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DIAGRAMS FOR STRESS AND DEFLECTION PREDICTION IN CROSS-LAMINATED TIMBER (CLT) PANELS WITH NON-CLASSICAL BOUNDARY CONDITIONS

Abstract:

Invention of cross-laminated timber (CLT) was a big milestone for building with wood. Due to novelty of CLT and timber's complex mechanical behavior, the existing design codes cover only rectangular CLT panels, simply supported along 2 parallel or all 4 edges, making numerical methods necessary in other cases. This paper presents a practical engineering tool for stress and deflection prediction of CLT panels with non-classical boundary conditions, based on the software for the computational analysis of laminar composites, previously developed by the authors. Diagrams applicable in engineering practice are developed for some common cases. The presented methodology could be a basis for more detailed design handbooks and guidelines for various layouts of CLT panels and different types of loadings.

Keywords: cross-laminated timber, stress, deflection, Eurocode 5, layerwise plate theory

ДИЈАГРАМИ ЗА ПРОРАЧУН НАПОНА И УГИБА CLT-ПАНЕЛА СА НЕСТАНДАРДНИМ УСЛОВИМА ОСЛАЊАЊА

Сажетак:

Појава унакрсно-ламелираног дрвета (CLT) представља прекретницу у градњи дрветом. Како је CLT нов материјал са сложеним механичким понашањем, постојећи стандарди за прорачун покривају само слободно ослоњене правоугаоне панеле. Стога су нумеричке методе неопходне у осталим случајевима. У овом раду је представљен практичан инжењерски алат за прорачун напона и угиба CLT панела са нестандарним условима ослањања, базиран на (од стране аутора) раније развијеном програму за прорачун ламинатних композита. Дати су дијаграми за поједине случајеве који су примењиви у инжењерској пракси. Ова методологија може бити основа за детаљније смернице при прорачуну CLT панела различитих облика, услова ослањања и оптерећења.

Кључне ријечи: унакрсно-ламелирано дрво, напон, угиб, Еврокод 5, слојевита теорија плоча

1. INTRODUCTION

The cross-laminated timber (CLT) is rapidly spreading in most European countries. It is an innovative material, in which timber boards are assembled in layers and glued together crosswise in order to form massive timber wall and floor panels. Since timber is an anisotropic material, gluing laminations at right angles allows for the panels to have better strength and stiffness properties in both directions compared to traditional timber. In addition, CLT has good insulation properties inherited from wood and good behavior in case of fire.

There is an ongoing trend that CLT continuously shifts the limits for tall timber buildings [1, 2]. In the early 2000s, construction with CLT increased drastically due to the green building tendency. Typical building types from CLT include multi-family apartments, multi-storey business or administrative buildings.

Mechanical properties and design procedures for CLT have been regulated via international European Technical Approvals (ETAs) started in 2006. The first activities standardizing CLT in Europe began in 2008 and the first European product standard for CLT, EN 16351 [3], has recently passed the formal vote. CLT is going to be included in the European timber design code Eurocode 5 [4]. One of the reasons for the slow progress in the development of timber design codes, and in particular, for the difficulties to fully understand the mechanics of timber materials, lies mainly in the highly complex nature of wood microstructure [5].

CLT panels are generally produced in a rectangular shape and foreseen to be line supported in one or both directions. However, as a consequence of architectural design requirements, CLT panels are often manufactured in shapes that are irregular and/or that have openings (e.g. for staircases or chimneys). Furthermore, due to structural demands, the panels are not always simply line supported on two sides, but they can have different support systems depending on the vertical members that transfer the load from the slabs to the foundations. For example, balconies are designed as cantilever slabs, while façade columns represent point supports, as shown in Figure 1.

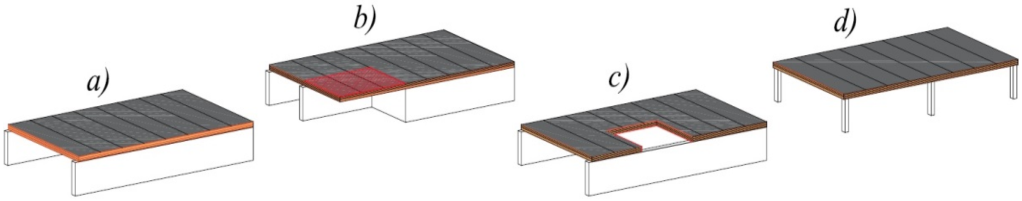

Figure 1. *Different support systems of CLT floors: (a) standard beam-like structure (b) balcony; (c) rectangular opening; (d) point supports*

1.1. Analytical design methods for clt panels

For determining stress state of CLT panels in bending (normal - σ_m, and shear stresses - τ_V), there are several commonly used analytical procedures. Most common is Gamma method implemented in Annex B of Eurocode 5 [4] and pro:Holz recommendations [6]. Besides this method, other analytical procedures are: composite theory of Blass [7], shear analogy method [8], Timoshenko [9] or laminated beam theories [10]. These methods use a simplified approach, treating the 2D structure as a beam system, which do not completely take advantage of the cross-lamination.

Gamma method accounts for the horizontal shear deformation occurring in the cross-layers and vertical shear deformation in the longitudinal layers. Longitudinal layers are taken as beam elements connected with „imaginary“ fasteners that have stiffness equal to that of rolling shear of cross layers. The stiffness properties are defined using the effective moment of inertia $I_{0,ef}$ that depends on the section properties and the connection efficiency factor γ. The Gamma method can be used for a maximum of 5 layers and it is recommended for span-to-depth (*a/h*) ratio greater than 30. If the CLT slab has more than 5 layers, the Extended Gamma Method (EGM) is required.

In the composite theory (*k*-method), strength and stiffness properties of single layers are taken into account using the so-called "composition factors" (k_i) [7]. In the method, Bernoulli's hypothesis and linear stress-strain relationship are assumed. Therefore, it doesn't consider shear deformation and it can only be used for $a/h > 30$.

1.2. Deflection prediction for clt panels

The deflection of CLT panels is derived according to Eurocode 5 [4]. Final deflection w_{fin} results from both the instantaneous (w_{inst}) and creep deflection (w_{creep}):

$$w_{inst} = w_G + \sum_{i \geq 1} \psi_{2,i} \cdot w_{Q,i}, \quad w_{creep} = k_{def} \cdot w_{inst}, \quad w_{fin} = w_{inst} \cdot \left(1 + k_{def}\right) \leq \frac{l}{300} \tag{1}$$

where w_G and w_Q are the deflections from permanent and variable loads, respectively, k_{def} is deformation factor, ψ_2 is factor for quasi-permanent value of a variable action and l is the shortest span of the CLT slab. Factor k_{def} evaluates creep deformation and takes into account the relevant service class and material type. Factor ψ_2 depends from the type of a variable action considered, that is, from its duration.

Low specific weight of timber is a disadvantage when it comes to serviceability of elements loaded out of plane, such as CLT floors, due to the possibility of uncomfortable deflections and vibrations in these elements [11]. Since serviceability limit state usually governs design of timber floors both stresses and deflections are considered in this paper.

The aim of the paper is to provide practical engineering tool for stress and deflection prediction of CLT panels with non-classical boundary conditions. Based on the previously developed software for the computational analysis of laminar composites, diagrams applicable in engineering practice are developed for some common cases. The methodology presented in the paper could serve as a basis for the development of more detailed handbooks and guidelines.

2. FINITE ELEMENT MODEL FOR STRESS AND DEFLECTION PREDICTION IN CLT PANELS

The existing analytical 2D procedures for CLT analysis are limited to the simple rectangular panels, simply supported along 2 parallel or all 4 edges. Consequently, there is a necessity to use numerical methods for the analysis of CLT panels in many cases of everyday engineering practice.

The possible solution for the above issue is the application of finite element model based on the full-layerwise theory (FLWT) of Reddy [10]. It was initially developed for the analysis of composite laminates with a thickness of h, made of n orthotropic layers. In the FLWT, the displacement field (u, v, w) of an arbitrary point (x,y,z) of the laminate is given as:

$$u(x,y,z) = \sum_{I=1}^{N} U^I(x,y)\Phi^I(z),\ v(x,y,z) = \sum_{I=1}^{N} V^I(x,y)\Phi^I(z),\ w(x,y,z) = \sum_{I=1}^{N} W^I(x,y)\Phi^I(z) \tag{2}$$

where $U^I(x,y)$, $V^I(x,y)$ and $W^I(x,y)$ are the displacement components in the I^{th} numerical layer of the plate in directions x, y and z, respectively, while N is the number of interfaces between the layers including both top and bottom surfaces. $\Phi^I(z)$ are selected to be linear layerwise continuous functions of the z-coordinate [10]. Piece-wise linear variation of all three displacement components through the plate thickness is imposed, leading to the 3D stress description of all material layers. The stresses in the k^{th} layer may be computed from the well-known lamina 3-D constitutive equations. Based on the FLWT, the displacement finite element model is derived using an assumed interpolation of the displacement field [12, 13]. Element stiffness matrix and force vector are obtained using 2-D Gauss-Legendre quadrature for quadrilateral domains. Quadratic serendipity (Q8) layered quadrilateral elements have been used in the paper. To avoid shear locking, reduced integration is used (2×2 Gauss points). The assumed piecewise linear interpolation of displacement field through the laminate thickness provide discontinuous stresses across the interface between adjacent layers. Once the nodal displacements are obtained, the stresses σ_x, σ_y, σ_z, τ_{yz}, τ_{xz} and τ_{xy} can be computed from the constitutive relations of every layer k [12, 13]. Since the interlaminar stresses τ_{yz}, τ_{xz} and σ_z calculated in this way do not satisfy continuous distribution through the laminate thickness, they are re-computed assuming the quadratic distribution within each layer for every stress component [12, 13].

3. DIAGRAMS FOR STRESS AND DEFLECTION PREDICTION IN CLT PANELS

3.1. Overview

Due to stress concentration occurrence around the openings, the driving factor for the design of CLT panels could be both stress or deformation criteria. This requires employment of refined numerical methods when designing complex-shape panels. Since one of the biggest downsides of CLT structures is price, savings could be made with more accurate design procedures like the one presented in this paper.

In this part, the finite element model presented in the previous section is used for the stress and deflection prediction in CLT panels. For the proposed numerical method to be used in structural design, it needs to be practical, which is achieved through definition of easily applicable diagrams.

Manufacturing process and transportation limitations define the panel size that can be delivered to construction sites. Therefore, panel-to panel connections, used to connect panels along their longitudinal edges and transfer in-plane forces, are mostly established on site. These connections should allow for quick and easy assembly and give almost unlimited possibilities for panels length-to-width ratios. There are various panel-to-panel connections, such as internal splines, single surface splines, double surface splines, half-lapped joint, tube connection system, etc, as shown in Figure 2.

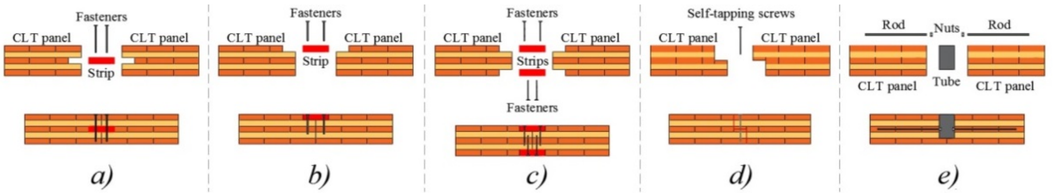

Figure 2. *Various types of CLT panel connections*

The influence of connections within the panel could be introduced in the proposed model by introducing the narrow strip of the finite elements with reduced elastic material properties, to account for the disconnection between two connected panels. This influence is not considered in the paper.

3.2. Rectangular clt panels with non-classical boundary conditions

First, rectangular CLT panels with different combinations of boundary conditions are considered (Figure 3a-c). The panels are exposed to uniformly distributed loading q_0 on the top surface. The 3-layer (L/h = 20), 5-layer (L/h = 20 and L/h = 30) and 7-layer panels (L/h = 20 and L/h = 30), are analysed (Figure 3d). The span-to-thickness ratios (L/h) have been selected in order to cover wide range of possible practical problems.

Table 1. *Mechanical properties for C24 timber class*

E_L	E_T=E_R	G_{LT}=G_{LR}	G_{RT}	ν_{LT}	ν_{LR}	ν_{RT}
11000 N/mm^2	370 N/mm^2	690 N/mm^2	69 N/mm^2	0.49	0.39	0.64

All layers are of equal thickness. Each layer is considered as a C24 unidirectional lamina, with the material properties given in Table 1 [14, 15]. The fiber direction of the outside layers for all CLT panels is parallel to the span L, while transverse layers are parallel to the B direction.

In the finite element model, boundary conditions are defined in edge nodes as: U^1=W^1=0 for the L-edges, and V^1=W^1=0 for edges parallel to B. Element size of L/10 was used. The laminas are modeled as single numerical layer, adopting the linear distribution of displacements along the lamina thickness.

Using the finite element models of the considered panels, plots of the dimensionless transverse deflection $\overline{w} = w \cdot E_T h^3 / \left(q_0 L^4\right) \cdot 1000$, dimensionless normal stress at the bottom interface in the center of the panel $\overline{\sigma}_0 = \sigma_0 \cdot h / \left(q_0 L\right)$ and dimensionless transverse shear stress at the mid surface along the B-edge of the panel $\overline{\tau}_{xz} = \tau_{xz} \cdot h / \left(q_0 L\right)$ are generated and illustrated in Figures 4-9.

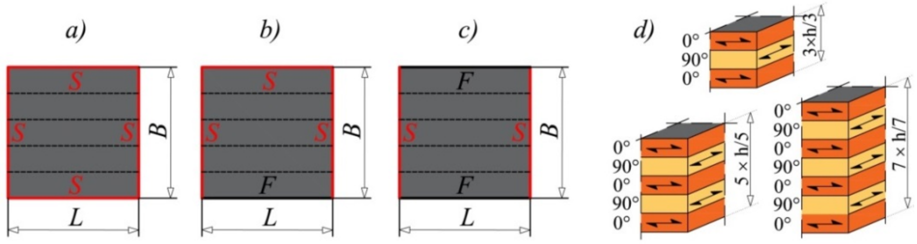

Figure 3. *(a-c): layouts of considered rectangular CLT panels with various boundary conditions; (d): considered stacking sequences. S – simply supported, F – free.*

The selected points are those where stresses and deflections reach maximum values. The obtained results are compared against the results from the same models, with boundary conditions corresponding to the plate simply supported only along B-edges (the scenario which is covered the most in technical approvals for CLT [4, 6]).

To account for the variety of B/L and L/h ratios, the following geometry ranges were used for the calculation: L = (2.7-6.3m), B = (2.7-31.5m), h_{plate} = (9-21cm), h_{layer} = 3cm.

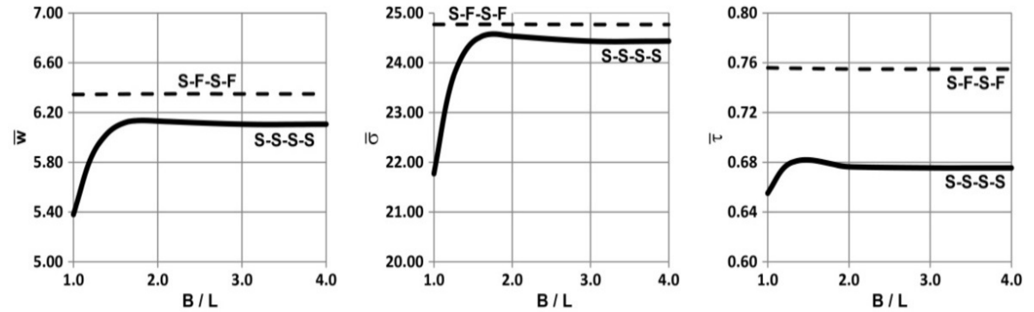

Figure 4. *Dimensionless transverse deflection, normal and transverse shear stress of 3-layer rectangular CLT panel, considering various B/L ratios and different boundary conditions*

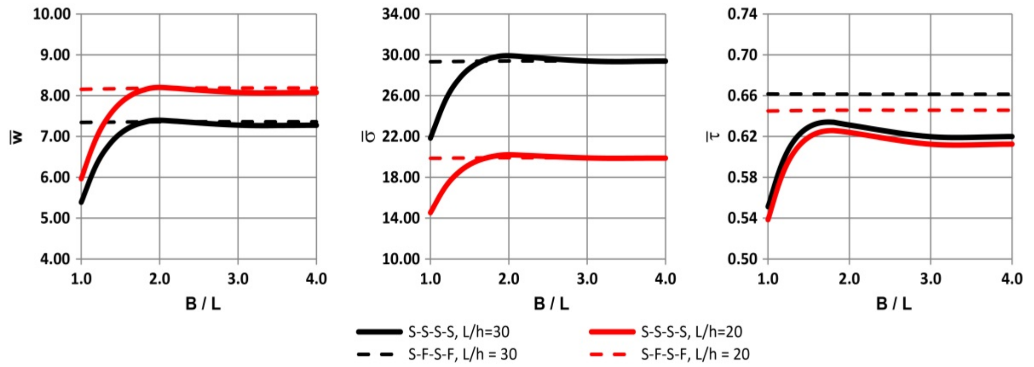

Figure 5. *Dimensionless transverse deflection, normal and transverse shear stress of 5-layer rectangular CLT panel, considering various B/L and L/h ratios and different boundary conditions*

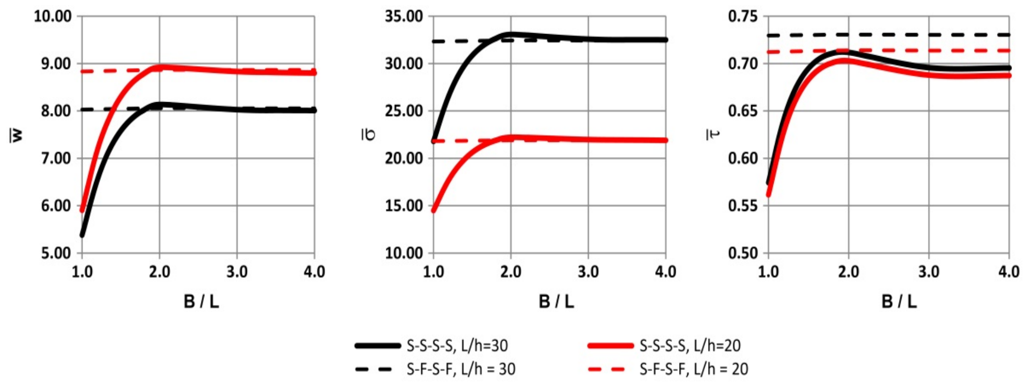

Figure 6. *Dimensionless transverse deflection, normal and transverse shear stress of 7-layer rectangular CLT panel, considering various B/L and L/h ratios and different boundary conditions*

The presented diagrams (Figure 4-6) confirm the justification of using the beam-like models for the design of CLT panels with B/L higher than 2. However, savings could be made for square-like panels, both for the deflection criterion (up to 15% for the 3-layer, 26% for the 5-layer and 32% for the 7-layer panel), and normal stress criterion (12% for the 3-layer, 25% for the 5-layer and 27% for the 7-layer panel).

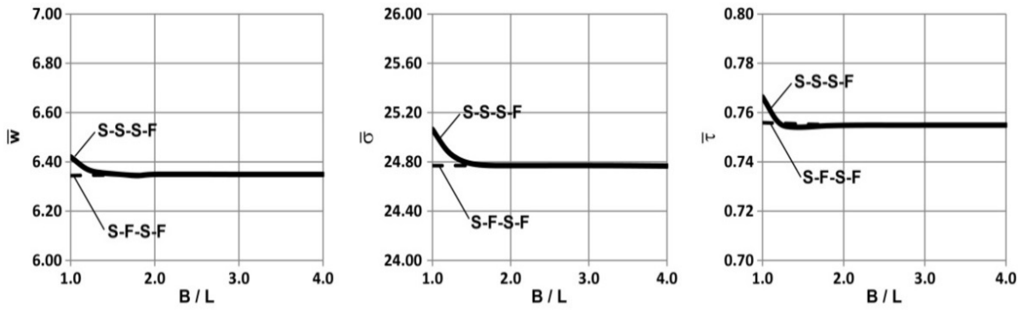

Figure 7. *Dimensionless transverse deflection, normal and transverse shear stress of 3-layer rectangular CLT panel, considering various B/L ratios and different boundary conditions*

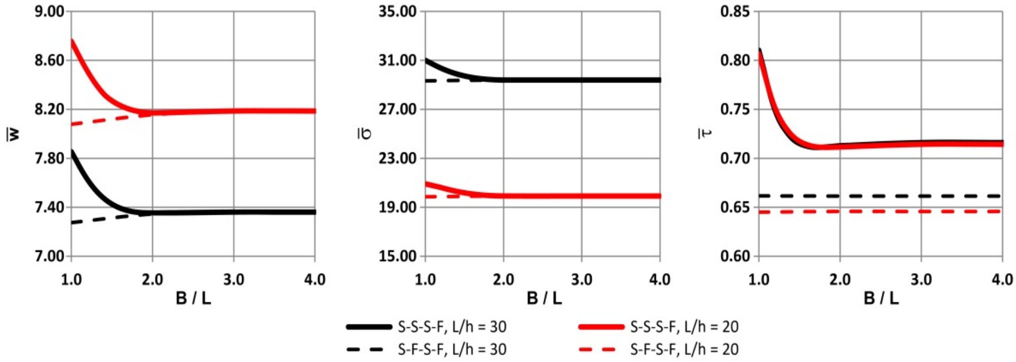

Figure 8. *Dimensionless transverse deflection, normal and transverse shear stress of 5-layer rectangular CLT panel, considering various B/L and L/h ratios and different boundary conditions*

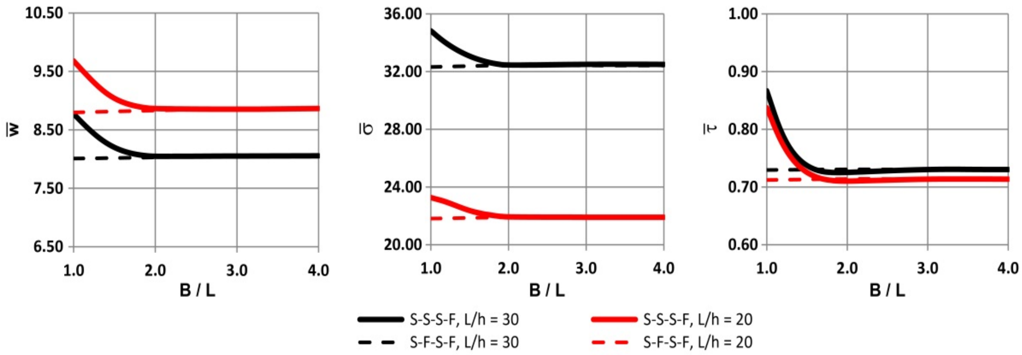

Figure 9. *Dimensionless transverse deflection, normal and transverse shear stress of 7-layer rectangular CLT panel, considering various B/L and L/h ratios and different boundary conditions*

Obviously, the effects of the transverse shear deformation are more pronounced for thick panels, illustrated by the higher relative differences in dimensionless deflections for the 7-layer than for the other considered types of panels. The diagrams shown in Figures 7-9 also confirm the justification of using the beam-like models, for *B*/*L*>2. The cross lamination effect is pronounced for square-like panels, resulting in the slight increase of $\overline{w}$, $\overline{\sigma}_0$ and $\overline{\tau}_{xz}$.

3.3. Complex-shape clt panels

The FEM-based model will be used for bending analysis of CLT panels with balcony (Figure 10). The analytical procedures [4-10] are not completely applicable in this situation, or may lead to the conservative values while designing the CLT floor. When such CLT panels are considered, the driving factor for the design could be both stress and deformation criterion.

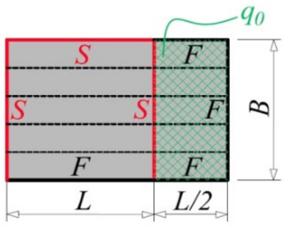

Figure 10. *Layout of CLT panel with balcony*

The panels are exposed to uniformly distributed loading q_0 on the top surface of the balcony, according to Figure 10. Note that for the panels loaded unlike the Figure 10 scheme, diagrams presented in Figures 7-9 are applicable, while for the loading over the entire panel, superposition principle could be applied. The 5-layer (L/h = 20 and L/h = 30) and 7-layer panels (L/h = 20 and L/h = 30), are considered, with the material properties elaborated in Section 3.2. The modeling approach and assignment of boundary conditions is the same as in the previous examples.

To account for the variety of B/L and L/h ratios, the following geometry ranges were used for the calculation: L = (4.5-6.3m), B = (4.5-31.5m), h_{plate} = (15-21cm), h_{layer} = 3cm.

Using the finite element models of the considered panels, plots of the dimensionless transverse deflection $\overline{w}\,(L/2,\ B/2)$ – plate center and $\overline{w}\,(3L/2,\ B/2)$ – end of the cantilever, dimensionless normal stress at the top interface $\overline{\sigma}_0\,(L,\ B/2)$ and dimensionless transverse shear stress $\overline{\tau}_{xz}\,(L,\ 0)$ at the mid surface are generated and illustrated in Figures 11-12.

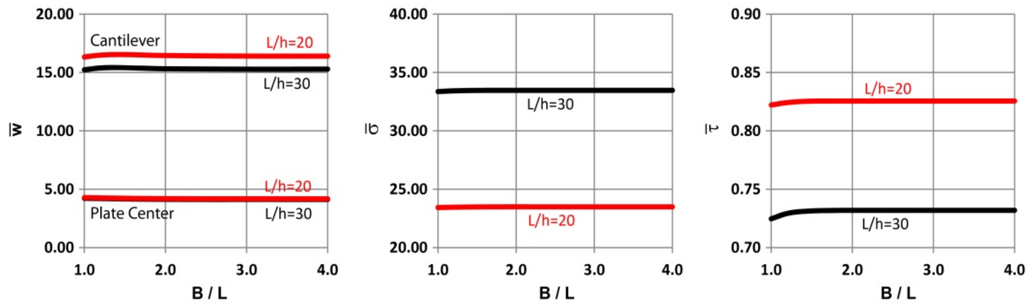

Figure 11. *Dimensionless transverse deflection, normal and transverse shear stress of 5-layer CLT panel with balcony, considering various B/L and L/h ratios*

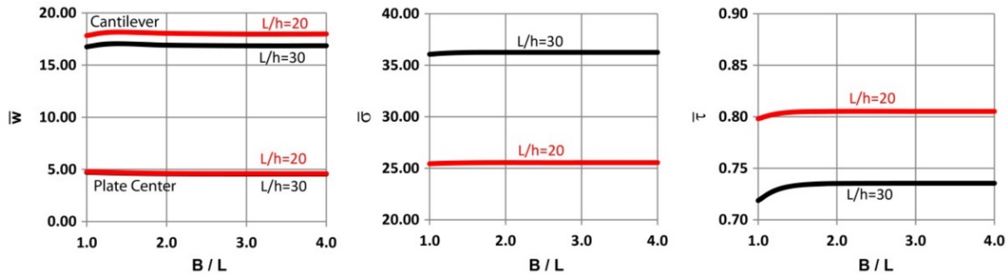

Figure 12. *Dimensionless transverse deflection, normal and transverse shear stress of 7-layer CLT panel with balcony, considering various B/L and L/h ratios*

4. CONCLUSIONS

The paper provides diagrams for the prediction of stresses and deflections in cross-laminated timber (CLT) panels with non-classical boundary conditions, under uniformly distributed loading, derived based on the previously developed software for the computational analysis of laminar composites. The methodology presented in the paper could serve as a basis for the development of more detailed handbooks and guidelines, covering various layouts of CLT panels, different types of loadings and introducing a variety of timber classes. Beside the presented diagrams applicable in static analysis of such structures, the same methodology could be applied for systematic overview of dynamic properties of CLT panels.

Based on the conducted research, some conclusions are derived:

- The justification of using the beam-like (S-F-S-F) models for the design of CLT panels is confirmed for the ratios *B*/*L* higher than 2.
- The effects of transverse shear deformation must be accounted by using refined plate theories when analyzing the thick (i.e. 7-layer) panels.
- Considerable savings could be made for square-like panels ($L \sim B$), both for the deflection and stress criteria, by introducing the effects of 2-way load carrying mechanism.
- Using the superposition principle, more layouts of CLT floors could be designed by combining the diagrams for simple combinations of boundary conditions.

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