3rd International Scientific Conference

Treći međunarodni naučni skup

moNGeometrija 2012

Proceedings

Zbornik radova

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ISBN 978-86-7892-405-7

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UNDER THE AUSPICES OF:

Republic of Serbia

Ministry of Science and Technological Development

Autonomous Province of Vojvodina

Faculty of Technical Sciences, Novi Sad

Faculty of Agriculture, Novi Sad

Higher Technical School of Professional Studies in Novi Sad

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Publisher | *Izdavač*

Faculty of Technical Sciences
Novi Sad

Fakultet tehničkih nauka, Novi Sad

Serbian Society for
Geometry and Graphics

Srpsko udruženje za geometriju i grafiku
SUGIG

Editor-in-Chief | *Glavni urednik*
Ph.D. Ratko Obradović

Design| *dizajn*
M.Sc. Igor Kekeljević

Title of Publication
PROCEEDINGS | *Zbornik radova*

Reviewers | *Recezenti*
Ph. D. Ratko Obradović
Ph. D. Radovan Štulić
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Text formatting| *formatiranje teksta*
M.Arch. Lea Škrinjar
M.Arch. Marko Jovanović

ISBN 978-86-7892-405-7

Numbers of copies printed | *Tiraž*: 200

Printing | Štampa: **Faculty of Technical Sciences, Novi Sad**
Fakultet tehničkih nauka, Novi Sad, Trg Dositeja Obradovića 6

CONCAVE ANTIPRISMS OF SECOND SORT WITH REGULAR POLYGONAL BASES

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Abstract

The paper discusses determining the positions and spatial coordinates of the vertices of polyhedral structures - concave antiprisms of second sort. These polyhedra originate from folding the double row strip of equilateral triangles, closed by two identical regular polygons, the principle akin to the way of the concave cupolae of second sort formation. The paper considers constructive - geometrical solution, including the method of solving the problem using the mechanisms, by application of SolidWorks software package.

Key words: concave antiprism, regular polygon, triangular net, mechanism

1. INTRODUCTION

Concave antiprism of second sort is a polyhedron which lateral surface is consisted of double –row strip of equilateral triangles. The triangles are aranged in such an array to form a segment of isomorphic triangular tiling in a plane net (Fig.1), simmilarly to the concave cupolae of second sort [3], [4], [5], and polygrammatic antiprisms [1]. By folding and pasting the adjacent edges, we obtain a closed, ring-like fragment of polyhedral surface consisted of spatial hexa-triangular cells, interconnected by the triangles. The number of unit cells – the spatial hexagons, is determined by the number n of base polygon sides.

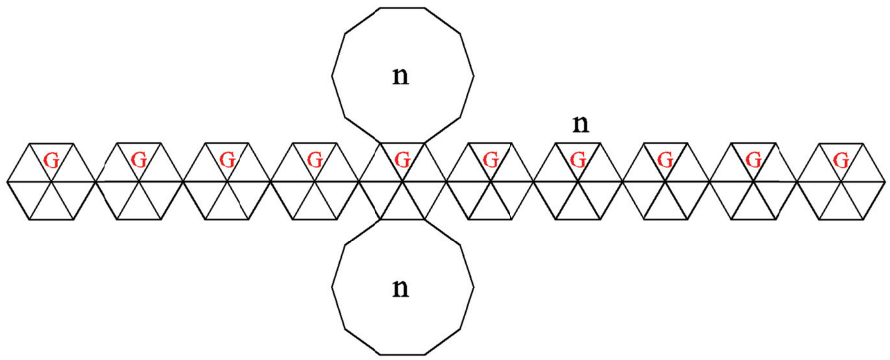

Figure 1 – A net of concave antiprism of second sort

Knowing that six equilateral triangles around a common point can not form the vertex of a convex polyhedral figure, we conclude that the fragment formed in such a manner must be concave polyhedral surface. Furthermore, there is always two variants of concave antiprisms of second sort with the same polygonal base: a) with internal vertices G of spatial hexa-triangular cell ABCDEFG, b) with external vertices G of spatial hexa-triangular cell ABCDEFG, Fig. 2.

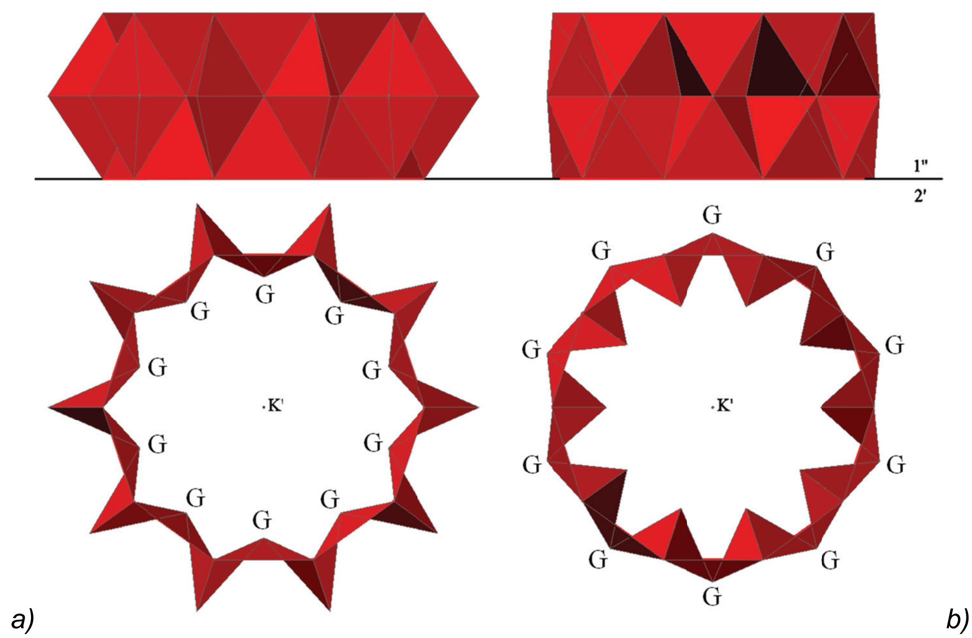

Figure 2 - Concave antiprism of second sort: a) with internal vertex G of spatial hexa-triangular cell ABCDEFG, b) with external vertex G of spatial hexa-triangular cell ABCDEFG

 ISBN 978-86-7892-405-7

2. CONSTRUCTIVE GEOMETRIC METHOD OF FINDING PARAMETERS

In the Fig. 3 and 4, we present just one segment of concave antiprism of II sort's net: the spatial hexa-triangular cell ABCDEF. It is formed by six equilateral triangles arranged arround common vertex G. In order to define parameters of these solids, it is neccessery to set some initial conditions which have to be fulfilled by the hexa-triangular cell, so that their radial array arround the common vertical axis *k* could enclose a convergent geometric space. These conditions are:

- the plane α is vertical symmetry plane of the edges AB and DE, while the vertex G belongs to the plane α.
- vertical plane β is determined by the axis *k* and by the vertices B, C and D
- edges AB and DE are horizontal and in the common vertical plane,
- edges CG and FG belong to the common horizontal plane which is set at the half height of the spatial hexa-triangular cell ABCDEFG.

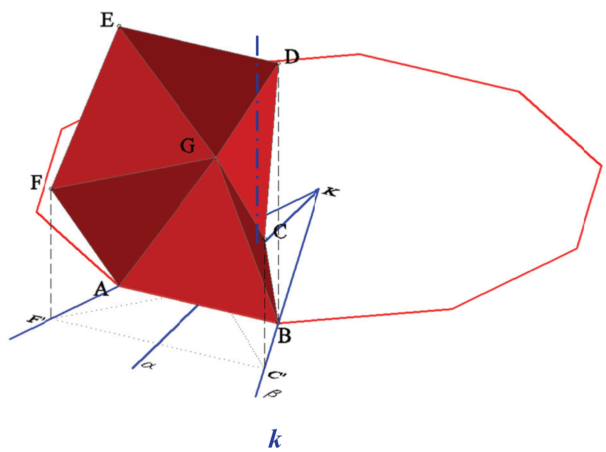

Figure 3 - Unit hexa-triangular cell ABCDEFG with internal vertex G

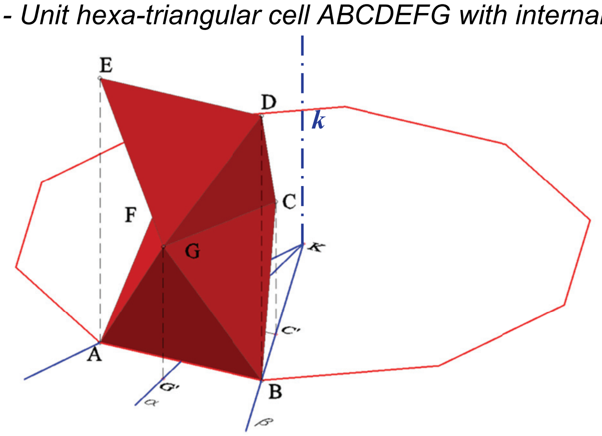

Figure 4 - Unit hexa-triangular cell ABCDEFG with external vertex G

To answer the question: how to find the height of the concave antiprism, we have to realize that the occurred problem is of a higher order than allowing it to be exactly solvable by classical accessories - a compass and straightedge. The reason lies in the observations that the spatial hexa-triangular cell ABCDEFG acts as a mechanism, which characteristics will be explained bellow. We need to define the movement of the mechanism in respect to the above requirements.

The point C of spatial hexa-triangular cell ABCDEFG is set on a sphere of radius *r=a* (edge of the equilateral triangle), centered at the vertex G. The point C is also located on a sphere of radius r=a, centered at the point B. Horizontal plane whose height is equal to the height of the point G, intersects the sphere by circles whose intersection points give the positions of point C. Based on these assumptions, we can look up for the point C by iterations of C_n positions, depending on the assumed arbitrary initial position of the central vertex of G_n. In Fig. 5. a fragment of the trajectory of the vertex C is shown, generated in the described manner. From the condition that C must belong to the plane β as well, we obtain the required positions the vertices C and G, and thus the height of the antiprism in transformational plane (1^{β} β').

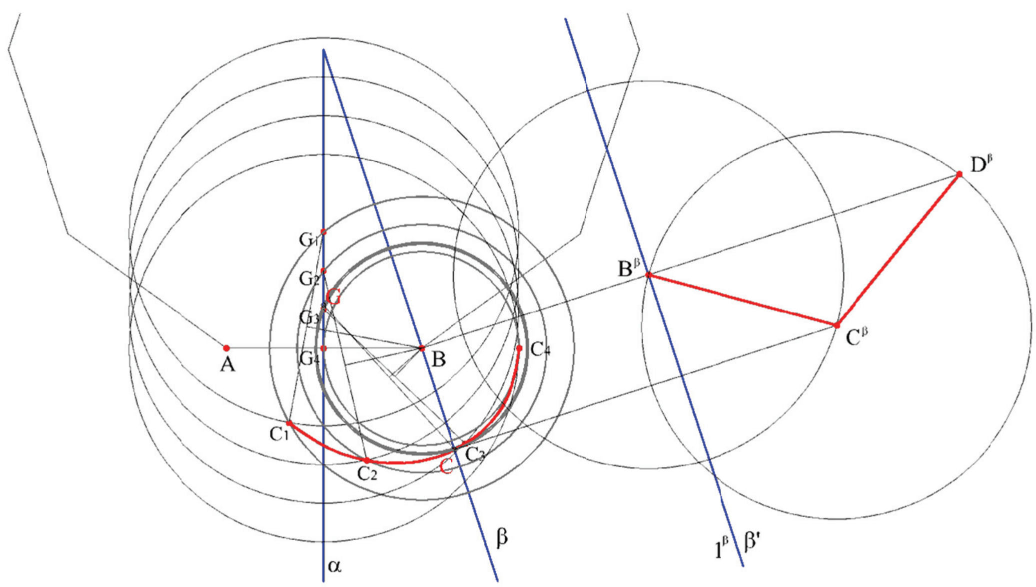

Figure 5 - The origin of the vertex C trajectory, and construction of the height of concave antiprism of second sort

The figure 6 gives the whole trajectory of the vertex C, from the initial height h=0 ($A=C_1$) to the maximum height (G_6C_6). We see that the plane β intersects the trajectory twice, which corresponds to our assumption that for the same base we get two concave antiprisms types: with the internal and with the external vertex G. When the vertex G is in the interior of the antiprism, the vertex C is in the exterior, and vice versa. Fig. 6 shows the top projection of trajectory which is obtained as a section of a sphere and a quartic surface - Bohemian Dome. The sphere is of the radius *r=a*, centered at the point B. Bohemian Dome consists of horizontal circles of radius *r=a*, which move along the directrix, circle, a cross section of the given sphere and the vertical plane α. From the spatial model (Fig. 7) of the vertex C trajectory, we can directly read off the height of the vertex C, and therefore the whole height of the antiprism.

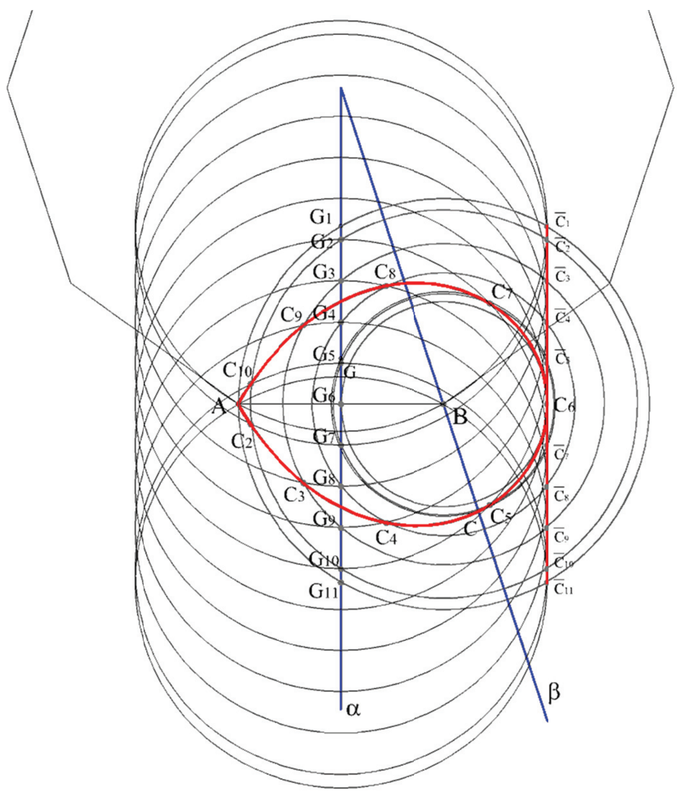

Figure 6 - The trajectory of the vertex C

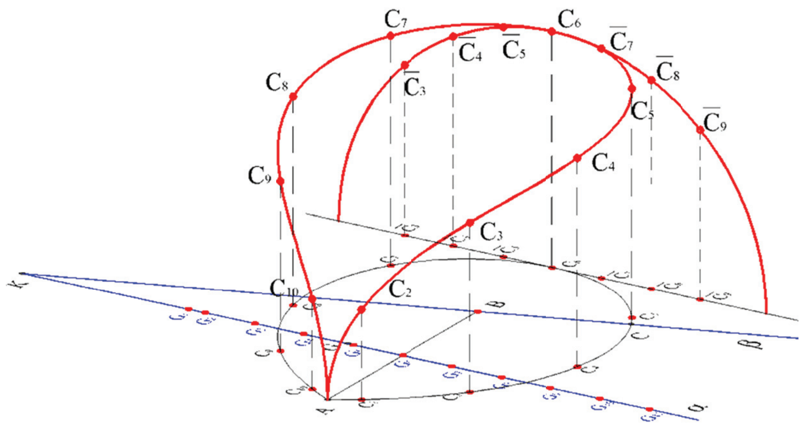

Figure 7- 3D model of the vertex C trajectory

In addition to constructive-geometric methods in solving the problem of finding the Concave antiprisms' (of second sort) height, we may apply analytical methods as well. By setting an appropriate algorithm and using iterative numerical methods, we obtain values of required parameters with satisfactorily small error (the authors have applied Microsoft Excel for the iterations). The observed error is 0.0000004, for the appointed edge size *a=100*.

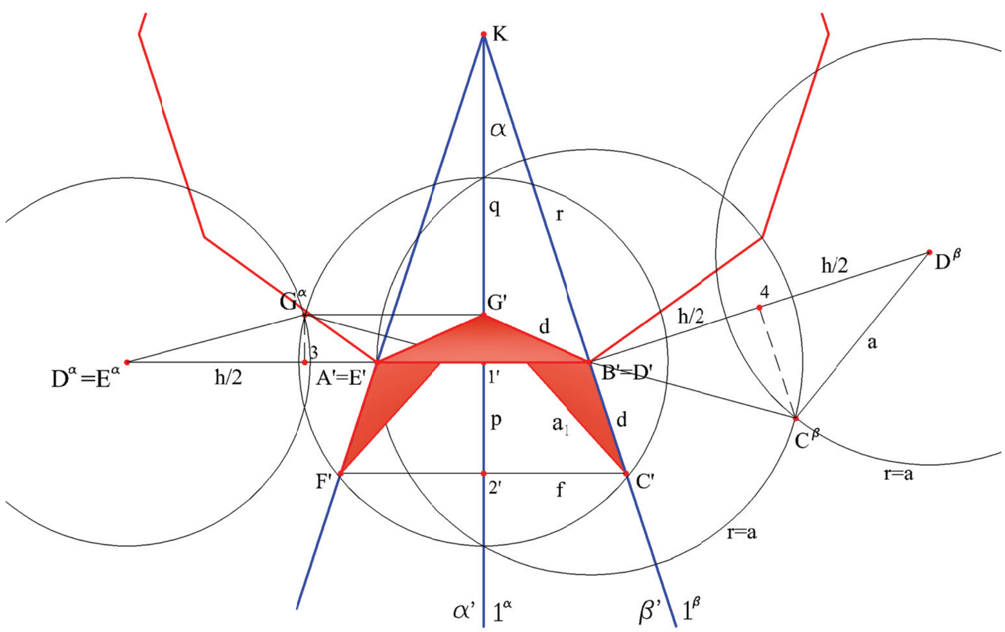

Figure 8 - Parameters and measurements of spatial hexa-triangular cell ABCDEFG with internal vertex G

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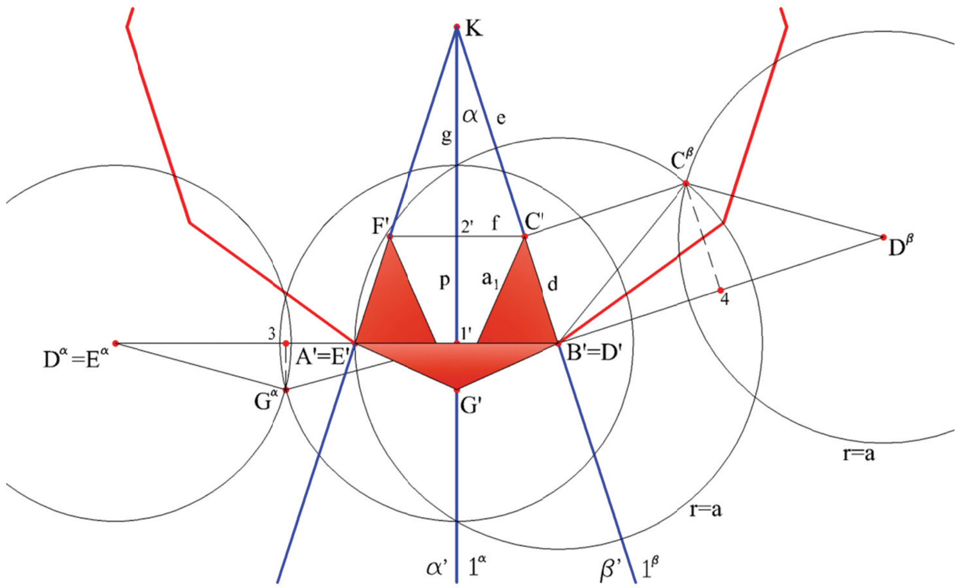


Figure 9 - Parameters and measurements of spatial hexa-triangular cell ABCDEFG with external vertex G

3. ALGORITHM FOR FINDING SOLIDS' PARAMETERS

The algorithm created according to the fig. 8, for determining the parameters of Concave antiprism of second sort with internal vertex G, differs from the algorithm for the antiprism with the external vertex G (Fig. 9) only in equations. (8) and (10). This algorithm is valid for any regular n-sided polygonal basis. Therefore, based on the Fig. 8 and Fig. 9, we can apply the following algorithm:

(n) the number of base polygon vertices	n=10
(h) the lateral ring height	h=167.384928
(a) the side of the base polygon	a=100

$a = \frac{p}{n}$	α=18	(1)
$K'A' = K'B' = r = \frac{a}{2\sin a}$	$K'A' = K'B' = r =$ 161.8033989	(2)
$K'1' = q = r \times \cos a$	$K'1' = q =$ 153.8841769	(3)
$D^a G^a = \frac{a}{2}\sqrt{3}$	$D^a G^a = \frac{a}{2}\sqrt{3}$	(4)
$G'1' = b = \frac{1}{2}\sqrt{3a^2 - h^2}$	$G'1' = b =$ 22.26143458	(5)

$C^bD^b = a$ $\qquad$ $C^bD^b = a$ (6)

$C'B' = B'G' = d = \frac{1}{2}\sqrt{4a^2 - h^2}$ $\qquad$ $C'B' = B'G' = d = 54.73181405$ (7)

$K'C' = e = r + d$ $\qquad$ $K'C' = e = 216.5352129$ (8)

$K'C' = e = r - d$ - for hexa-triangular cell ABCDEFG with internal vertex G

$2'C' = f = e \times \sin a$ $\qquad$ $2'C' = f = 66.91306068$ (9)

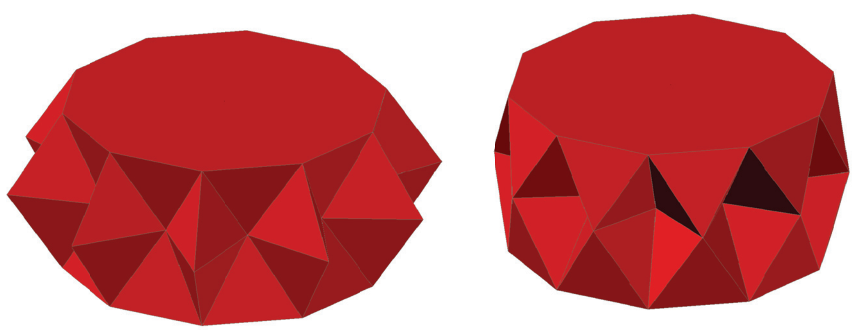
$1'2' = p = \sqrt{e^2 - f^2} - q$ $\qquad$ $1'2' = p = 52.0530484$ (10)

$1'2' = p = q - \sqrt{e^2 - f^2}$ - for hexa-triangular cell ABCDEFG with external vertex G

$G'C' = a_1 = \sqrt{(p + b)^2 + f^2}$ $\qquad$ $G'C' = a_1 = 100.0000004$ (11)

$D = a - a_1$ $\qquad$ $D = 0.0000004$ (12)

In the Fig. 10, we present an axonometric view on the both cases of concave antiprism of second sort: a) with the interior vertex G, and b) with the exterior vertex G, modeled according to the given algorithm.



a) b)

Fig. 10 – Axonometric view of the two types of decagonal concave antiprism of second sort

4. MECHANICAL INTERPRETATION OF THE GEOMETRIC CONSTRUCTION

The geometrical construction exposed in this paper can be interpreted mechanically and realized by one particularly synthesized mechanism. Top and oblique projection of that mechanism is shown on Fig.11 and Fig.12 respectively.

 ISBN 978-86-7892-405-7

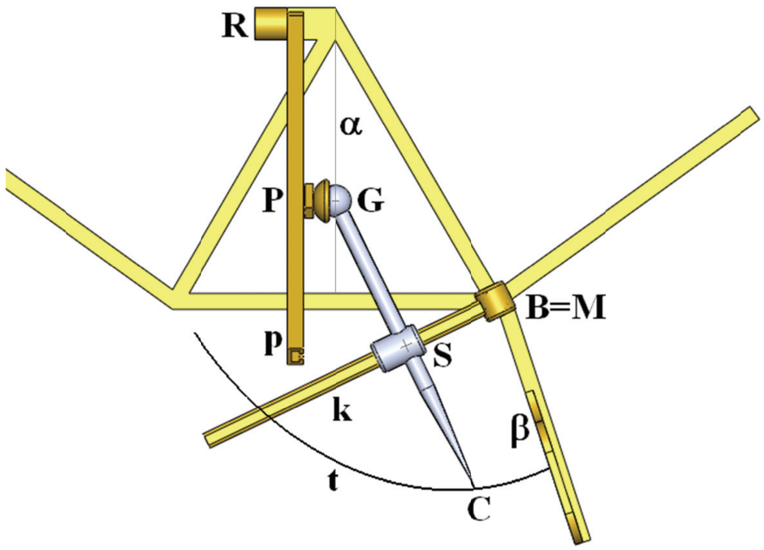


Figure 11- Top projection of the mechanism

The mechanism comprises the revolute joints R and M, antiprismatic joints P and S, spherical joint G, arm p, lever GC and crank k. The mobility (degree(s) of freedom) of this linkage system is 1. Antiprismatic joint P can slide along the groove of the arm p, and arm p can rotate about the axis of the revolute joint R. Antiprismatic joint P holds the spherical joint G, in which the lever GC is attached. Crank k is always held in horizontal position by the revolute joint M and lever GC is connected with the crank k under the right angle by the antiprismatic joint S in such a way that geometrical center of joint S is mid point of the lever GC. Thus, lever GC is also held in horizontal position by its joints and crank k. The complete mechanism is driven by a rotary motor in the point B which rotates the crank k about the vertical axis of the revolute joint M. During the mechanism motion, center of the spherical joint G remains in vertical plane α and the point C generates the plane curve. The mechanism motion lasts until the lever GC touches the surface of the vertical panel β. This contact point is the intersection point between trajectory t of the point C and plane β and represents the required solution of the geometrical problem modeled by this mechanism.

3D model of this mechanism is accomplished by the using of Solid Works application and motion simulation as well as the motion analysis of the mechanism model is obtained by appropriate Solid Works sub-routines.

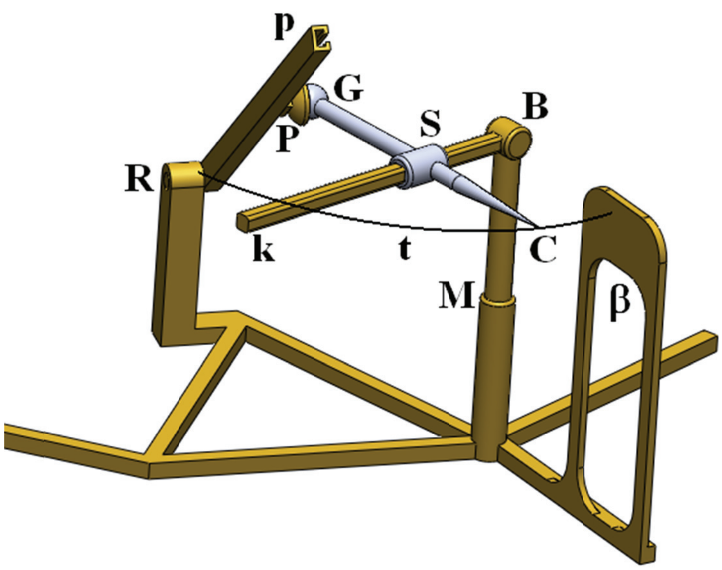


Figure 12 – Oblique projection of the mechanism

5. CONCLUSIONS

Concave antiprisms of second sort are polyhedra obtained by joining two identical n-sided polygons in parallel planes with the lateral surface consisted of n spatial hexa-triangular cells, folded out of the plane net of equilateral triangles. Concave antiprisms of second sort are the group of infinite number of members. These antiprisms are akin to the concave cupolae of second sort, and to the polygrammatic antiprisms.

We can solve the problem of finding the concave antiprisms' (of second sort) parameters, in several ways:

- we can solve the problem using constructive-geometric method, by finding the trajectory of the vertex C. The intersection of this trajectory and the plane of incidence (β) for the vertex C, we find the solution of the vertex C position.
- We can solve the problem analyticaly, by seting an algorithm. Implementing iterations conducted by computer tools (Microsoft Exel), we get all the required data.
- Also, we can observe the problem as a mechanical and by setting a mechanism (in SolidWorks software package) we can get both the trajectory and the vertices parameters.

ACKNOWLEDGEMENT: Research is supported by the Ministry of Science and Education of the Republic of Serbia, Grant No. III 44006.

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