the total Lagrangian formulation is used to derive the weak form of the problem. The weak form or nonlinear integral equilibrium equations are discretized using isoparametric finite element approximation. The obtained nonlinear incremental algebric equilibrium equations are solved using the Newton Raphson's method. The originally coded MATLAB computer program for the finite element solution is used to investigate the effects of geometrical nonlinearity on displacement field of thick anisotropic laminated composite plate.*

1. Introduction

The aim of the author's research on composite materials so far was to implement Layerwise theory of Reddy or Generalized Layerwise Plate Theory-GLPT [1] on different levels of analysis of laminated composite plates. The previous work has been concerned with the linear analysis [2, 3], and the linear laminated plate element of GLPT has been formulated, while in the present paper the GLPT nonlinear continuum plate element with von Karman geometrical nonlinearity is presented. Two main reasons have driven the present authors to formulate continuum element based on GLPT. The first reason was to formulate general numerical model capable to include different levels nonlinearity, like large strain geometrical nonlinearity or material nonlinearity, in the future. The second reason was to represent the more sophisticated way to derive the tangent stiffness matrix (with less computational cost), from the one using laminate element approach.

2. Theoretical formulation

2.1 Displacement field

In the LW theory of Reddy [1] or Generalized Layerwise Plate Theory (GLPT) displacement field is assumed as:

$$\begin{aligned} u_1(x,y,z) &= u(x,y) + \sum_{I=1}^{N+1} U^I(x,y)\cdot \Phi^I(z) \\ u_2(x,y,z) &= v(x,y) + \sum_{I=1}^{N+1} V^I(x,y)\cdot \Phi^I(z), \\ u_3(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

2.2 Strain-displacement relations

The Green Lagrange strain tensor associated with the displacement field (1) can be computed using von Karman strain-displacement relation [3] to include geometric nonlinearities as follows:

$$\{\varepsilon\} = \begin{Bmatrix} \{\varepsilon^0\} + \{\varepsilon^m\} \\ \{\varepsilon^l\} \end{Bmatrix} \tag{2}$$

2.3 Constitutive equations

For Hook's elastic material, the stress-strain relations for k-th orthotropic lamina have the following form:

$$\{\sigma\}^{(k)} = [Q]^{(k)} \cdot \{\varepsilon\}^{(k)}. \tag{3}$$

where $\boldsymbol{\sigma}^{(k)} = \{\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}\}^{(k)T}$ and $\boldsymbol{\varepsilon}^{(k)} = \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^{(k)T}$ are stress and strain components respectively, and $\mathbf{Q}_{ij}^{(k)}$ are transformed elastic coefficients, of k-th lamina in global coordinates [3].

2.4 Incremental equilibrium equations

The geometrically nonlinear problem of laminated composite plates subjected to external loading may be obtained using incremental continuum formulation:

$$\int_{^2V} \delta({}_2e) \cdot {}^2\sigma \cdot d^2V = \int_{^2V} \delta u \cdot {}^2f \cdot d^2V + \int_{^2S} \delta u \cdot {}^2t \cdot d^2S \tag{4}$$

3. Finite Element Model

Discretization of the weak form Eq. (4) with the assumed displacement field Eq. (1) gives the following nonlinear incremental finite element GLPT model equations:

$$\left([\mathbf{K}_{NL}]^e + [\mathbf{K}_G]^e\right) \cdot \{\bar{\mathbf{d}}\}^e = \{{}^2_0\mathbf{R}\}^e - \{{}^1_0\mathbf{R}\}^e \tag{5}$$

3. Example

A nonlinear bending of square simply supported (SS1) general quasi-isotropic eight layer $(0/45/\text{-}45/90)_s$, laminated plate with $a = b = 1$ and $h = 0.1$, made of material:

$$E_1/E_2 = 40,\ G_{12}/E_2 = 0.6,\ G_{13}/E_2 = 0.6,\ G_{23}/E_2 = 0.5,\ \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{6}$$

subjected to uniform transverse pressure is analyzed.

q_0	Linear Argyris and Tanek(1994)	Nonlinear Argyris and Tanek(1994)	Present
50	0.2717	0.2691	0.2980
100	0.5435	0.4862	0.5582
150	0.8152	0.6573	0.7276
200	1.0870	0.7975	0.8631
250	1.3587	0.9179	0.9780

TABLE 1. Central displacement versus load of square simply supported (SS1) general quasi-isotropic $(0/45/\text{-}45/90)_s$ laminated plate with $a/h = 10$.

3. References

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