25th National and 2nd International Scientific Conference

moNGeometrija 2010

Serbia | Belgrade | June 24th - 27th | www.mongeometrija.org

UNDER THE AUSPICES OF:

Republic of Serbia – Ministry of Science and Technological Development

Faculty of Architecture in Belgrade

Faculty of Mechanical Engineering in Belgrade

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Faculty of Transport and Traffic Engineering in Belgrade

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25th National and 2nd International Scientific Conference

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Publisher | *Izdavač*

Faculty of Architecture in Belgrade
Arhitektonski fakultet u Beogradu

Serbian Society for
Geometry and Graphics
Srpsko udruženje za
geometriju i grafiku
SUGIG

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Title of Publication

PROCEEDINGS | *BILTEN*

Reviewers | *Recezenti*

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Text formatting

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ISBN 978-86-7924-040-8

Numbers of copies printed | Tiraž: 100

Printing | Štampa:

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CONIC SECTIONS OF A TYPE OF EGG CURVE BASED CONOID

Marija Obradović[95]
Branko Malešević[96]
Maja Petrović[97]

RESUME

A cubic egg curve obtained by Hügelschäffer's construction, can be spatially interpreted as a plane section of a type of a conoid set through a specially chosen 4-th order intersecting curve of two quadrics: right cylinder and cone. That implies that the apex of a cone must lay on the axis of a cylinder in order to obtain one sheet surface. This type of conoid will be of 4-th order, and will exclude plane sections by conics. We consider a special case of forming an akin conoid that would include also conic sections. If the apex of the cone is set off the cylinder axis, there would appear a double conoid, as a surface set through the intersection curve of the quadrics. Its plane section will be a double egg curve obtained by generalized Hügelschäffer's construction. In case that cylinder and cone would intersect by a degenerated 4-th degree space curve on two conics (circle and ellipse), there would emerge double egg curve, as a plane section of the double conoid. The curve degenerates onto ellipse and a quartic curve - Granville's egg. We also gave a mathematical condition of degeneration of the base double egg curve.

Key words: conoid, Hügelschäffer's construction, ellipse, egg curve.

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1. INTRODUCTION

If we adopt an egg-shaped curve of 3rd order - a cubic hyperbolic parabola - to be a directrix $\mathbf{d_1}$ of a ruled surface, together with a straight **line** $\mathbf{d_2}$ which is parallel to the curve's linear asymptote, and infinitely distant straight line $\mathbf{d_3}$ which has a common infinite point with its parabolic asymptote, we get (when we eliminate two planes, which will occur as degeneration products) a conoid of 4th order (**Figure 1**). The oval part of the curve $\mathbf{d_1}$ can be obtained by Hügelschäffer's construction.

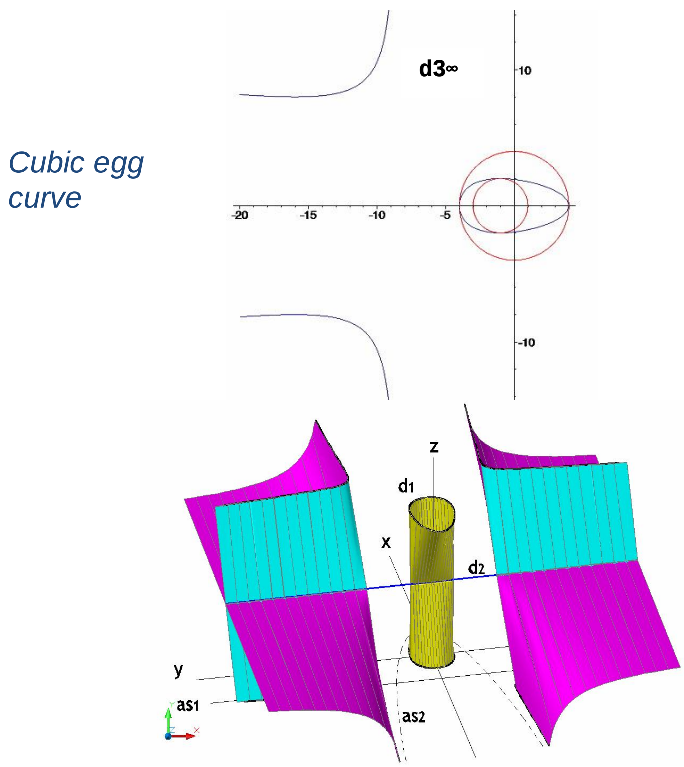


Figure 1. *Cubic Egg Curve And Conoid Obtained With This Curve As The Directrix*

We proved in out previous research [2] that it is not possible for such a conoid to have conic sections. In this study, we investigated a type of conoid with a directrix curve obtained by ***generalized*** Hügelschäffer's construction, which would give conic curve as a part of its plane intersection.

2. HÜGELSCHÄFFER'S CONSTRUCTION OF EGG CURVE

The construction we start from is the well known ellipse construction using the concentric circles, which is transposed into an egg curve construction, by displacing the centre of the minor circle, as done by mathematician Fritz Hügelschäffer (**figure 2**). The obtained egg curve is a cubic curve**,** actually, a part of the right cubic hyperbolic parabola of type A [2].

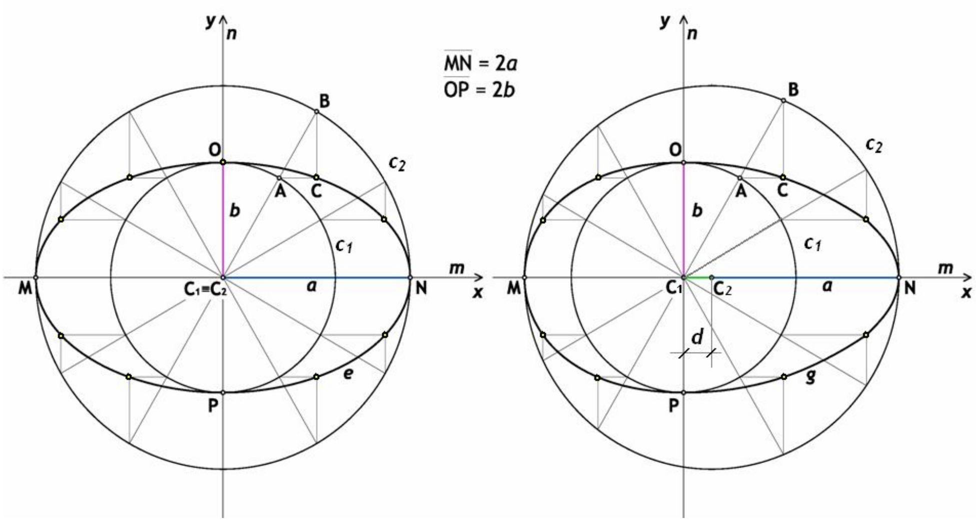


Figure 2*: Ellipse Construction and Egg Curve Construction*

Ellipse equation:

$$e: \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Cubic egg curve general equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\, f(x) = 1$$

Distortion of the equation for the value **d**:

$$f(x) = 1 + \frac{2dx+d^2}{a^2}$$

Hügelschäffer's egg curve equation:

$$g: \quad b^2x^2+ a^2y^2+2dxy^2+d^2y^2-a^2b^2 = 0$$

So, by the eccentricity of the centre C_1 of the minor circle for the value **d**, the degree of the gained curve rises from two (for the ellipse) to tree (for the Hügelschäffer's egg curve).

We can consider the generalization of this construction, when the center of the pencil of straight lines of the transformation is not coincidental with center of the small circle **C1**. In this paper we will deal with only the case when the center **V'** moves linearly along the x axis. We will do spatial interpretation of this generalization, and also give mathematical support to the claim.

3. SPATIAL INTERPRETATION - STARTING SUPPOSITIONS

Spatial interpretation of the construction will be given using conoid surface whose plane intersection curve is adopted for the basis.

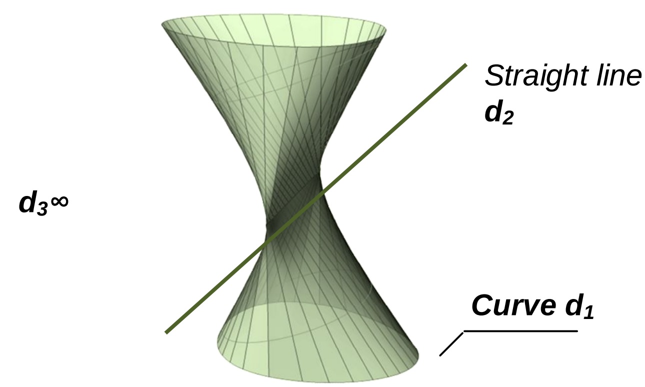


Figure 3: *Cubic Egg Curve Based Conoid*

Conoid is a ruled surface, obtained by moving the generatrix along three directrices:

A plane curve – $\mathbf{d_1}$ **(of order n1)**

A straight line (or curve) - $\mathbf{d_2}$ (of order **n2**) and

An infinte straight line – $\mathbf{d_3}$ **(of order 1)** which determines the direction plane (**figure 3**).

The order of the conoid can be expressed by the equation: ***2n1n2*** [12]***.***

Let us suppose that the minor circle (**c_1**) is a base of a right cylinder with center **C_1**(0,0,0), and the major circle (**c_2**) is a base of an oblique cone with center **C_2**(d,0,0), which apex would be set on the z axis, on the hight ***h***: **V*(0,0,h).***

a) Let us start from a circle ***ζ*** with radius **R_1** and the center **C** on **z** axis, on the altitude ***m*** **(figure 4 -a).**

b) Through this circle ***ζ*** it is possible to set a cylinder of rotation. Its base will be the circle **c_1 = *ζ*** in the plane **x-y.**

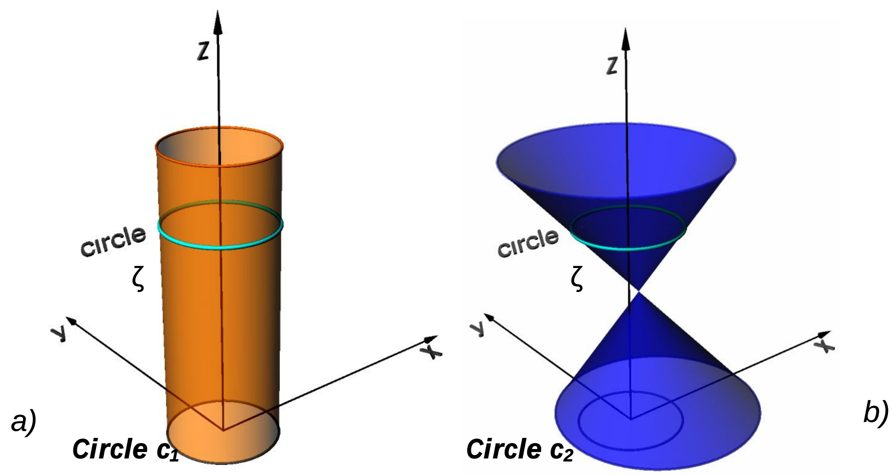


Figure 4*: a) Cylinder of Rotation Trough the Circle c1 and the Circle ζ= c1*
b) Oblique Cone That Consists the Circle ζ, With Apex V (d, 0, h)

c) Through the circle **ζ** it is also possible to set an oblique cone, with the apex ***V***, on the altitude ***h*** and the base circle **c_2** with radius **R_2,** and center **C_2** which will be shifted for some value **d**, in the direction of **x** axis (**figure 4- b**).

The general case of intersection of a cylinder and a cone is a fourth degree spatial curve (**figure 5**). Starting from such an intersection curve as a directrix of a surface, and with the condition that the vertex **V** of the cone is set on the axis of the cylinder, we can obtain a right conoid **(figure 6)**. The straight line directrix will be the line d_2 set trough the apex ***V*** of the oblique cone, and the directrix plane, **x-z**, determines the position of generatrices [1].

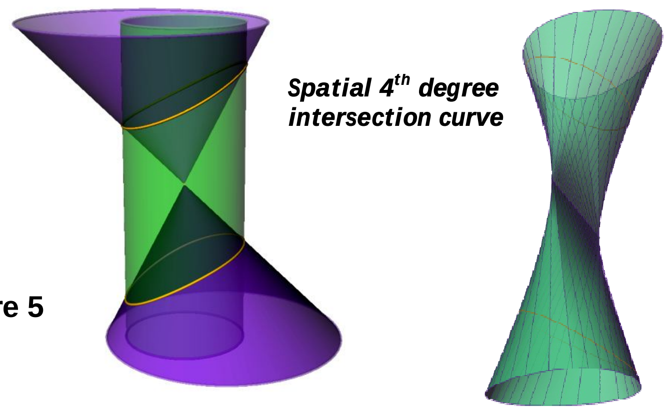


Figure 5

Figure 6

Figure 5 and 6*: Cone and Cylinder intersection curve*

The planar curve directrix, can be accomplished as a section by plane **x-y** of the base circles $\mathbf{c_1}$ and $\mathbf{c_2}$ when we get exactly the Hügelschäffer's egg curve. In this manner, we obtain a fourth degree surface – an egg curve based conoid [2].

The special case of cone and cylinder intersection will be the one that implies a degenerated spatial curve of fourth degree on two conics.

It occurs when the surfaces (quadrics) have two common tangential planes. In current case, tangential pair of planes is imaginary, so the spatial curve will degenerate onto a starting circle **ζ** and another conic. The other conic must be convergent, because there can not exist a second degree curve on a cylinder of rotation that would be divergent (opened), so it must be an ellipse, assigned as **ε** on **figure 7**.

In this case, it is not possible for the vertex (***V***) of an oblique cone to be set on the right cylinder axis [4].

Instead of a unique spatial four degree curve, now we take a degenerated curve on two conics, circle **ζ**, and ellipse **ε,** we generate a new conoid, and investigate a type of a surface obtained in this manner.

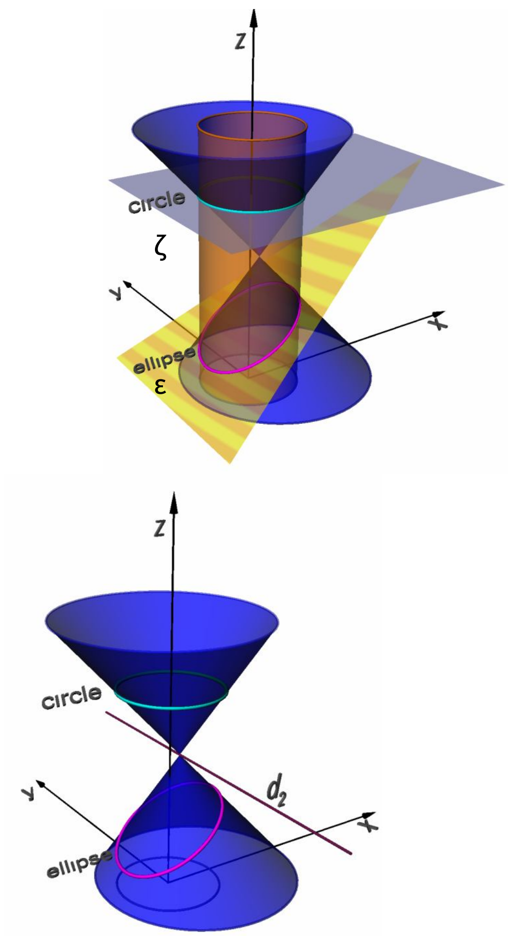


Figure 7*: Cone And Cylinder Intersecting Curve Degenerated To A Circle And An Ellipse*

4. CONOID OBTAINED BY USING A SPECIAL CASE OF THE CYLINDER - CONE INTERSECTION AS THE DIRECTRIX CURVE

If we set a surface trough the conics that are derived from the degenerated intersection of a cone and a cylinder, directed by another straight line d_2 set in the vertex ***V***, shifted for the value **m** from the cylinder vertex, and a direction plane ***x-z***, instead of one unique surface, as shown in the initial case, we get two branches of conoids[98]. One of them K_1, will pass trough the circle **ζ** and the double straight line d_2, (**figure 8.a.**) while the other one, K_2, will pass trough the ellipse **ε**, and the same double straight line d_2 (**Figure 8.b**).

Now, we get two separate conoids K_1 and K_2, directed by two conic: the circle **ζ** and the ellipse **ε**, set in two different planes. The whole set of planes parallel to the plane of circle **ζ (=c1)** will intersect the conoid K_1 by the conics, and conoid K_2 by the 4^{th} degree curves, and the whole set of planes parallel to the plane of ellipse **ε** will set the conoid K_2 by the conics, and conoid K_1 by the 4^{th} degree curves (**figure 8**).

[98] Because of the central symmetry of the intersection points, according to the apex ***V***, which is now disturbed.

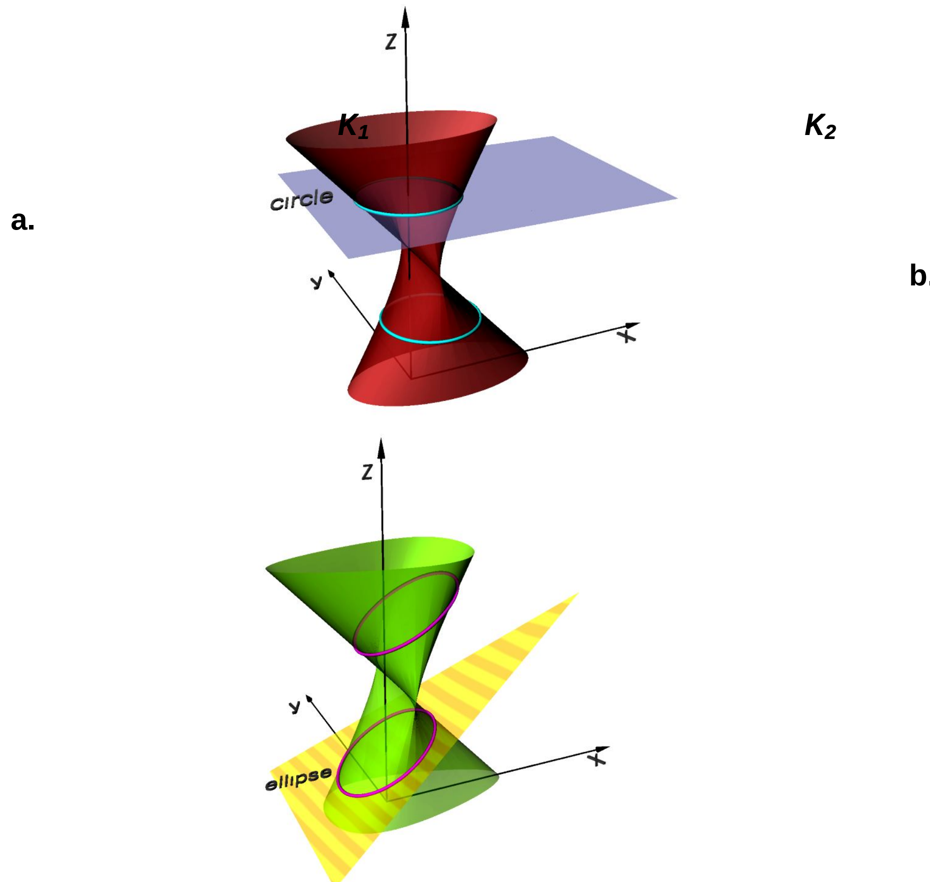


Figure 8*: Two conoids obtained by two basic curves – ellipse and circle*

Altogether, there will exist a double conoid, consisting of two **4th** degree conoids: **2nd** degree conic, and two straight lines, which produces: **2x2x1x1=4** order of the ruled surface, and passing trough the same line **d2**, so it will be a fourfold line of double conoid (**figure 9**).

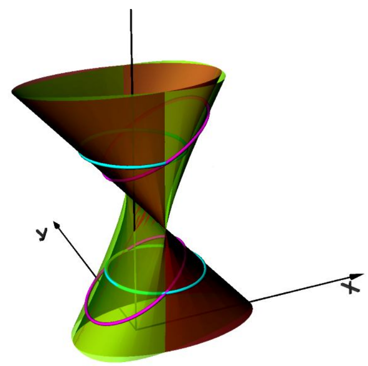
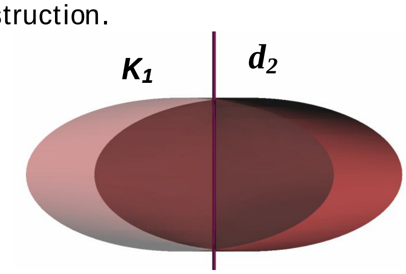


Figure 9**: Double conoid – **K1** and **K2

So, the conoid of **8th** degree is degenerated to two conoids of **4th** degree, because the plane directrix curve of **4th** degree desintegrated on two conics.

5. SPATIAL INTERPRETATION OF THE DOUBLE EGG CURVE

By using the example of those two formerly obtained conoids, we can give a spatial interpretation of an generalized Hügelschäffer's construction. We will investigate the plane section of the obtained surface by **x-y** plane, and observe the aberrations of the Hügelschäffer's construction.



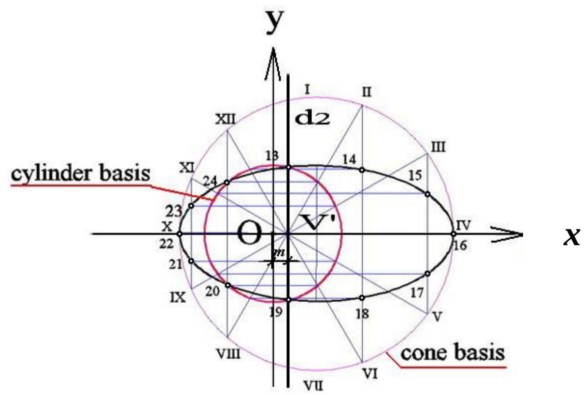


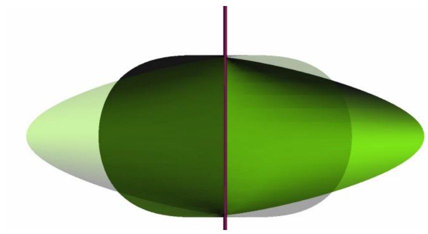
Figure 10: *Circle based conoid and its basic curve (ellipse) obtained by Hügelschäffer's construction of egg curve*

Top views of the conoids, shows the generatrices of the cone as rays of construction lines intersecting in the apex **V'** in the Hugelschaffer's construction of the egg curve. The set of lines parallel to axis **x** (as plane **x-z**) represents the generatrices of the conoid. The lines parallel to the axis **y** represent the intersection lines of the plane **x-y** and the pencil of planes set trough the line $\mathbf{d_2}$, intersecting cone by the generatrices (V-1, V-2…V-12[99]) (**figures 10** and **11)**.

In the intersection points (1-24) of those section lines of the planes, and the generatrices of the conoids, we get the points of new egg curve, a double egg curve based on Hugelschaffer's construction. The parts of the curves are given separately on the **figures 10** and **11** for enhanced visibility of the construction.

d_2

[99] (12 chosen points on the major circle $\mathbf{c_2}$, are taken for an example)



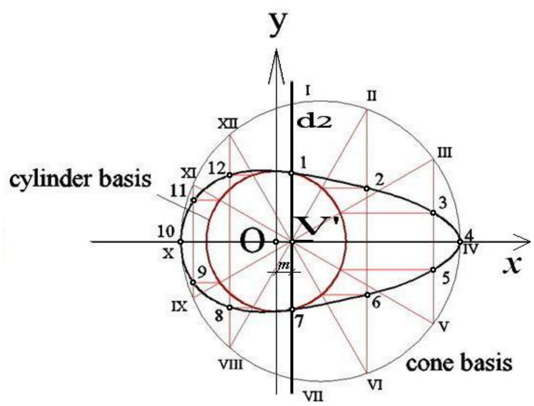


Figure 11: *Ellipse based conoid and its basic curve (egg shaped) obtained by Hügelschäffer's construction of egg curve*

We will notice that the conoid $\mathbf{K_1}$ obtained by using circle $\boldsymbol{\zeta}$ as directrix curve will give an ***ellipse e*** [6] as the part of the double egg curve, and the conoid $\mathbf{K_2}$ obtained by using ellipse $\boldsymbol{\varepsilon}$ as directrix curve, will give an egg shaped curve ***g*** of fourth degree [8].

This occurs because the ellipse $\boldsymbol{\varepsilon}$ and directix line $\mathbf{d_2}$ will generate 4^{th} degree conoid, which only the set of planes parallel to the plane of the ellipse $\boldsymbol{\varepsilon}$ will intersect by ***conic sections*** (ellipse or circle) and two more conjunctively imaginary lines set through the intersection point ($\mathbf{Y_\infty}$) of the plane of ellipse $\boldsymbol{\varepsilon}$ and the double straight line $\mathbf{d_2}$**.** In the same manner, the $\mathbf{4^{th}}$ degree conoid with the directix circle $\boldsymbol{\zeta}$, can only be intersected by the planes parallel to the accepted plane **x-y** of the circular section, so to obtain conics (circles and ellipses) and two more conjunctively imaginary lines that complete the degree of the intersection. These imaginary lines are set through the same infinite point ($\mathbf{Y_\infty}$) of the line $\mathbf{d_2}$**.**

In general case, the plane ***x-y*** would intersect the obtained double conoid of $\mathbf{8^{th}}$ **degree** by an octic curve degenerated onto sextic curve and two imaginary generatrices in the point ***Y∞*** [9]. In the considered case, the sextic curve degenerates onto ellipse ***e*** and a quartic curve ***g***, Granville's egg, that can be obtained by ***generalized*** Hugelschaffer's construction of the egg curve.

In the following part we will give the geometric conditions by which the double egg curve obtained by Hugelschaffer's construction will degenerate to an ellipse and an oval.

We can determine the conditions just by observing the top view projection, and even more, determine the condition of degeneration of the cylinder - cone base curve of intersection, regardless of the coordinate **z** of the apex **V**.

6. CONDITIONS OF DEGENERATION OF DOUBLE EGG CURVE ON A OVAL AND AN ELLIPSE

In general case, if we set the generalized Hügelschäffer's construction using two arbitrary chosen circles with centers displaced for value **d**, and then shift the centre **V'** of radial rays for value ***m*** along **x** axis set through the centre $\mathbf{C_1}$, in which is set the origin **O**, we get a **6th** degree double egg curve and the infinitely distant double straight line (6+2=8) because the arbitrary chosen apex **V** does not guarantee the degeneration of the cone – cylinder intersection curve. But, if we set conditions by which one of the curves will be ellipse, we define the conditions of degeneration of the basic sextic curve, and at the same time we make sure that the cylinder and the cone seen in the top view of the Hügelschäffer's construction, intersect by degenerated curve on two conics:

The condition is that the intersection of the projection of the double straight line $\mathbf{d_2}$ and the circle $\mathbf{c_1}$, point **E** (**figure 12**), must belong to ellipse with parameters: $\mathbf{a=R_2}$**,** $\mathbf{b=R_1}$**,** and center in $\mathbf{C_2}$**.**

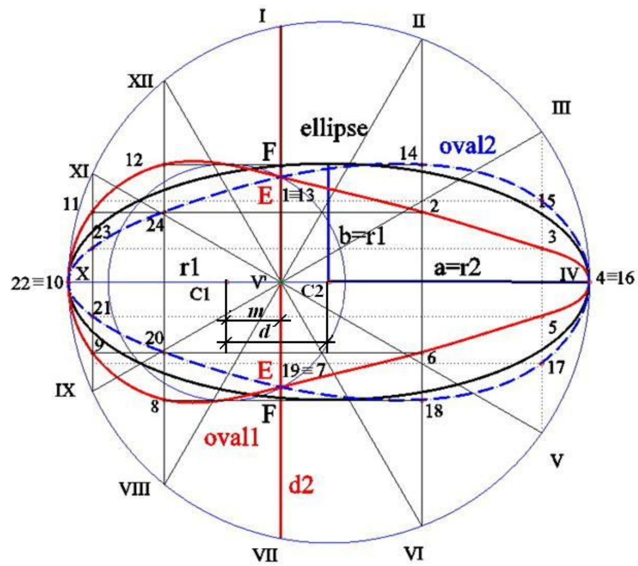


Figure 12: *Double Egg Curve As the Base of an Conoidal Surface With Non-degenerated Base Curve of Cone- Cylinder Intersection*

This case, point **E** will represent the (double) generatrix of the conoid, set vertically through the point **1≡13** (symmetrically **7≡19**) of the base cylinder circle $\mathbf{c_1}$, and intersecting the double straight line $\mathbf{d_2}$, seen as the point in the top view projection.

Let's mark intersection point of line $\mathbf{d_2}$ and circle $\mathbf{c_1}$ as **E**, and intersection point(s) of line $\mathbf{d_2}$ and ellipse as **F**. If **E=F**, then we have degeneration of the double egg curve - on ellipse and another egg shaped oval of fourth degree (**Figure 13**).

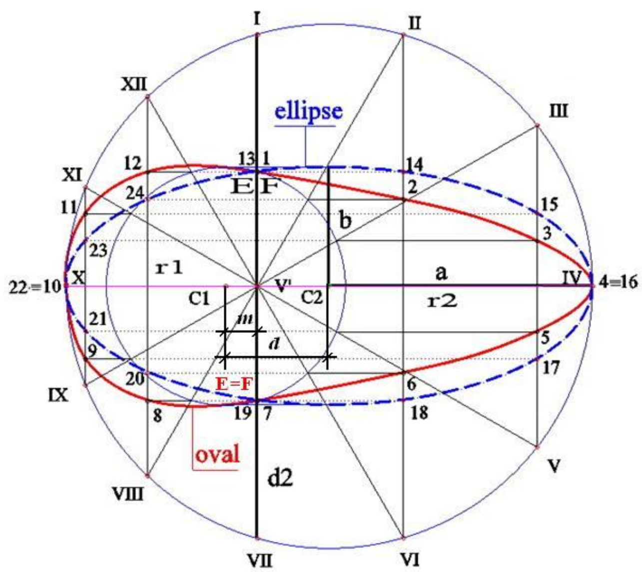


Figure 13*: Double Egg Curve Degenerated On Ellipse and An Oval Curve*

So, we showed by geometrical construction, that the double egg curve of 6-th order is degenerated onto an ellipse and Granville's egg, a fourth degree curve.

7. MATHEMATICAL GENERALISATION OF HÜGELSCHÄFFER'S CONSTRUCTION

Let us assign circles $c_1=c(C_1, R_1)$ and $c_2=c(C_2, R_2)$, with centers $C_1\ (0,0)$ and $C_2\ (d,0)$, and radii **$R1$** and **$R2$**, respectively. Next, let's set straight line $y=k(x-m)$ through the point $V'(m, 0)$, which intersects the circle c_1 in the point $P(x, z)$, and intersects the circle c_2 in the point $Q\ (x_1, z)$ as shown in the **figure 14**.The equation of the geometrical loci of points $T(x, y)$ is obtained by elimination of factors x_1, z and k from the system:

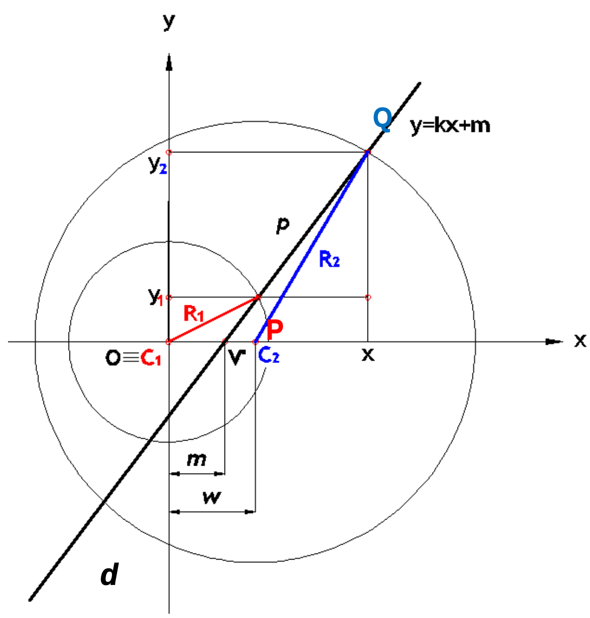


Figure 14: *Parameters That Figures In the Curve Equation*

$$x_1^2 + y^2 = R_1^2$$
$$y = k(x_1 - m)$$
$$(x-d)^2 + z^2 = R_2^2$$
$$z = k(x - m)$$

The requested elimination is accomplished in Maple, by using command:

eliminate$(\{(x_1)^2 + y^2 = (R1)^2,\ y=k(x_1-m),\ (x-d)^2+(z)^2=(R2)^2,\ z=k(x-m)\},\ \{x_1,z,k\});$

which is based on Groebner basis technique [2]. After additional simplification, in Maple we obtain a sixth order curve:

$$f(x,y) = M_{4,2}x^4y^2+M_{2,4}x^2y^4+M_{4,0}x^4+M_{3,2}x^3y^2+M_{0,4}y^4+ M_{3,0}x^3+M_{2,2}x^2y^2+M_{1,2}xy^2+M_{2,0}x^2+M_{0,2}y^2+M_{1,0}x+M_{0,0}=0$$

Whereat:

$$M_{4,2}=4m^2, \qquad M_{2,4}=4(d-m)^2,$$

$$M_{3,2}=-4m^3-12dm^2-4R_1^2m+4R_1^2d, \qquad M_{1,4}=4(d-m)(m^2+R_2^2-d^2),$$

$$M_{4,0}=(R_1^2-m^2)^2, \qquad \begin{aligned} M_{2,2}&=2m^4+8dm^3+(14d^2+2R_1^2-6R_2^2)m^2, \\ &+8dR_1^2m+R_1^2(2R_2^2-10d^2) \end{aligned}$$

$$M_{0,4}=(m^2+R_2^2-d^2)^2,$$

$$M_{3,0}=-4d(R_1^2-m^2)^2, \qquad M_{1,2}=\begin{array}{l} -4dm^4+4(R_2^2-d^2)m^3 \\ -4d(2d^2+R_1^2-2R_2^2)m^2 \\ +4R_1^2(R_2^2-d^2)m+8dR_1^2(d^2-R_2^2) \end{array},$$

$$M_{2,0}=-2(R_2^2-3d^2)^2(R_1^2-m^2)^2, \qquad M_{0,2}=\begin{array}{l} -2(R_2^2-d^2)^2(m^4+(R_1^2-R_2^2+d^2)m^2 \\ +R_1^2(R_2^2-d^2)) \end{array},$$

$$M_{1,0}=4d(R_2^2-d^2)(R_1^2-m^2),$$

$$M_{0,0}=(R_2^2-d^2)^2(R_1^2-m^2)^2.$$

Let us notice that if $\boldsymbol{m \neq 0}$ or $\boldsymbol{m \neq d}$ is fulfilled, then for the coefficients it is valid: $\boldsymbol{M_{4,2}=4m^2 \neq 0}$ **or** $\boldsymbol{M_{2,4}= 4(d-m)^2 \neq 0}$, i.e. observed curve is a **6-th order curve**. At the contrary, if $\boldsymbol{m=d=0}$, then the observed curve, based on the analytic forms of coefficients transits to a curve of fourth order:

$$R_1^4x^4 + R_2^4y^4 + 2R_1^2R_2^2x^2y^2 - 2R_1^4R_2^2x^2 - 2R_1^2R_2^2y^2 + R_1^4R_2^4 = 0$$

$$\Leftrightarrow \quad \left(\frac{x^2}{R_2^2} + \frac{y^2}{R_1^2} - 1\right)^2 = 0.$$

which determines a double ellipse in the observed plane.

8. THE CASE OF CURVE DEGENERATION

Let as assume that ***m≠0***, or ***m≠d***, then it is case of **6-th** order curve. So, if we observe ellipse *e*:

$$e(x,y) = \frac{(x-d)^2}{R_2^2} + \frac{y^2}{R_1^2} - 1 = 0$$

and determine the value of parameter ***m***, for which the point $M = (m, \sqrt{R_1^2 - m^2})$ of the observed curve belongs to ellipse, then:

$$e(m, \sqrt{R_1^2 - m^2}) = 0 \qquad \Leftrightarrow \qquad m = \frac{R_1 d}{R_1 \pm R_2}$$

is valid.

In this case, the initial curve ***f(x,y)=0*** decomposes onto union of an ellipse ***e(x,y)=0*** and a curve of 4-th order:

$$\begin{aligned} g(x,y) = -4d^2(R_1 \pm R_2)^2x^2y^2 + 4d(R_1 \pm R_2)(\mp R_2(R_1 \pm R_2)^2 + d^2(\pm R_2 + 2R_1))xy^2 \\ -(\mp R_2(R_1 \pm R_2)^2 + d^2(\pm R_2 + 2R_1))^2y^2 - R_1^2((R_1 \pm R_2)^2 - d^2)x^2 \\ + 2R_1^2 d((R_1 \pm R_2)^2 - d^2)x + R_1^2(R_2^2 - d^2)(((R_1 \pm R_2)^2 - d^2)) = 0. \end{aligned}$$

By application of affine transformation:

$$(x,y) \rightarrow (2(R_1 \pm R_2)x + 2R_1d^4 - 2R_1(R_1 \pm R_2)^2d^2, y)$$

We obtain transformation of the equation:

$$g(x,y) = 0 \rightarrow x^2y^2 + c^2(x-a)(x-b) = 0,$$

where:

$$a = \pm R_2(R_1 \pm R_2 - d)^2,$$
$$b = \pm R_2(R1 \pm R_2 + d)^2,$$
$$c = \frac{R_1((R_1 \pm R_2)^2 - d^2)}{2(R_1 \pm R_2)d}.$$

is true.

In this manner, we're reducing the equation on canonical equation of Granville's egg curve [3] (Torsten Sillke, Granville's egg - quartic (Granville 1929) $\boldsymbol{x^2 y^2 + c^2 (x-a)(x-b) = 0}$, with $\boldsymbol{0 < a < b}$).

Due to the condition $\boldsymbol{0 < a < b,}$ in the previous we only considered the case of positive sign $\boldsymbol{(+R_1)}$ and thus transformed curve gets visually an egg shape.

9. CONSLUSIONS

- In this paper we appointed a connection between the Hugelschaffer's construction of the egg curve, and the variation of the same construction, which gives a double egg curve.
- We used a ruled surface that has the double egg curve as the directrix curve, as the model for spatial interpretation of the generalized Hugelschaffer's construction.
- It is shown that such a surface can be degenerated to a double conoid, and we found the conditions of the degeneration.
- We proved that a conoid which has a Granville'e egg (a curve that can be obtained as a generalized Hugelschaffer's egg curve) as the directrix curve, has a set of planes that intersect it by the conics – ellipses.
- This paper may be a starting point for some further investigations, such as finding the exact plane(s) that intersects the conoid $\mathbf{K_2}$, with the Granville's egg as the directrix curve, by the circles.

ACKNOWLEDGEMENTS

The first author is supported by the project MNTRS No.16009. The second author is supported in part by the project MNTRS, Grant No. ON144020.

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