09 | Research paper I Оригинални научни рад
ISSN 2566-4484 I Doi 10.7251/STP2014077O
pages 77-90

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EVALUATION AND TAILORING OF GLOBAL GEOPOTENTIAL MODELS IN THE DETERMINATION OF GRAVITY FIELD IN SERBIA

Abstract:
Within this paper, we evaluated the quality of three Global Geopotential Models entitled: EGM96, EGM2008, and GOCO05c. The models were evaluated by using 1001 terrestrial discrete values of height anomalies determined by Global Navigation Satellite Systems and normal heights, which we considered to be true values within this research. In addition to the quality evaluation, we tailored the models by using more than 80000 free air anomalies. The results obtained from the evaluation and tailoring indicate that by using the GOCO05c it is possible to determine a set of anomaly heights across Serbia, which are in agreement with terrestrial values with an average value of -7 cm, the standard deviation of ±9 cm and with the range of 44 cm.

Keywords: Global Geopotential Model, Tailoring, height anomalies

ОЦЕНА И ТЕЈЛОРОВАЊЕ ГЛОБАЛНИХ ГЕОПОТЕНЦИЈАЛНИХ МОДЕЛА ПРИ ОДРЕЂИВАЊУ ГРАВИТАЦИОНОГ ПОЉА НА ТЕРИТОРИЈИ СРБИЈЕ

Сажетак:
У оквиру рада оцењен је квалитет три глобална гепотенцијална модела: EGM96, EGM2008 i GOCO05s. Квалитет модела је тестиран коришћењем 1001 условно тачних вредности дискретних вредности аномалија висина које су претходно одређене применом (комбиновањем) Глобалних навигационих сателитских система и нормалних висина. Поред наведеног, модели су и прилагођени територији Србије коришћењем више од 80000 аномалија слободног ваздуха. Резултати добијени из наведених одређивања указују да се применом модела GOCO05c могу одредити аномалије висина на територији Србије које апроксимирају терестрички одређене аномалије висина са средњом вредношћу од -7 cm, стандардном девијацијом од ±9 cm и то у оквиру распона од 44 cm.

Кључне ријечи: Глобални геопотенцијални модел, прилагођавање, аномалије висина

1. INTRODUCTION

Earth's gravity potential can be represented in the form of addition of two scalar functions 5.[4]:

$$W = V + \Phi, \tag{1}$$

where V is the gravitational potential generated by the Earth's mass, and Φ is centrifugal potential generated by angular rotation of the Earth's body.

From the other side, potential W can be described as the addition of normal potential U and anomalies potential T 5.[4]:

$$W = U + T. \tag{2}$$

When normal potential U is adequately defined 5.[6] than determination of potential W can be treated trough functionals of anomaly potential: *anomaly height* and *free air anomaly*.

For any point P situated on the physical surface of the Earth height anomaly ζ (Figure 1) is given by Bruns equation

$$\zeta_P = \frac{T_P}{\gamma_Q}, \tag{3}$$

and free air anomaly Δg_p (Figure 2) as

$$\Delta g_p = g_p - \gamma_Q\,, \tag{4}$$

where T_P is anomaly potential at point P, g_P is gravity value measured at the surface of the Earth and γ_Q is normal gravity value at the telluroid 5.[10].

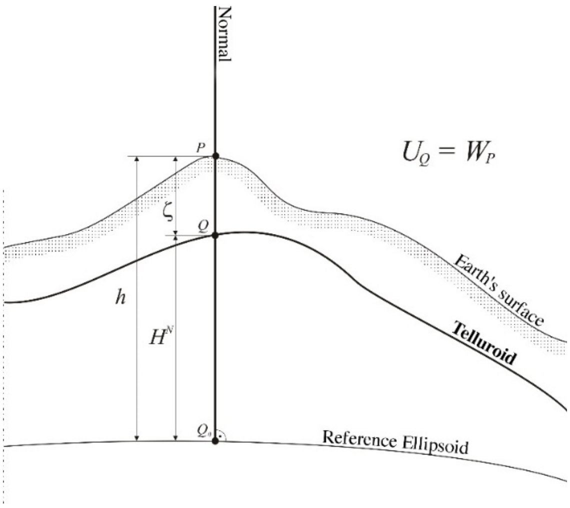

Figure 1. *Physical surface of the Earth, normal height, anomaly height, telluroid and reference (level) ellipsoid*

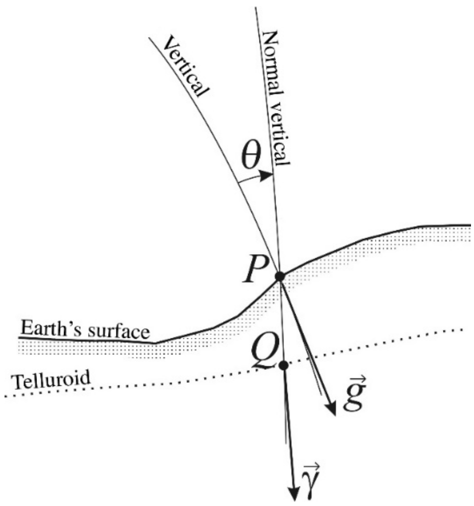

Figure 2. *Gravity and normal gravity vectors at the physical surface of the Earth*

The value of normal gravity at telluroid is given by

$$\gamma_Q = \gamma_0 \left(1 - \frac{2}{a^{REF}}\left(1 + f^{REF} + m_\gamma - 2f^{REF}\sin^2\varphi'\right)H_P^N + \frac{3}{(a^{REF})^2}\left(H_P^N\right)^2\right), \tag{5}$$

where γ_0 is the normal gravity value on the reference ellipsoid, a^{REFF} semi-major of the reference ellipsoid, f^{REF} flattering of the ellipsoid, m_γ centrifugal force divided by gravitational acceleration at the equator, φ' ellipsoidal latitude representing by ellipsoid normal and H_P^N normal height. Gravity value on the reference ellipsoid is defined by the well-known formula of Somigliana 5.[10]:

$$\gamma_0 = \frac{1 + k \sin^2 \varphi'}{\sqrt{1 - (e^{REFF})^2 \ \sin^2 \varphi'}}, \tag{6}$$

$$k = \frac{b^{REFF}}{a^{REFF}} \frac{\gamma_a}{\gamma_b} - 1, \tag{7}$$

and m_γ as centrifugal force divided by gravitational acceleration at the equator by:

$$m_\gamma = \frac{(\omega^{REFF})^2 \ (a^{REFF})^2 b^{REFF}}{GM^{REFF}}, \tag{8}$$

where ω^{REFF} is the rotational velocity of the reference ellipsoid, b^{REFF} small semi-major of the reference ellipsoid, GM^{REFF} is gravitational constant times the total mass of Earth for reference ellipsoid and γ_a and γ_b are normal gravity at the equator and at the pole, respectively.

The height anomaly ζ_P defined by equation (1*)* can be treated in a purely geometrical way in definition of normal heights H^N by (Figure 1):

$$H_P^N = h_P - \zeta_P \,, \tag{9}$$

where h_P is the ellipsoidal height at point P. The ellipsoid height in the last equation can be nowadays determined by GNSS (Global Navigation Satellite System) technology and the anomaly height can be determined, besides lots of other geodetic approaches, by methods or techniques 5.[6], 5.[12], by using *only* appropriately selected Global Geopotential Model (GGM).

This approach gives us an opportunity to replace, to some admissible level of accuracy, complicated and hard works in the determination of normal heights throughout the creation of the levelling networks of high precision, by the elegant, fast and economical way.

2. GLOBAL GEOPOTENTIAL MODELS

During the last 50 years, large numbers of GGMs are created by a great number of individual researches or by a lot of different institutions across the world. From more than 170 publicly available GGMs for this research, we decided to test just three of all of them, entitled: EGM96, EGM2008, and GOCO05c.

The *Earth Gravitational Model 1996* (EGM96) is a set of coefficients of spherical harmonic expansion of gravity potential completed to degree and order N=M=360 5.[2]. The Model has been developed in the United States of America (USA) as a result of the cooperation of the following institutions: National Imagery and Mapping Agency – NIMA, National Aeronautics and Space Agency Goddard Space Flight Center – NASA GSFC and Ohio State University – OSU. EGM96 is a composite solution consisting of the following parts: Low-degree combination model to degree 70, Model with BD (block-diagonal) approach from degree 71 to 359 and Model with NQ (Numerical Quadrature) approach at degree 360. This model is based on a large amount of data, such as surface gravity data, Satellite-to-Satellite tracking data (TDRSS, GPS), altimeter data from ERS-1 and GEOSAT Geodetic Mission and conventional tracking data, including observations acquired by SLR, TRANET, and DORIS systems. The model contains about 130000 coefficients of spherical harmonic expansion and the estimated values of their standard deviations. The model was used to compute geoid undulations accurate to better than one meter and realize WGS84 as a true three-dimensional reference system. EGM96 was also used for advanced geophysical researches, gravity field modeling and for precise local and global geoid computation.

The official *Earth Gravitational Model 2008* (EGM2008) has been publicly released by the National Geospatial-Intelligence Agency (NGA) EGM Development Team 5.[8]. The model represents a spherical harmonic model of the Earth's gravitational potential, which is developed by the least-squares, the combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5 arc-minute equiangular grid. This grid was formed by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where only lower resolution gravity data were available, their spectral content was supplemented with gravitational information implied by the topography. EGM2008 is complete to degree and order 2159 and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high-quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/Leveling values are on the order from ±5 cm to ±10 cm. EGM2008 vertical deflections over the USA and Australia are within ±1.1 arc-seconds to ±1.3 arc-seconds of independent astrogeodetic values. These results indicate that EGM2008 performs comparably with contemporary detailed regional geoid models. EGM2008 performs equally well with other GRACE-based gravitational models in orbit computations. Compared to EGM96, EGM2008 represents an improvement by a factor of six in terms of resolution, and by factors of three to six in terms of accuracy, depending on gravitational quantity and geographic area. At the time it was developed, EGM2008 was a milestone and a new paradigm in global gravity field modeling.

GOCO05c (*Gravity Observation Combination*) is a GGM which represents a gravity field model computed as a combined solution of a satellite-only model and a global data set of gravity anomalies, constituting a global 15'x15' grid. It is fully resolved up to the 720 degree and order based on fully normal equation systems. It has been elaborated by the GOCO group (TU Munich, Bohn University, TU Graz, Austrian Academy of Science, University of Bern). GOCO05c represents the first model that's applying regionally varying weights. Also, GOCO05c is the first combined gravity field model that is independent of EGM2008 that contains GOCE data of the whole mission period. Data sets used for this model are data from gravity field model GOCO05s, fully resolved up to degree and order 280, Altimetric Gravity Anomalies and Terrestrial Gravity Anomalies. Having in mind that it had to ensure a stable, normal equation system which has to be a complete observation data grid 15'x15', the remaining 20% of Earth's surface missing data had to be filled with fill-in data sets such as the NIMA96 data set of the Defense Mapping Agency and the Goddard Space Flight Center data. As this data couldn't cover all the missing data, the remaining regions were filled with band-limited gravity anomalies computed from GOCO05s up to degree and order 220. The GOCO05c model combination model was computed by a rigorous solution of the fully occupied normal equation system and not on block-diagonal approximation. A detailed description of the estimated method used for the estimation of spherical coefficients for this model can be found in 5.[1]. In GOCO05c model the transition from satellite to terrestrial data takes place between degrees 140 and 240, depending on the quality of terrestrial data. As it was said before, it is independent of EGM2008 with the exception of two coefficients $\bar{C}_{720,0}$ $\bar{C}_{720,720}$, which have been used for regularization. It has been shown that GOCO05c model can at least achieve a level of accuracy of existing high-resolution models. On the other hand, it has to be mentioned that GOCO05c model should not be applied in geophysical interpretations in a region where fill-in data have been used since in some areas it does not meet high-accuracy requirements.

All three models are publicly available due to the activity of the International Center for Global Geopotential Model Services (ICGEM) established under the authority of the International Association of Geodesy (IAG) via the International Centre for Global Earth Models website.

3. MATHEMATICAL BACKGROUND AND APPLICATION OF GGM

Gravity potential W can be expanded in the functions of the spherical harmonics:

$$\begin{aligned} W(r,\theta,\lambda) = {} & \frac{GM^{REF}}{a^{REF}} \sum_{n=0}^{N_{max}} \left(\frac{a^{REF}}{r}\right)^{n+1} \sum_{m=0}^{n} \begin{pmatrix} \bar{C}_{nm}^{ELL}\cos(m\lambda) \\ +\, \bar{S}_{nm}^{ELL}\sin(m\lambda) \end{pmatrix} \bar{P}_{nm}\cos(\theta) \\ & + \frac{1}{2}(\omega^{REF})^2 r^2 \sin^2\theta, \end{aligned} \tag{10}$$

where r, θ and λ are the spherical coordinates, n and m are degree and order of spherical harmonic coefficients, $\bar{C}_{nm}^{ELL}$ and $\bar{S}_{nm}^{ELL}$ are normalized spherical harmonic coefficients referring to the constants of the reference ellipsoid,

$$\begin{Bmatrix} \bar{C}_{nm}^{ELL} \\ \bar{S}_{nm}^{ELL} \end{Bmatrix} = \left(\frac{GM^{GGM}}{GM^{REF}}\right)\left(\frac{a^{GGM}}{a^{REF}}\right)^{n} \begin{Bmatrix} \bar{C}_{nm}^{GGM} \\ \bar{S}_{nm}^{GGM} \end{Bmatrix}, \tag{11}$$

$\bar{C}_{nm}^{GGM}$ and $\bar{S}_{nm}^{GGM}$ are normalized spherical harmonic coefficients referring to the constants of selected GGM, GM^{GGM} is gravitational constant times the total mass of the selected GGM, a^{GGM} is the equatorial radius for the selected GGM and $\overline{P}_{nm}\cos(\theta)$ are normalized associated Legendre functions of degree n and order m defined by equations:

$$\overline{P}_{n0}\cos(\theta) = \sqrt{2n+1}\,P_{n0}\cos(\theta)\,, \tag{12}$$

$$\overline{P}_{nm}\cos(\theta) = \sqrt{\frac{2(2n+1)(n-m)!}{(n+m)!}}\,P_{nm}\cos(\theta), \tag{13}$$

where $P_{nm}\cos(\theta)$ are ordinary Legendre functions defined by

$$P_{nm}(t) = \frac{1}{2^n n!}(1-t^2)^{\frac{m}{2}}\frac{d^{n+m}}{dt^{n+m}}(t^2-1)\,, \tag{14}$$

where $t = \cos(\theta)$.

The normal potential U also can be expanded in an analogous way into spherical harmonics:

$$U(r,\theta,\lambda) = \frac{GM^{REF}}{a^{REF}}\sum_{n=0(2)}^{8}\left(\frac{a^{REF}}{r}\right)^{n+1}\overline{C}_n^{REF}\,\overline{P}_n\cos(\theta) + \frac{1}{2}(\omega^{REF})^2 r^2\sin^2\theta, \tag{15}$$

where coefficients $\overline{C}_n^{REF}$ are given by

$$\overline{C}_{2k}^{REFF} = \left(-1^k\right)\frac{3(e^{REFF})^{2k}}{(2k+3)(2k+1)\sqrt{4k+1}}\left[1+\frac{2}{3}k\left(1-\frac{m_\gamma\, e'^{REFF}}{3q_0}\right)\right], \tag{16}$$

where e'^{REFF} is the second numerical eccentricity od reference ellipsoid and

$$q_0 = \frac{1}{2}\left[\left(1+\frac{3}{(e'^{REFF})^2}\right)\tan^{-1}(e'^{REFF}) - \frac{3}{e'^{REFF}}\right], \tag{17}$$

In order to obtain of anomaly height ζ_P at point P using equations (10) and (15) it is necessary, by definition of Molodensky, to find point Q at the normal vertical where the normal potential U is equal to potential W at the point P e.g. to find a point Q in which is satisfied conditions:

$$U_Q = W_P, \tag{18}$$

the anomaly height ζ_p is simply the distance along the normal vertical between the point P and the point Q.

All of these equations are applied for calculation of anomaly height at 1001 points (Figure 3) located at the surface of the Earth where the terrestrial value of anomaly heights are determined by

$$\zeta_P^{GPS/dh} = h_p - H_P^N, \tag{19}$$

where h_p is ellipsoidal height determined by GNSS and H_P^N are normal height determined by the combination of geometrical levelling and measured gravity values. The basic statistical data for differences $\zeta_P^{GPS/dh}$ are shown in Table 1.

Figure 3. *Spatial distribution of GPS/dh points*

Table 1. *Basic statistical of $\zeta_P^{GPS/dh}$ (N = 1001, units: m)*

Parameter		**Minimum**	**Maximum**	**Average**	**Standard deviation**
$\zeta_P^{GPS/dh}$		42.33	46.41	44.64	0.85

After calculations of $\zeta_P^{GPS/dh}$ equations from (10) to (18) were applied for all of three selected GGMs and three sets of differences were calculated:

$$R_j = \zeta_i^j - \zeta_i^{GPS/dh} \tag{20}$$

where j *index* denotes one of the GGMs (EGM96, EGM2008 or GOCO05c) and i is the index which denotes running point, from 1 to 1001. The basic statistical data, the general shape of surfaces and histograms of calculated differences are given in the following table and figures.

Table 2. *Basic statistical data of differences R (N = 1001, units: m)*

Parameter	**Minimum**	**Maximum**	**Average**	**Standard deviation**	**Range**
R_{EGM96}	-2.02	0.38	-0.86	0.61	2.40
$R_{EGM2008}$	-0.49	0.17	-0.13	0.09	0.66
$\boldsymbol{R_{GOCO05c}}$	**-0.33**	**0.20**	**-0.05**	**0.10**	**0.54**

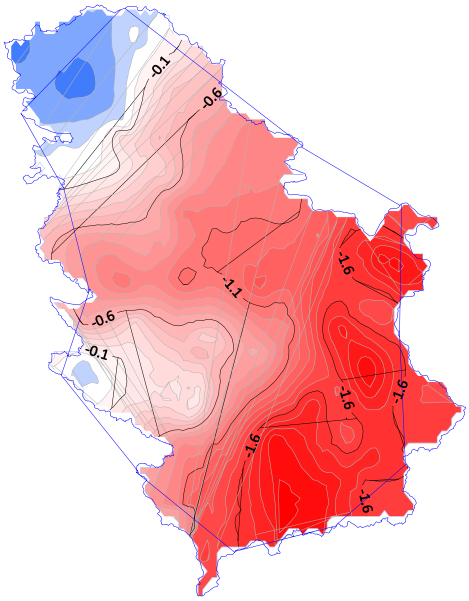

Figure 4. *General shape of the differences* R_{EGM96}

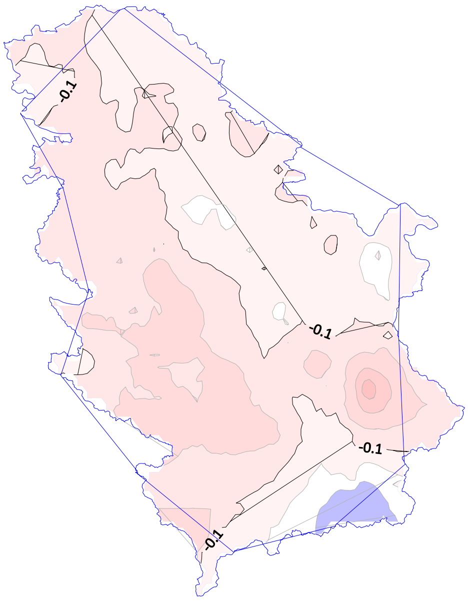

Figure 5. *General shape of the differences* $R_{EGM2008}$

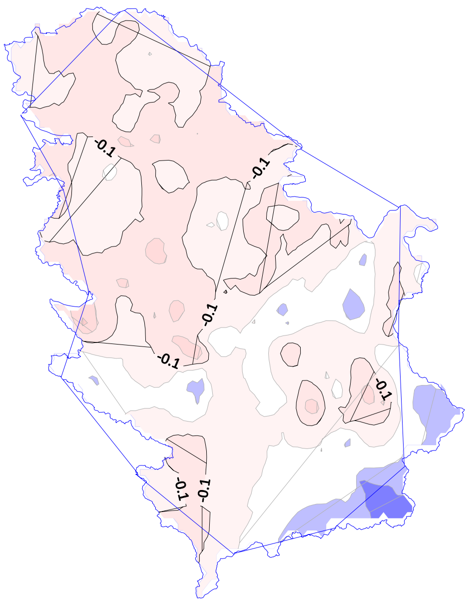

Figure 6. *General shape of the differences* $R_{GOCO05c}$

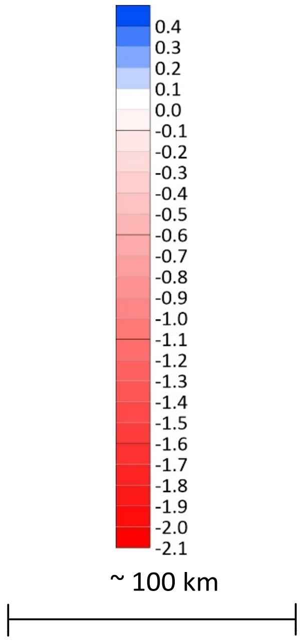

Figure 7. *Colors and scale for figures 4, 5 and 6*

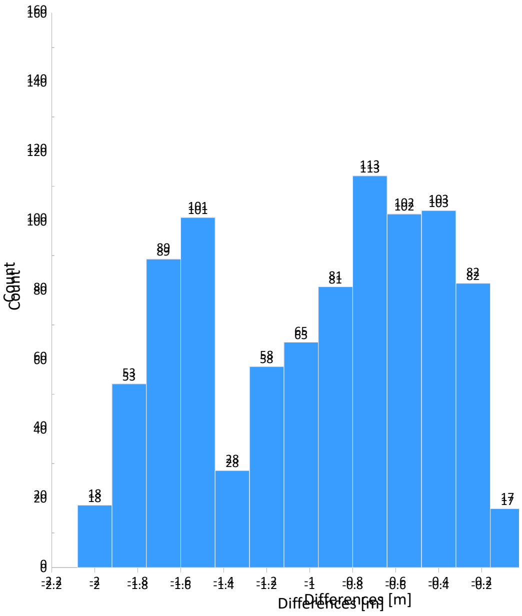

Figure 8. *Histogram of the differences* R_{EGM96}

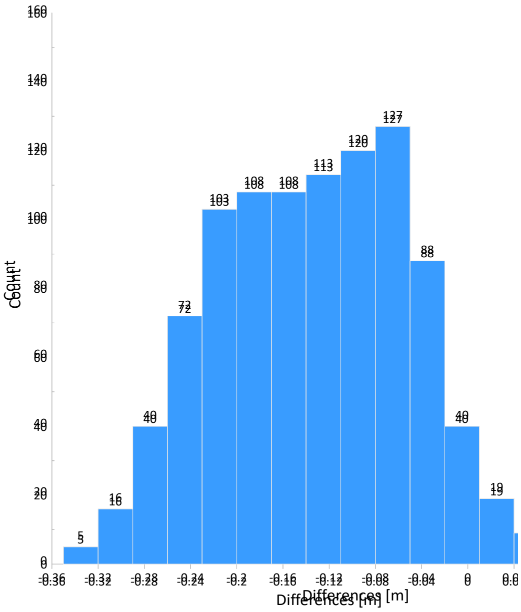

Figure 9. *Histogram of the differences* $R_{EGM2008}$

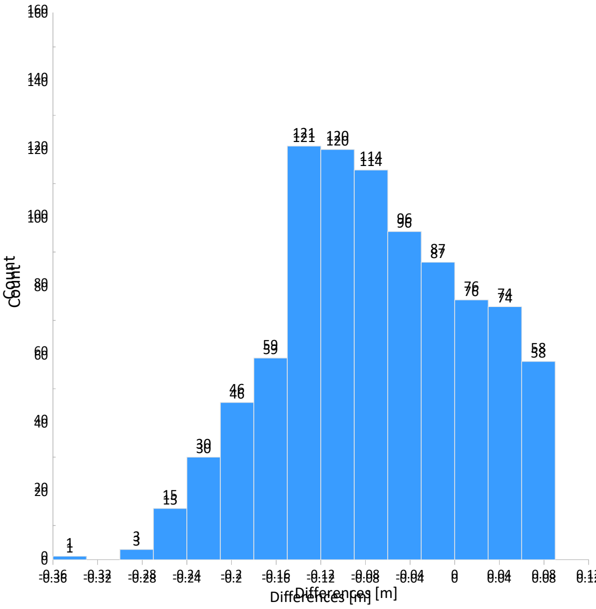

Figure 10. *Histogram of the differences* $R_{GOCO05c}$

4. TAILORING OF THE GLOBAL GEOPOTENTIAL MODEL

In this research, tailoring of the GGM meant adapting an existing set of coefficients of the GGM by using gravity anomalies from the territory of Serbia, in such a way that we can approximate the anomaly height at any point in the territory of Serbia with a new set of coefficients to the greatest possible extent.

Step by step description of the tailoring procedure was performed as follows.

From a database of the gravimetric survey of Serbia, more than 80000 measured values were collected (Figure 1) and the free air anomaly at all points was calculated by using equation (4). The territory of Serbia is divided into l quasi quadratic subareas Δ, bounded by latitudes and longitudes with spatial resolution of 5 arc minutes in the both directions (from the East to the West and from the South to the North). All free air anomalies were divided into l sets of anomalies in accordance to their positions across newly formed subareas. For each l sets, the average value of free air anomalies were calculated using the equation:

$$\overline{\Delta g} = \frac{1}{N_A} \sum_{i=1}^{N_A} \Delta g_i \, . \tag{21}$$

where N_A is the number of free air anomalies in the Δ subareas. We treated the calculated average value as the average value of the anomaly at the mean point of each Δ subarea.

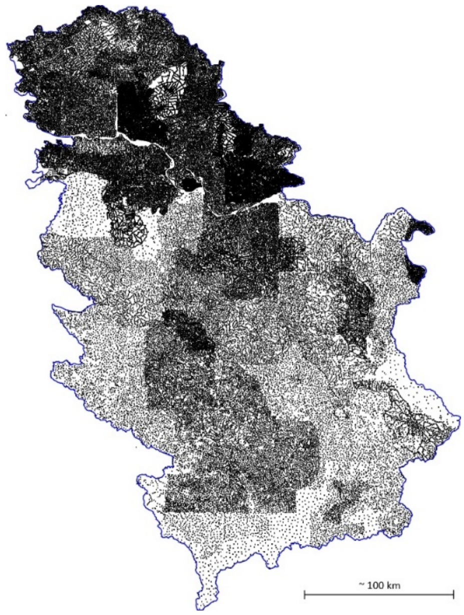

Figure 1: Spatial distribution of measured gravity values

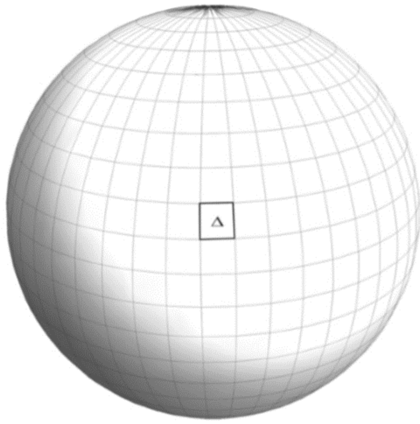

Figure 2: Δ area as the part of the Earth surface (or the part of the unit sphere)

Average free air anomaly, for any of Δ subareas, is also possible to calculate by using the coefficients of the GGM according to 5.[11]:

$$\overline{\Delta g}_{GGM} = \frac{GM}{r^2} \sum_{n=2}^{N_{max}} (n-1) \left(\frac{a}{r}\right)^n \beta_n \sum_{m=0}^{n} \begin{pmatrix} \overline{\Delta C}_{nm}^{ELL} \cos(m\lambda) \\ + \overline{S}_{nm}^{ELL} \sin(m\lambda) \end{pmatrix} \overline{P}_{nm} \cos(\theta), \tag{22}$$

where β_n is a function defined by 5.[9]

$$\beta_n = \frac{1}{1-\cos(\psi_0)} \int_{\cos(\psi_0)}^{1} P_n(t) dt, \tag{23}$$

and ψ_0 is the radius of the area Δ. For calculated average values, the difference is given by:

$$\delta = \overline{\Delta g} - \overline{\Delta g}_{GGM}, \tag{24}$$

and this difference we used as basic information for tailoring coefficients of a GGM to terrestrial anomalies at the territory of Serbia by the equation:

$$\begin{Bmatrix} \delta\bar{C}_{nm}^{ELL} \\ \delta\bar{S}_{nm}^{ELL} \end{Bmatrix} = \frac{1}{4\pi} \iint_{\sigma} \frac{\bar{r}^2}{GM} \left(\frac{\bar{r}}{a}\right)^n \frac{1}{(n-1)\beta_n} \delta \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix} \bar{P}_{nm} \cos(\theta)\, d\sigma, \tag{25}$$

For l areas Δ we can improve the tailoring of coefficients by summing the effects of all differences $\delta_i, i = 1, \dots, l$ by:

$$\begin{Bmatrix} \delta\bar{C}_{nm}^{ELL} \\ \delta\bar{S}_{nm}^{ELL} \end{Bmatrix} = \frac{1}{4\pi} \sum_{i=1}^{l} \frac{\bar{r}_i{}^2}{GM} \left(\frac{\bar{r}_i}{a}\right)^n \frac{1}{(n-1)\beta_{n,i}} \delta_i \iint_{\Delta\sigma} \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix} \bar{P}_{nm} \cos(\theta)\, d\sigma, \tag{26}$$

or equivalently by:

$$\begin{aligned} \begin{Bmatrix} \delta\bar{C}_{nm}^{ELL} \\ \delta\bar{S}_{nm}^{ELL} \end{Bmatrix} &= \frac{1}{4\pi} \sum_{i=1}^{l} \frac{\bar{r}_i{}^2}{GM} \left(\frac{\bar{r}_i}{a}\right)^n \frac{1}{(n-1)\beta_{n,i}} \delta_i \cdot \\ &\cdot \int_{\lambda_{W_i}}^{\lambda_{E_i}} \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix} d\lambda \int_{\theta_{N_i}}^{\theta_{S_i}} \bar{P}_{nm} \cos(\theta) \sin(\theta) d\theta\, d\sigma, \end{aligned} \tag{27}$$

and finally, calculate the new set of coefficients of adapted or tailored GGM:

$$\begin{Bmatrix} \bar{C}_{nm}^{ELL} \\ \bar{S}_{nm}^{ELL} \end{Bmatrix}_T = \begin{Bmatrix} \bar{C}_{nm}^{ELL} \\ \bar{S}_{nm}^{ELL} \end{Bmatrix} + \begin{Bmatrix} \delta\bar{C}_{nm}^{ELL} \\ \delta\bar{S}_{nm}^{ELL} \end{Bmatrix}. \tag{28}$$

This set of coefficients we used in the calculation the anomaly heights ζ and the differences R_j by equations from (10) and (18).

In addition, and in order to achieve better tailoring of the coefficients, the described procedure can be performed repeatedly (in iterations), forming after each cycle of new differences (*24*) and new coefficients (28), whereby it is necessary to introduce a criterion (or criteria) to limit the number of iterations.

In this research, we adopted criteria in the following way. After each iteration several sets of data were calculated:

1. Sets of differences with terrestrial anomaly heights (GPS/dh)

$$\left(\Delta\zeta_j\right)^i = \left(\zeta_j^{GGM}\right)^i - \zeta_i^{GPS/dh}, \tag{29}$$

   where *i* is the label of the current iteration and the *j* is the number of running difference of GPS/dh, $j = 1, \dots, 1001$,
2. Difference between the average values of the sets $\left(\Delta\zeta_j\right)^i$ and $\left(\Delta\zeta_j\right)^{i+1}$,
3. Difference between standard deviations of the sets $\left(\Delta\zeta_j\right)^i$ and $\left(\Delta\zeta_j\right)^{i+1}$,

For the last iteration, we adopt the one which satisfied the following conditions:

$$\left|\overline{(\Delta\zeta)^i} - \overline{(\Delta\zeta)^{i+1}}\right| \approx 1 \text{ cm},$$

$$\left|\sigma_{(\Delta\zeta)^{i}} - \sigma_{(\Delta\zeta)^{i+1}}\right| \approx 1\ \text{cm}.$$

where $\overline{(\Delta\zeta)}$ is the average value of the set and $\sigma_{\Delta\zeta}$ is its standard deviation.

By applying all the above equations selected GGMs were tailored. EGM96 was tailored after 4 iterations and EGM2008 and GOCO05c after only 3 iterations. The basic statistical data, the general shape of differences and histograms of differences for all tailored GGMs are shown in the following tables and figures.

Table 3. *Basic statistical data of $\Delta\zeta$ sets calculated by original and by tailored EGM96 coefficients throughout the iteration (number of data $N = 1001$)*

$N_{max} = 360, n_{min} = 37$					
Parameter	**EGM96**	**Iteration**			
		0	1	2	3
Minimum	-2.02	-1.41	-1.23	-1.17	-1.14
Maximum	0.38	0.04	0.01	0.02	0.02
Average	-0.86	-0.69	-0.61	-0.58	0.57
Standard deviation	0.61	0.27	0.24	0.23	0.23
Range	2.40	1.45	1.24	1.19	1.17

Table 4. *Basic statistical data of $\Delta\zeta$ sets calculated by original and by tailored EGM2008 coefficients throughout the iteration (number of data $N = 1001$)*

$N_{max} = 2190, n_{min} = 281$					
Parameter	**EGM2008**	**Iteration**			
		0	1	2	3
Minimum	-0.49	-0.34	-0.35	-0.35	
Maximum	0.17	0.12	0.11	0.10	
Average	-0.13	0.12	-0.13	-0.13	-
Standard deviation	0.09	0.09	0.09	0.09	
Range	0.66	0.46	0.45	0.45	

Table 5. *Basic statistical data of $\Delta\zeta$ sets calculated by original and by tailored GOCO05c coefficients throughout the iteration (number of data $N = 1001$)*

$N_{max} = 720, n_{min} = 281$					
Parameter	**GOCO05c**	**Iteration**			
		0	1	2	3
Minimum	-0.33	-0.33	-0.31	**-0.31**	-
Maximum	0.20	0.14	0.13	**0.13**	

Average	-0.05	-0.06	-0.07	**-0.07**	
Standard deviation	0.10	0.09	0.09	**0.09**	
Range	0.54	0.47	0.44	**0.44**	

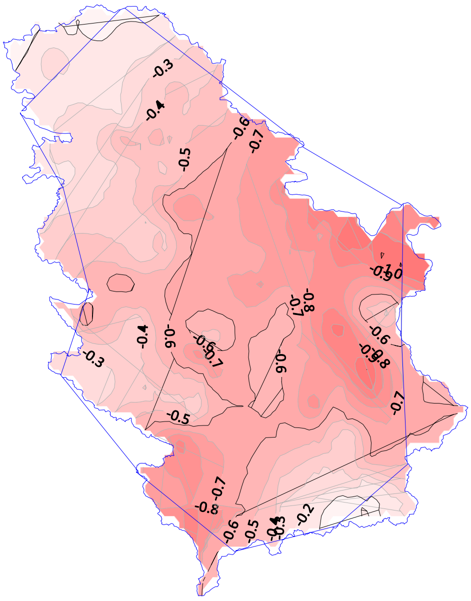

Figure 3: General shape of the differences R_{EGM96} *after tailoring of* EGM96

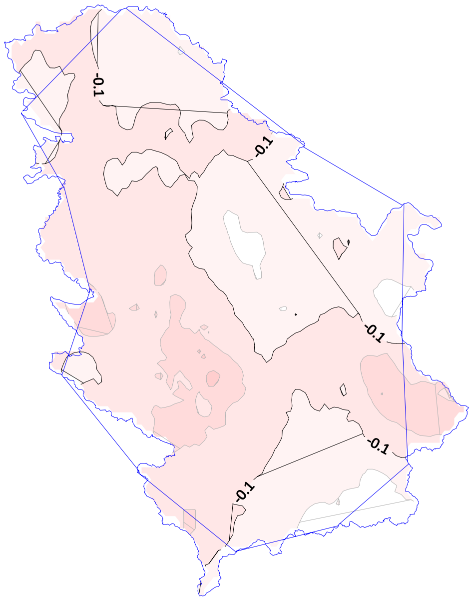

Figure 4: General shape of the differences $R_{EGM2008}$ *after tailoring of* EGM2008

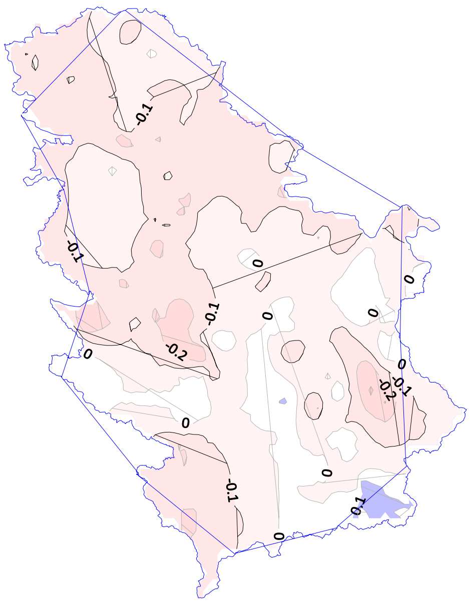

Figure 5: General shape of the differences $R_{GOCO05c}$ *after tailoring of* GOCO05c

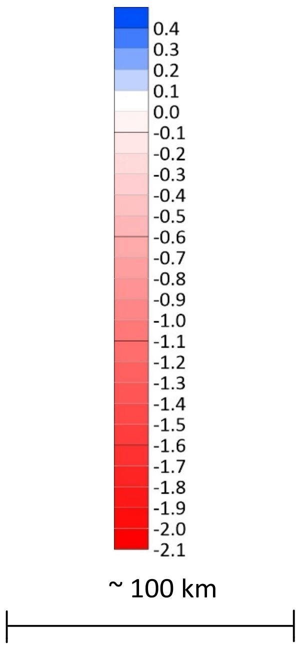

Figure 6: Colors and scale for figures 13, 14 and 15

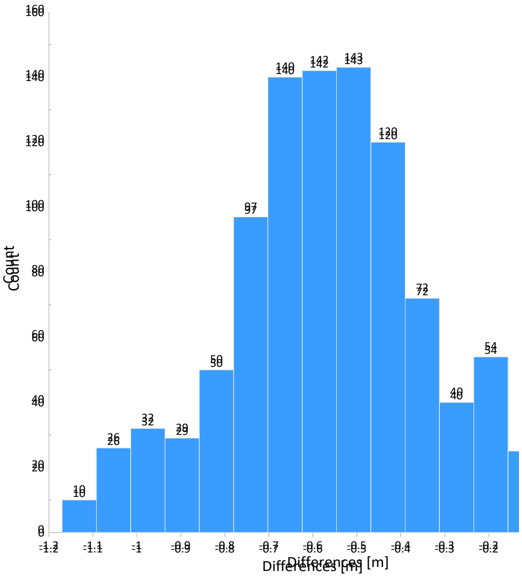

Figure 7: Histogram of the differences R_{EGM96} after tailoring of EGM96

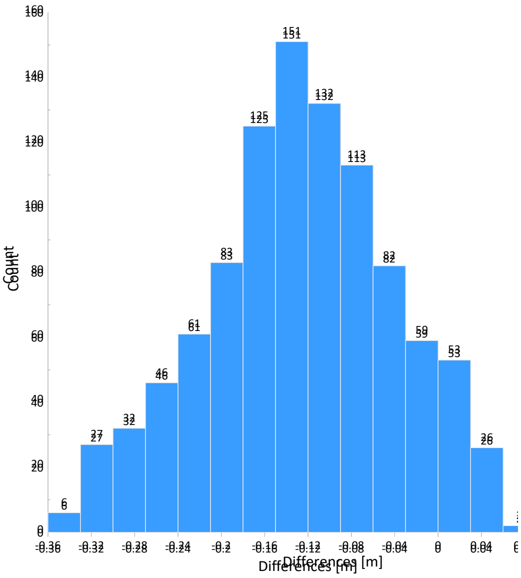

Figure 8: Histogram of the differences$R_{EGM2008}$ after tailoring of EGM2008

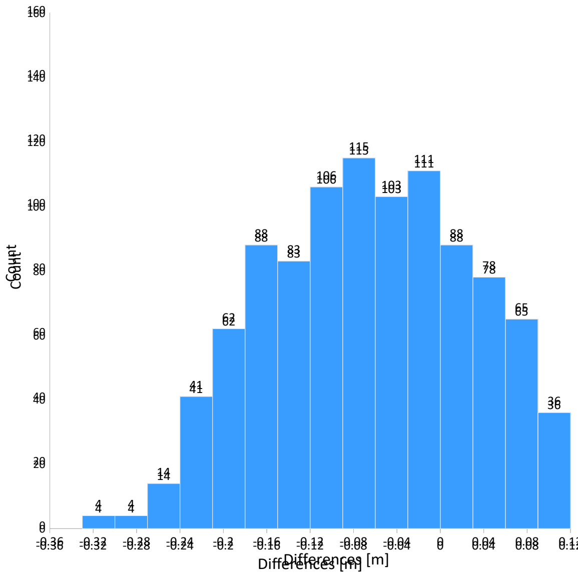

Figure 9: Histogram of the differences $R_{GOCO05c}$ after tailoring of GOCO05c

5. COMMENTS, CONCLUSIONS AND FUTURE ACTIVITIES

This paper presents research dedicated to the evaluation of the quality of three Global Geopotential Models: EGM96, EGM2008, and GOCO05c for the territory of Serbia. The evaluation was done by using 1001 terrestrial discrete values of height anomalies, determined by GNSS and normal heights, wherein further research these values were adopted as true values. From detailed quality evaluation, results have shown that the best approximation of height anomalies for the territory of Serbia gives GOCO05c. In addition to the quality evaluation, tailoring of models was performed using more than 80,000 free air anomalies, relatively regularly distributed over the territory. The models were tailored by free air anomalies but for the purposes of quality evaluation, the same 1001 terrestrial height anomalies were used. After performed evaluation and a final comparison of results, it can be concluded that with the Global Geopotential Model GOCO05c is possible to determine set of anomaly heights across the territory of Serbia, which are corresponding with terrestrial values with an average value of -7 cm, the standard deviation of ±9 cm within range of 44 cm.

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