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NONLINEAR ANALYSIS OF LAMINATED COMPOSITE PLATES

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Abstract

In this paper the geometrically nonlinear laminated finite element model is developed using the principle of virtual displacements (PVD). The 3D elasticity equations are reduced to 2D problem using kinematical assumptions based on assumed layerwise displacement field of Reddy. With the assumed displacement field, nonlinear Green-Lagrange small strain large displacements relations and linear orthotropic material properties for each lamina, the PVD is used to obtain the weak form of the problem. The weak form or nonlinear integral equilibrium equations are discretized using isoparametric finite element approximation. The nonlinear incremental algebric equilibrium equations are solved using the direct iteration procedure. The original MATLAB computer program is coded for finite element solution and is used to investigate the geometrical nonlinear effects on displacements of anisotropic laminated composite plates. The accuracy of the numerical model is verified by comparison with results from the literature and the linear solutions from the previous paper (Vuksanovic 2000, Ćetković et al. 2009). Appropriate conclusions are derived.

Keywords - geometrically nonlinear analysis, layerwise finite element model

1 INTRODUCTION

During the last decade, there has been increasing use of composites in the design of primary load carrying members in aerospace and automotive industry, ship building industry and bridge design. The low mass density associated with high tensile strength provides them with high strength to weight ratios and high specific modulus. As a result of their lightness, composites replaced most traditional materials without being constrained in slenderness and thickness. The second outstanding feature of composite laminates is their so called "controlled anisotropy" associated with manufacturing flexibility one has to control mechanical properties of composite laminates by adjusting at will the lamina orientation in the stacking sequence of the laminate.

The above mentioned features resulted in large weight savings and made possible the use of very thin composite plate elements. However these elements become susceptible to large deflections during their service life (Polat et al. 2007, Zhang et al. 2006). In such cases the geometry of structure is continually changing during the deformation and geometrically nonlinear analysis should be adopted. The geometrically nonlinear analysis seems also to be necessary for obtaining the structural response of unsymmetrical laminated composite materials (Zhang et al., 2003). Namely, the nonlinear response of these laminates is present even for small displacements, due to complex coupling between in-plane and out-of plane deformation.

In this paper the mathematical and numerical model for geometrically nonlinear, small strain, large displacements problem of laminated composite plates is presented. The 3D elasticity equations are reduced to 2D problem using kinematical assumptions based on layerwise displacement field of Reddy (GLPT). With the assumed displacement field, nonlinear Green-Lagrange small strain large displacements relations and linear orthotropic material properties for each lamina, the principle of virtual displacement (PVD) is used to derive the weak form of the problem. The weak form or nonlinear integral equilibrium equations are discretized using isoperimetric finite element approximation. The obtained nonlinear incremental algebric equilibrium equations are solved using direct iteration procedure. The originally coded MATLAB computer program for the finite element solution is used to investigate the effects of geometrical nonlinearity on displacements of anisotropic laminated composite plate. The accuracy of the numerical model is vefified by being compared with available results from the literature and the linear solutions from the previous paper (Ćetković et al. 2009). The appropriate conclusions are derived.

2 THEORETICAL FORMULATION

2.1 Displacement field

In the LW theory of Reddy (Reddy et al. 1989) or Generalized Layerwise Plate Theory (GLPT), in-plane displacements components (u,v) are interpolated through the thickness using 1D linear Lagrangian interpolation function $\Phi^I(z)$, while transverse displacement component w is assumed to be constant through the plate thickness.

$$\begin{aligned} u_1(x,y,z) &= u(x,y) + \sum_{I=1}^{N+1} U^I(x,y)\cdot\Phi^I(z) \\ u_2(x,y,z) &= v(x,y) + \sum_{I=1}^{N+1} V^I(x,y)\cdot\Phi^I(z), \\ u_3(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

2.2 Strain-displacement relations

The Green Lagrange strain tensor associated with the displacement field Eq.(1) is computed using von Karman strain-displacement relation to include geometric nonlinearities as follows:

$$\varepsilon_{xx} = \frac{\partial u_1}{\partial x} + \frac{1}{2}\left(\frac{\partial u_3}{\partial x}\right)^2 = \frac{\partial u}{\partial x} + \sum_{I=1}^{N+1}\frac{\partial U^I}{\partial x}\Phi^I + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,$$

$$\varepsilon_{yy} = \frac{\partial u_2}{\partial y} + \frac{1}{2}\left(\frac{\partial u_3}{\partial y}\right)^2 = \frac{\partial v}{\partial y} + \sum_{I=1}^{N+1} \frac{\partial V^I}{\partial y} \Phi^I + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$

$$\gamma_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} + \frac{\partial u_3}{\partial x}\frac{\partial u_3}{\partial y} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \sum_{I=1}^{N+1}\left(\frac{\partial U^I}{\partial y} + \frac{\partial V^I}{\partial x}\right)\Phi^I + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \tag{2}$$

$$\gamma_{xz} = \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} = \sum_{I=1}^{N+1} U^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial x},$$

$$\gamma_{yz} = \frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y} = \sum_{I=1}^{N+1} V^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial y}.$$

2.3 Constitutive equations

For Hook's elastic material, the stress-strain relations for k-th orthotropic lamina have the following form:

$$\{\sigma\}^{(k)} = [Q]^{(k)} \cdot \{\varepsilon\}^{(k)}. \tag{3}$$

where $\boldsymbol{\sigma}^{(k)} = \{\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}\}^{(k)T}$ and $\varepsilon^{(k)} = \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^{(k)T}$ are stress and strain components respectively, and $\mathbf{Q}_{ij}^{(k)}$ are transformed elastic coefficients, of k-th lamina in global coordinates (Cetkovic et al., 2009).

2.4 Equilibrium equations

Equilibrium equations may be obtained from the Principle of Virtual Displacements (PVD), in which sum of external virtual work done on the body and internal virtual work stored in the body should be equal zero:

$$\begin{aligned} 0 = & \iint_{\Omega}\left[\left(\{\delta\varepsilon^0\}^T + \{\delta\varepsilon^m\}^T\right)\{N^0\} + \{\delta\varepsilon^I\}^T\{N^I\} + \delta u_0\, q_x^0 + \delta v_0\, q_y^0 + \delta w_0\, q_z^0\right] dxdy \\ & - \oint_{\Gamma} \delta u_n\, N_{nn}\, ds - \oint_{\Gamma} \delta u_s\, N_{ns}\, ds - \oint_{\Gamma} \delta w_0 \left(Q_n + P_n\right) ds - \oint_{\Gamma} \delta U_n^I\, N_{nn}^I\, ds - \oint_{\Gamma} \delta U_s^I\, N_{ns}^I\, ds \end{aligned} \tag{4}$$

where $\{q_x^0, q_y^0, q_z^0\}$ is distributed load in x, y, z directions, while internal forces are:

$$\begin{Bmatrix} \{N^0\} \\ \{N^I\} \end{Bmatrix} = \begin{bmatrix} [\mathbf{A}] & [\mathbf{B}^\mathbf{I}] \\ [\mathbf{B}^\mathbf{I}] & \sum_{J=1}^{N} [\mathbf{D}^{\mathbf{JI}}] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} + \{\varepsilon^m\} \\ \{\varepsilon^I\} \end{Bmatrix} \tag{5}$$

where **A, B, B$^\mathbf{I}$, D$^\mathbf{JI}$** matrices are given in (Cetkovic, 2005).

3 FINITE ELEMENT MODEL

The GLPT finite element consists of middle surface plane and N+1 planes through the plate thickness Fig. 1. The element requires only the C^0 continuity of major unknowns, thus in each node only displacement components are adopted, that are (u, v, w) in the middle surface element nodes and (U^I, V^I) in the I-th plane element nodes. The generalized displacements over element Ω^e can be expressed as:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}^e = \begin{Bmatrix} \sum_{j=1}^{m} u_j \Psi_j \\ \sum_{j=1}^{m} v_j \Psi_j \\ \sum_{j=1}^{m} w_j \Psi_j \end{Bmatrix}^e = \sum_{j=1}^{m} [\Psi_j]^e \{d_j\}^e \quad \begin{Bmatrix} U^I \\ V^I \end{Bmatrix}^e = \begin{Bmatrix} \sum_{j=1}^{m} U_j^I \Psi_j \\ \sum_{j=1}^{m} V_j^I \Psi_j \end{Bmatrix}^e = \sum_{j=1}^{m} [\overline{\Psi}_j]^e \{d_j^I\}^e \tag{6}$$

where $\{\mathbf{d}_j\}^e = \{u_j^e \quad v_j^e \quad w_j^e\}^T, \{\mathbf{d}_j^I\}^e = \{U_j^I \quad V_j^I\}^T$ are displacement vectors, in the middle plane and I-th plane, respectively, Ψ_j^e are interpolation functions, while $[\boldsymbol{\Psi}_j]^e, [\overline{\boldsymbol{\Psi}}_j]^e$ are interpolation function matrix for the j-th node of the element Ω^e, given in (Ćetković et al,. 2009).

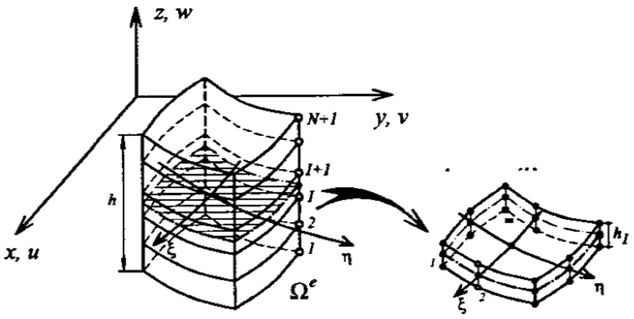

Fig. 1. Plate finite element with n layers and m nodes

Substituting element displacement field Eq.(6) in to weak form Eq.(4), the nonlinear laminated finite element is obtained:

$$[\mathbf{K}_{NL}]^e \cdot \{\mathbf{d}\}^e = \{\mathbf{f}\}^e \tag{7}$$

where secant stiffness matrix is:

$$[\mathbf{K}_{NL}]^e = \begin{bmatrix} [\mathbf{K}^{11}]^e & [\mathbf{K}^{12}]^e \\ [\mathbf{K}^{12}]^e & [\mathbf{K}^{22}]^e \end{bmatrix}$$

$$[\mathbf{K}^{11}]^e = \sum_{i=1}^{m}\sum_{j=1}^{n} \int_{\Omega^e} [\mathbf{H}_i^e]^T \cdot [\mathbf{A}] \cdot [\mathbf{H}_j^e] + \underline{[\mathbf{H}_i^e]^T \cdot [\mathbf{A}] \cdot [\mathbf{H}_{jNL}^e]} + \underline{2[\mathbf{H}_{iNL}^e]^T \cdot [\mathbf{A}] \cdot [\mathbf{H}_j^e]} + \underline{2[\mathbf{H}_{iNL}^e]^T \cdot [\mathbf{A}] \cdot [\mathbf{H}_{jNL}^e]} \; d$$

$$[\mathbf{K}^{12}]^e = \sum_{i=1}^{m}\sum_{j=1}^{n} \int_{\Omega^e} \left([\mathbf{H}_i^e]^T \cdot [\mathbf{B}^I] \cdot [\overline{\mathbf{H}}_j^e] + \underline{2[\mathbf{H}_{iNL}^e]^T \cdot [\mathbf{B}^I] \cdot [\overline{\mathbf{H}}_j^e]} \right) d\Omega^e$$

$$[\mathbf{K}^{21}]^e = \sum_{i=1}^{m}\sum_{j=1}^{n} \int_{\Omega^e} \left([\overline{\mathbf{H}}_i^e]^T \cdot [\mathbf{B}^I] \cdot [\mathbf{H}_j^e] + \underline{[\overline{\mathbf{H}}_i^e]^T \cdot [\mathbf{B}^I] \cdot [\mathbf{H}_{jNL}^e]} \right) d\Omega^e$$

$$[\mathbf{K}^{22}]^e = \sum_{i=1}^{m}\sum_{j=1}^{n} \int_{\Omega^e} \left[\overline{\mathbf{H}}_i^e\right]^T \cdot [\mathbf{D}^{II}] \cdot \left[\overline{\mathbf{H}}_j^e\right] d\Omega^e \tag{8}$$

and external force vectors $\{\mathbf{f}\}^e = \begin{Bmatrix} \{\mathbf{f}^0\}^e \\ \{\mathbf{f}^I\}^e \end{Bmatrix}$ are:

$$\{\mathbf{f}^0\}^e = \sum_{i=1}^{m}\left[\int_{\Omega^e} \left[\mathbf{\Psi}_i^e\right]^T \begin{Bmatrix} q_x^0 \\ q_y^0 \\ q_z^0 \end{Bmatrix} d\Omega^e + \oint_{\Gamma^e} \left[\mathbf{\Psi}_i^e\right]^T \begin{Bmatrix} N_{nn} \\ N_{ns} \\ Q_n + P_n \end{Bmatrix} d\Gamma^e\right]$$

$$\{\mathbf{f}^I\}^e = \sum_{i=1}^{m}\left[\int_{\Omega^e} \left[\mathbf{\Psi}_i^e\right]^T \begin{Bmatrix} q_x^I \\ q_y^I \end{Bmatrix} d\Omega^e + \oint_{\Gamma^e} \left[\mathbf{\Psi}_i^e\right]^T \begin{Bmatrix} N_{nn}^I \\ N_{ns}^I \end{Bmatrix} d\Gamma^e\right] \tag{9}$$

4 EXAMPLE

A nonlinear bending of square simply supported general quasi-isotropic $(0/45/-45/90)_s$, laminated plate with $a = b = 1$ and $h = 0.1$, made of material:

$$E_1/E_2 = 40,\ G_{12}/E_2 = 0.6,\ G_{13}/E_2 = 0.6,\ G_{12}/E_2 = 0.5,\ \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{10}$$

subjected to uniform transverse pressure is analyzed. Using the load parameter $\overline{P} = q_0 \cdot a^4/(E_2 h^4)$, the incremental load vector is chosen to be:

$$\{\Delta q\} = \{50, 50, 50, 50, 50\} \cdot \overline{P} \tag{11}$$

with convergence tolerance $\varepsilon = 0.01$ and acceleration parameter $\gamma = 0,8$.

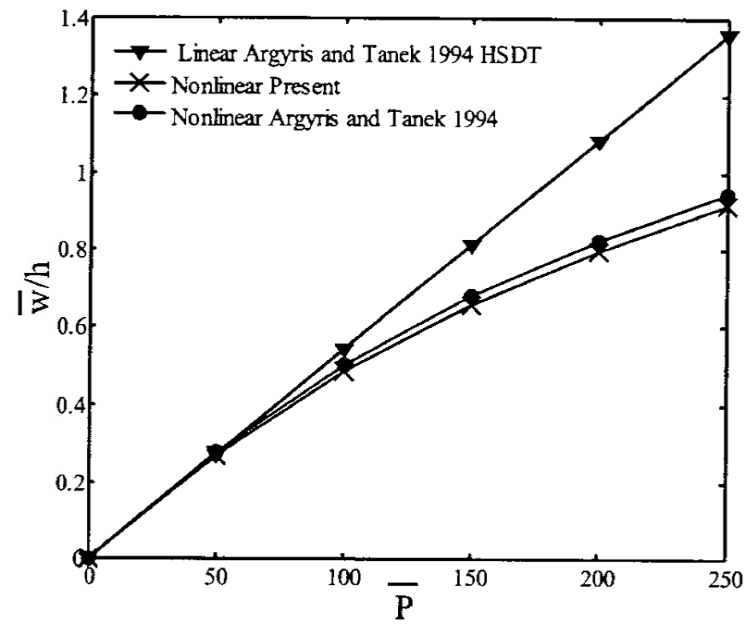

Fig. 2. Nonlinear bending of square simply supported general quasi-isotropic $(0/45/-45/90)_s$ laminated plate with $a/h = 10$; central displacement versus load parameter

A 2x2 quarter plate continuum GLPT model is compared with 8x8 full plate HSDT model (Argyris and Tanek, 1994). The results for linear and nonlinear deflections are presented in Fig. 2. It is shown that proposed GLPT model closely agree with HSDT model form literature, with the faster convergence (Cetkovic et al., 2011).

5 CONCLUSION

In this paper a laminated layerwise finite element model for geometrically nonlinear small strain, large deflection analysis of laminated composite plates is derived using the PVD. The accuracy of the model is verified by comparison with the results from the literature and close agreement is achieved.

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