ABOUT PLANAR SECTIONS OF A TYPE OF EGG CURVE BASED CONOID

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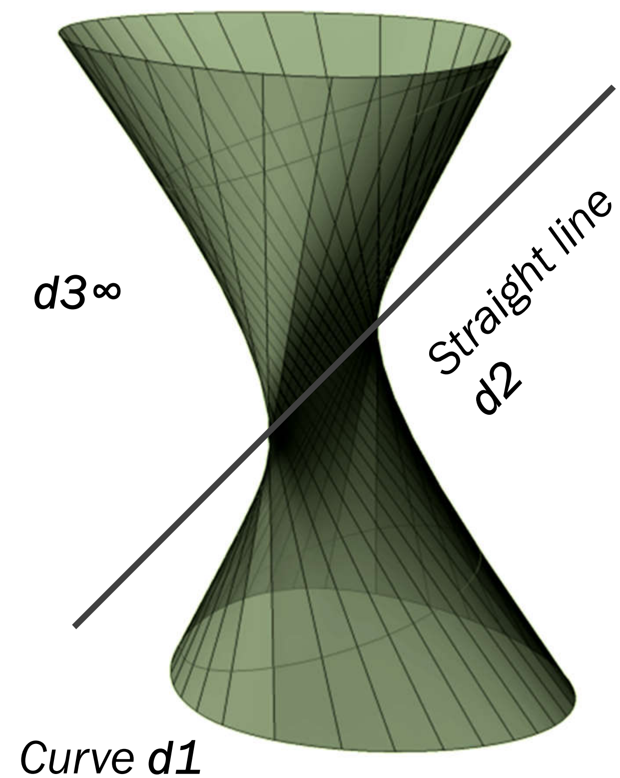
Introduction

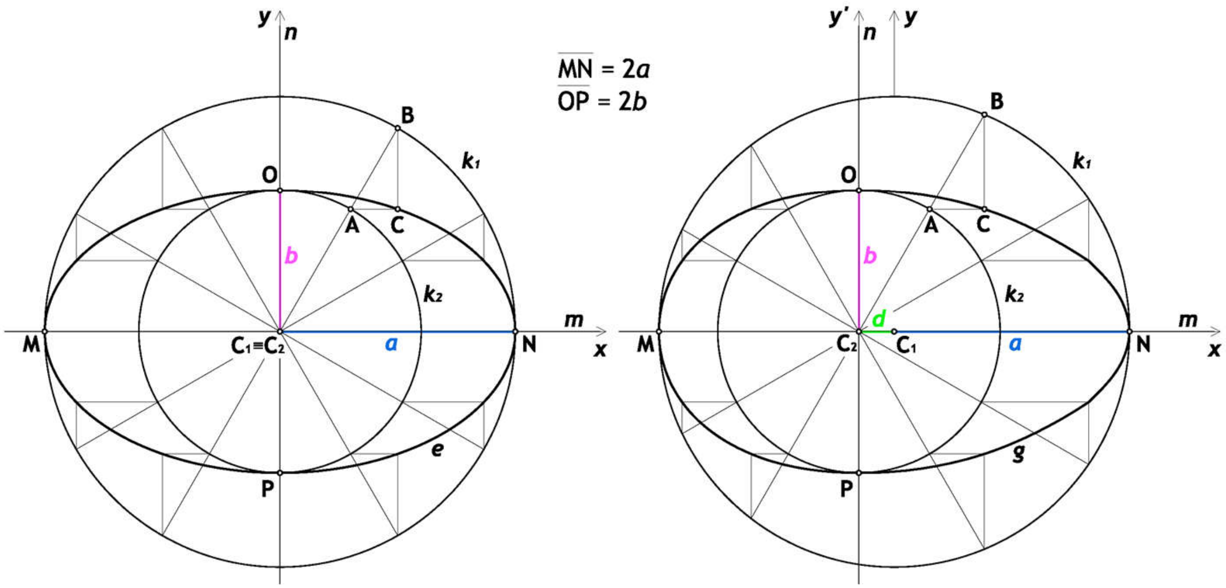
- Starting from a type of conoid which is based on a cubic egg curve obtained by Hügelschäffer's construction, it is considered a possible occurrence of related type of conoid, which would include conic curve as a part of its plane section.
- The solution is accomplished by constructively – geometrical methods, supported by Rhinoceros software package.
- **Conoid** is a ruled surface, obtained by moving the generatrix along three directrices:
- A plane curve – d1
- A straight line - d2 and
- An infinte straight line – d3

which determines the direction plane, as well as the positions of the generatrices..



Cubic egg curve based ***conoid***

Hügelschäffer's Construction Of Egg Curve



Ellipse construction *and* *Egg curve construction*

The starting constructions is the well known ellipse construction using the concentric circles which is transposed into a egg curve construction, by displacing the centre of the minor circle. The obtained egg curve is a cubic curve.

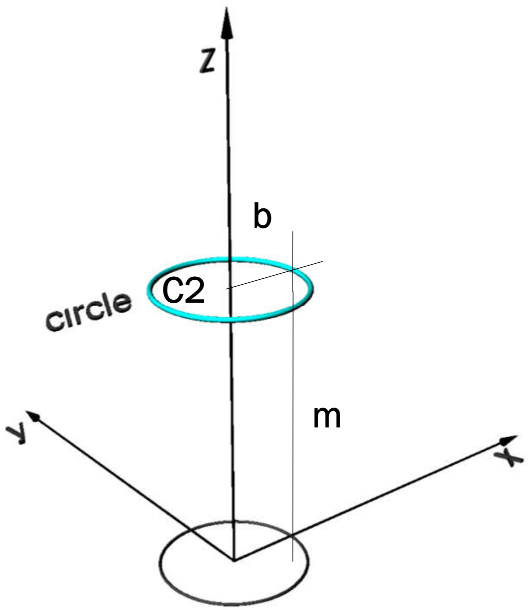
Ellipse equation: $e: \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

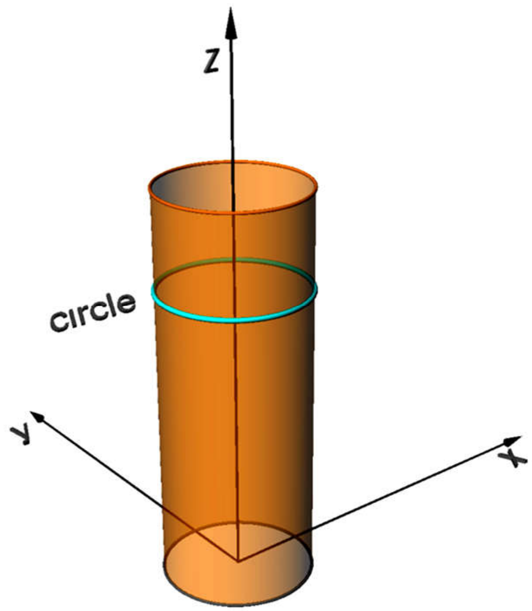
Cubic egg curve general equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} f(x) = 1$

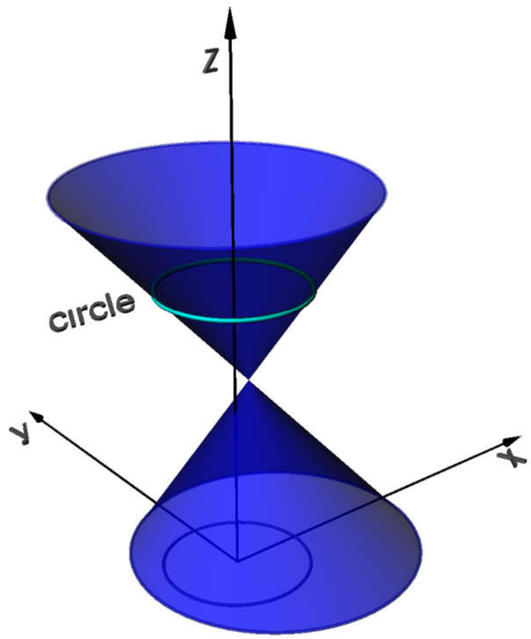
Distortion of the equation for the value d: $f(x) = 1 + \frac{2dx+d^2}{a^2}$

Hügelschäffer's egg curve equation: $g: \quad b^2x^2 + a^2y^2 + 2dxy^2 + d^2y^2 - a^2b^2 = 0$

Starting Suppositions





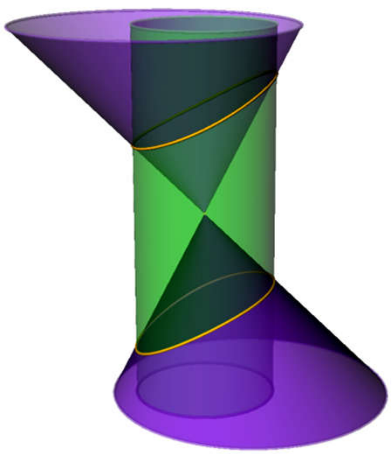


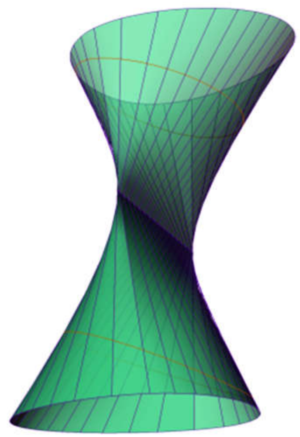
1. Let us start from a circle with radius **b** and the center **C**2 on **z** axis, on the altitude **m.**

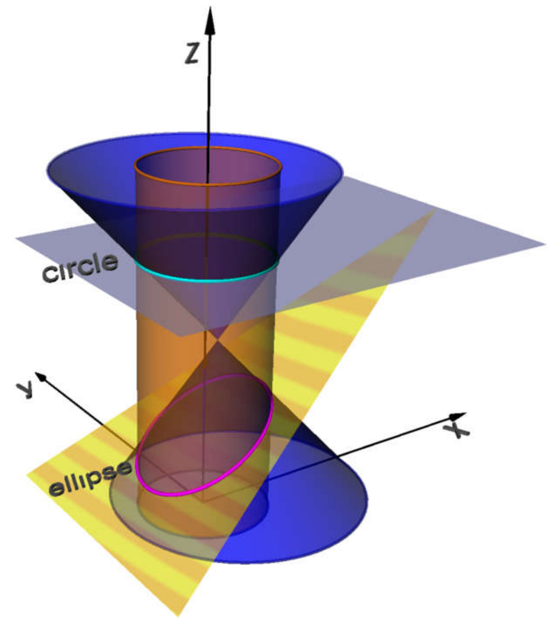
2. Through this circle **C**2 it is possible to set a cylinder of rotation. Its basis will also be a circle with radius **b**, and will occur in the plane **x-y.**

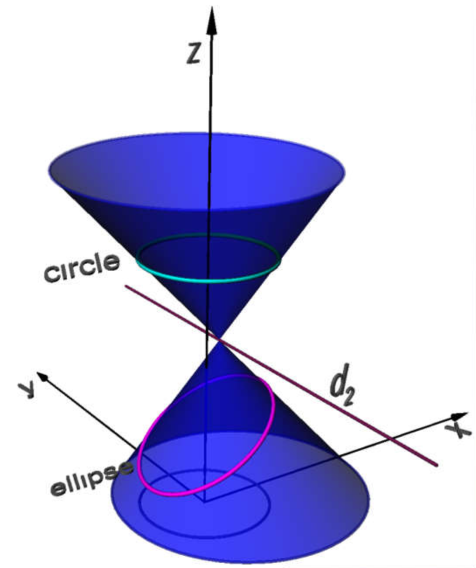
3. Through the circle with radius **b** it Is also possible to set an oblique cone, with the apex **V(r,0,h)**, on the altitude **h** and the base of radius **a,** Which center **C**1 will be shifted for some value **d**, in the direction of **x** axis.

Generating A Conoid Based On The Intersection Curve Of A Cone And Cylinder







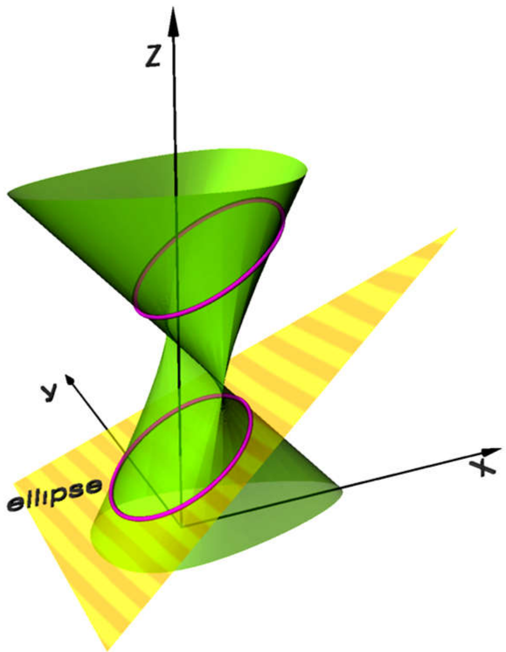


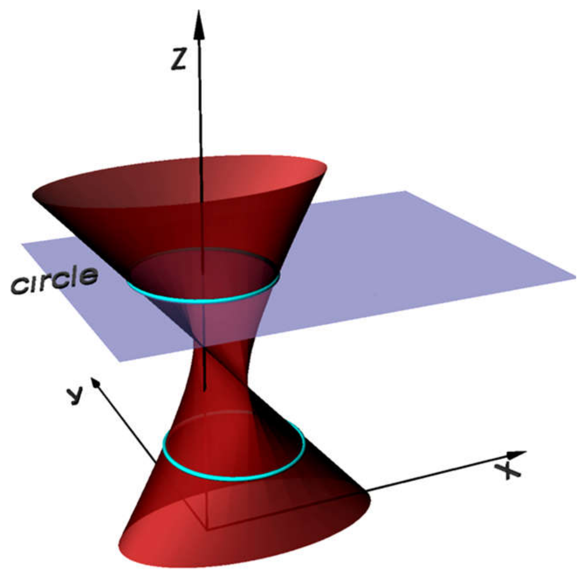
The general case of intersection of a cylinder and a cone is a fourth degree spatial curve.

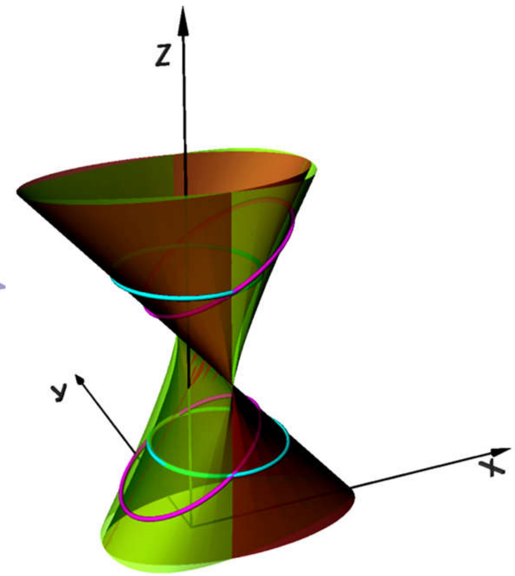
Starting from such an intersection curve, as a directrix of a surface, we can obtain a conoid.
The straight line directrix will be the line set trough the apex of the oblique cone, and the directrix plane determines the position of generatrices. The plane curve directrix, can be accepted as the plane section by basis x-y.

The special case of cone and cylinder intersection will be the one we supposed earlier, the one that represents the degenerated spatialn curve of fourth degree. It occures when the surfaces (quadrucs|) have a common tangential planes. In our case, the spatial curve will degenerate onto a circle and an ellipse.
Starting from such a intersection, we generate a new conoid.

Conoid Obtained By Using The Special Case Of The Surface Intersection As The Directrix Curve







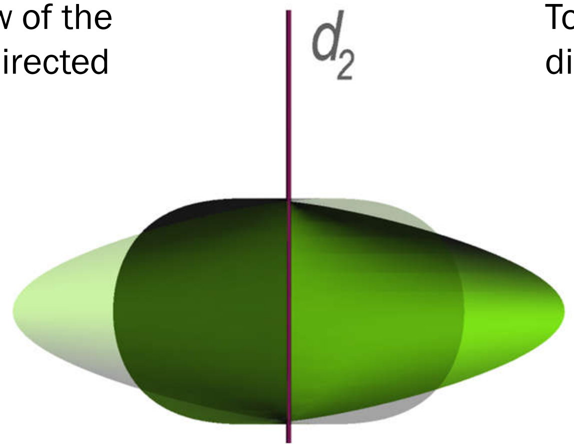
If we set a surface trough the curves that are derived from the intersection of a cone and a circle, guided by another straight line in the vertex V, and a direction plane x-z, we get two branches of conoids.

One of them will pass trough the ellipse And the double straight line d2, while The other will pass trough the circle, and The same double straight line d2. Because of the symmetry of those curves, according to the plane x-y, there will appear Another ellipse, and another circle, rotated by 180°.

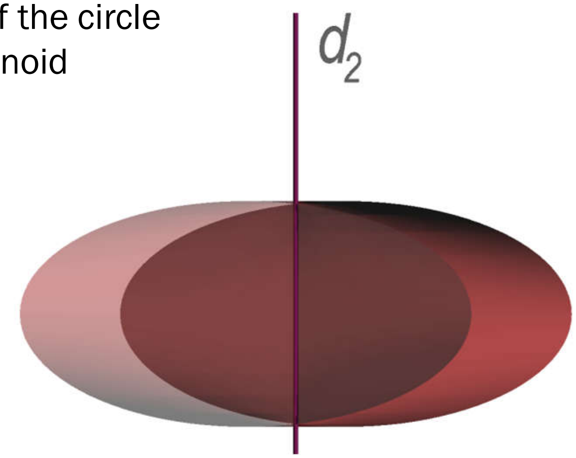
Altogether, there will exist twofold surface consisted of two branches of conoids, passing trough the same line d2, so it will be a fourfold line. The surface will be of the eight degree.

Spatial interpretation of the double egg curve

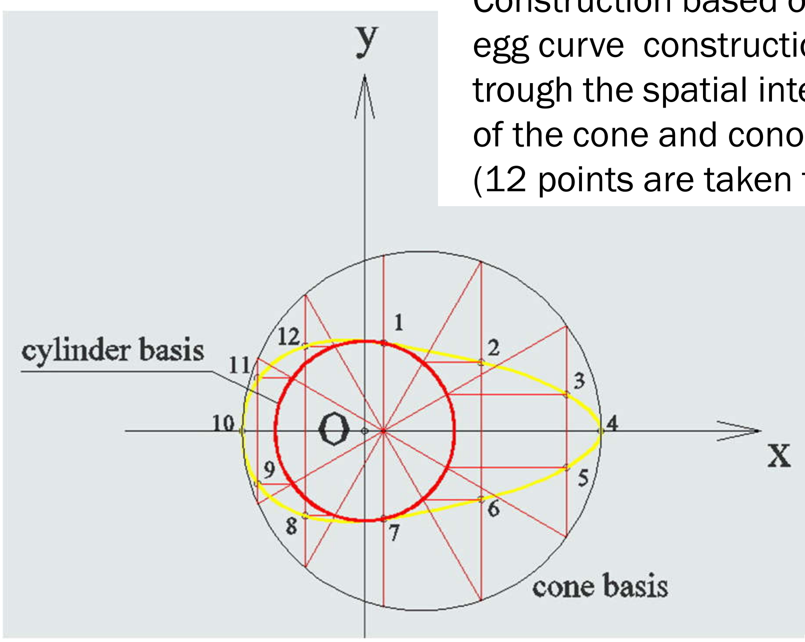
Top view of the ellipse directed conoid

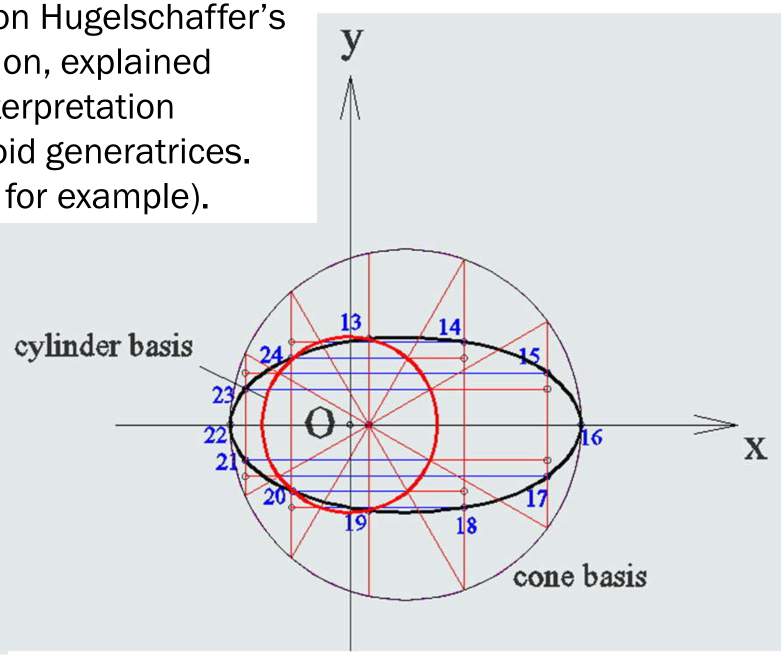


Top view of the circle directed conoid



Construction based on Hugelschaffer's egg curve construction, explained trough the spatial interpretation of the cone and conoid generatrices. (12 points are taken for example).



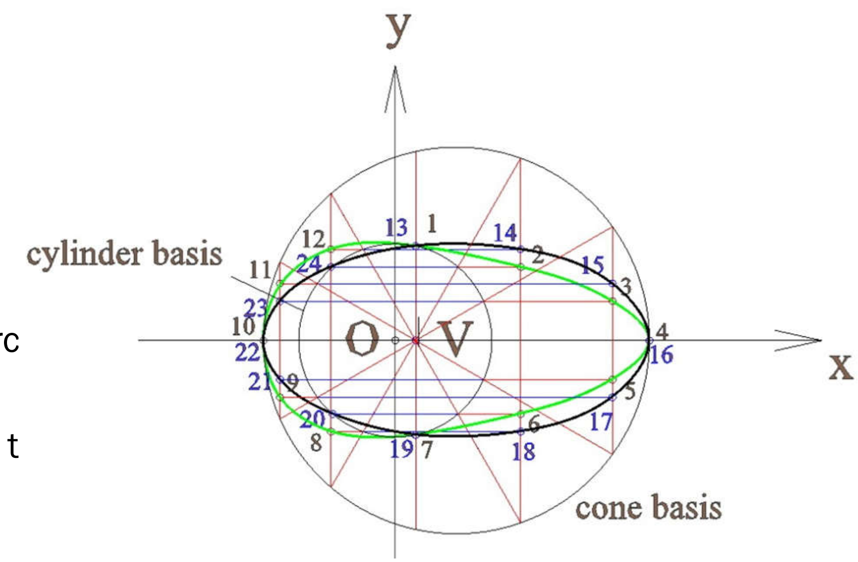


The Formulas For Intersection Of An Oblique Cone And A Cylinder

- We start from the following parameters:

- b – radius of cylinder of rotation
- a - radius of basis of oblique cone
- d – distance between the centers of the circ bases of the cylinder and the cone
- m – altitude of the circle of intersection of t cylinder and the cone

- h – the altitude of the cone apex: $h = \frac{ma}{a+b}$
- r= $\frac{bd}{a+b}$ - distance of the V', projection of the cone apex V, and center C2 (0,0,0)
- According to the geometrical principles, the choice of the cone base radius is not arbitrary, in order to achieve the case of degenerated intersection on two conics.



General equation of an oblique cone:

$(p(z-m) - hx)^2 + (q(z-m) - hy)^2 = (z-m-h)^2 b^2$

(p,q) coordinates of the center of the cone: p = d; q= 0

General equation of a cylinder of revolution:
$(x-s)^2 + (y-t)^2 - b$

(s, t) coordinate of the center of the cylinder: s = 0; t = 0

Application of the Groebner bases on solving the intersection of two quadrics

- By application of the Groebner bases, it is proven that the oblique cone and the cylinder of rotation are intersecting by common planes (1) and (2).
- (1) **$m-z=0$**
- (2) **$-b^2 m^3 + 2b^2m^2h - d^2h^2m + (2dhm^2 - 2dh^2m)x + (d^2h^2 - b^2m^2)z = 0$**
- Besides, Groebner bases provide even additional equation of the pencil of surfaces that can be set trough the intersection curve of two quadrics, as the fundamental curve:

Coefficient by z^2:
$(bm - dh)^2 (bm+dh)^2$

Coefficient by z:
$-2m(bm+dh)(bm-dh)(-d^2h^2+2hmb^2 - b^2m^2)$

Coefficient by y^2:
$4d^2h^2m^2(h-m)^2$

Free coefficient
m^2 (-bm+2bh-dh) (bm-dh) bm+dh) (-bm+bh+dh)

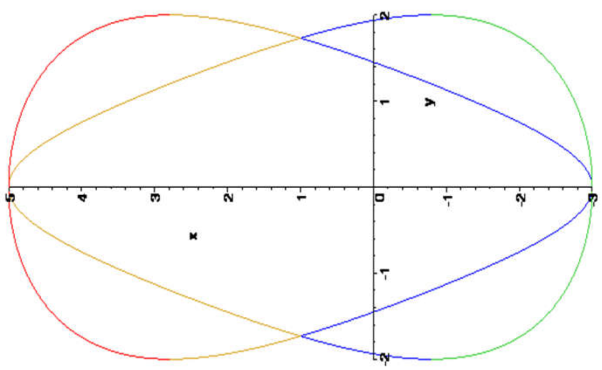
Equation of the pencil of quadrics:

$$b^4m^6 + 4h^2m^4b^4 - 4hm^5b^4 + 4d^2h^3m^3b^2 - 2d^2h^2m^4b^2 - 4d^2h^4m^2b^2 +$$
$$+ d^4h^4m^2 + (d^4h^4 + b^4m^4 - 2d^2h^2b^2m^2)z^2 +$$
$$+ (-4b^4m^4h + 2b^4m^5 + 4d^2h^3b^2m^2 - 2d^4h^4m)z +$$
$$+ (4d^2h^4m^2 - 8d^2h^3m^3 + 4d^2h^2m^4)y^2$$

The Equation Of A Double Egg Curve And The Equation Of A Double Egg Conoid

- **Double egg curve formula:**
- $M_{00} + M_{10}x + M_{02}y^2 + M_{40}x^4 + M_{20}x^2 + M_{30}x^3 + M_{12}xy^2 + M_{42}x^4y^2 + M_{32}x^3y^2 + M_{22}x^2y^2 + M_{04}y^4 = 0$
- Coefficients M_{ij} are in function of parameters **a, b, d** and **m.**

Certain coefficients M_{ij} can be factorized.



Double egg curve

Double egg conoid formula:

$M_{044}y^4z^4 + M_{043}y^4z^3 + M_{042}y^4z^2 + M_{041}y^4z + M_{021}y^2z + M_{202}x^2z^2 + M_{212}x^2yz^2 + M_{122}xy^2z^2 + \ldots + M_{000} = 0$

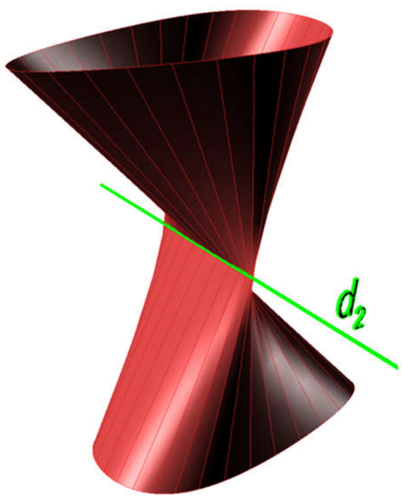
Coefficients M_{ijk} are in function of parameters

a, b, d and **m.**

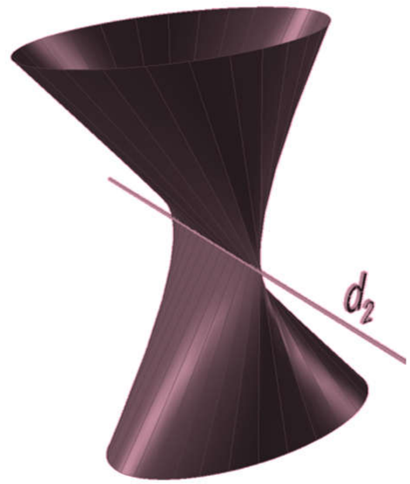
Certain coefficients M_{ijk} also can be factorized.

Double Egg Curve Based Conoid

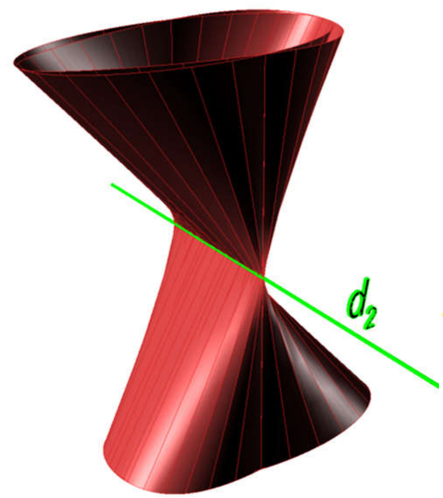
Conoid As The Surface Obtained By The Doble Egg Curve As The Directrix



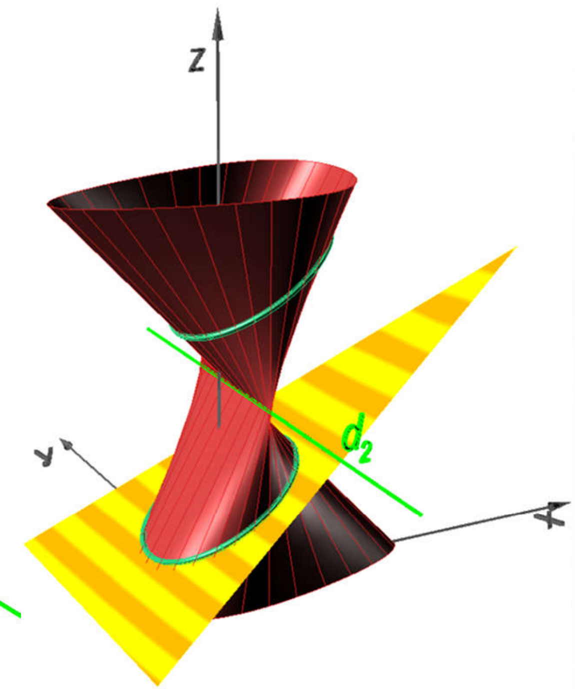
Conoide branch 1



Conoide branch 2



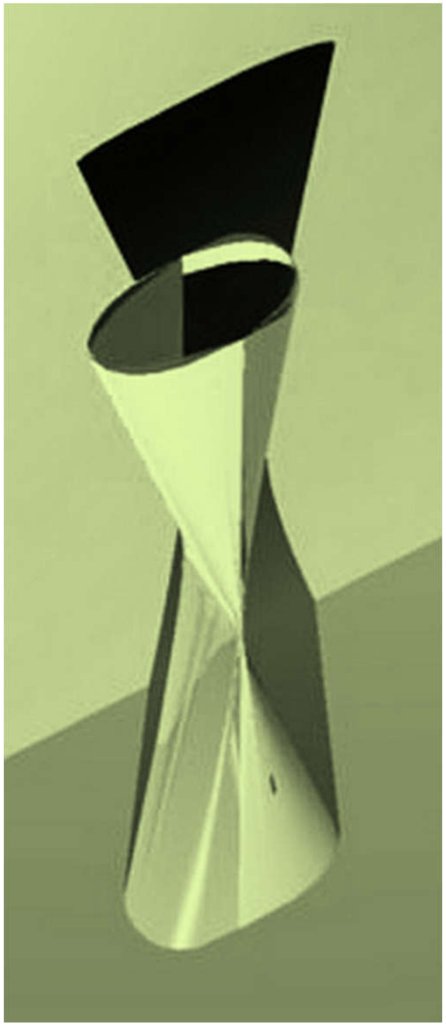
The whole conoidal surface

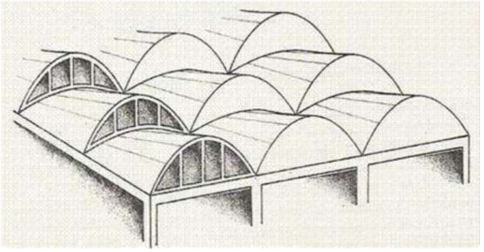


Plane intersection of the conoid branch 1 by ellipse

So, we can accept the double egg curve as the directrix of the conoid.
Each branch of the curve will form the branch of the conodal surface.
We obtain the double egg conoid that can be intersected by four planes
By the degenerated curve, which will imply a conic : two of them will consist
The firstly recognised ellipse, ad the remaining two will concict the circle
we started from.

The Application Of The Conoidal Surfaces





Roof constructions
Based on the geometry
Of conoidal surfaces

Conoidal surfaces have a great field of implementation on various segments of engineering and design. We will mention just the most common application as the roof construction, as well as a most suitable shape of some parts of aircrafts or ships.

Conclusions

- In this paper we appointed a connection between the Hugelschaffer's construction of the egg curve, and the variation of the same construction, which gives a double egg curve.
- We generated a ruled surface, conoid, that has the double egg curve as the directrix curve.
- It is shown that such a surface will be the double conoid, as well.
- This type of conoidal surface can be intersected by four different planes that will give a degenerated intersection curve - on a conic, and the remaining curve of higher degree.
- We found a very functional connection between the methods of Descriptive, Projective Geometry and Groebner bases method.

- Literature:

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