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COMPARISON OF DIFFERENT METHODS FOR VISCOELASTIC ANALYSIS OF COMPOSITE BEAMS

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Abstract:

The paper presents and compares four methods for visco-elastic analysis of steel-concrete composite beams. The method denoted as "exact" is based on the use of linear integro-differential operators and besides inevitable approximations of the rheological properties of constituent materials does not introduce other mathematical simplifications. The underlying assumption of the simplified method is that unknown deformations change linearly with the concrete creep function. In the paper, the results of the analysis of continuous composite beam obtained using the mentioned two methods are compared with results of widely used the Effective modulus method and its modified form proposed by Eurocode 4. Results have shown that the simplified method gives solutions closest to the “exact” analysis method.

Key words: viscoelastic analysis, composite beams, matrix stiffness method, linear integral operators

1. Introduction

Analysis of steel-concrete composite beams is a more challenging problem in comparison with the analysis of traditional steel or concrete beams because of different rheological properties of constituent materials: steel and concrete. Over the last several decades, a number of studies investigated long-term behavior of composite beams and different methods are proposed for analysis. This paper compares four analysis methods. The first method is previously developed the "exact" method that uses mathematical theory of linear integral operators [1-3]. Mandel [4] was the first one who used operators in the aging linear viscoelasticity and presented the integral relations using the linear integro-differential operators. Bazant and Huet [5] extended these operators to matrix and tensor integro-differential operators. Prof Lazic [4] was the first one who used linear integral operators for force based analysis of composite and prestressed beams. The “exact” method presented in [2, 3] uses the same operators but derives displacement based method for analysis of composite steel-concrete and prestressed beams. In this method, displacements are unknown and ultimate equations are nonhomogeneous integral equations that can be solved in closed form only for specific creep functions. For creep functions of the hereditary theory and the aging theory, under the assumption of constants concrete modulus of elasticity, these equations can be solved applying Laplace transformations.

The second method is a simplified method that follows the above described “exact” analysis method [1]. However, the solution procedure is simplified introducing the assumption that unknown displacements, in time, are linear functions of the concrete creep function. In this case, the nonhomogeneous integral equations transform into simple algebraic equations. Therefore, the method is more suitable for practical application. In addition, a high level of accuracy of the "exact" analysis method is preserved.

Besides these two methods, the paper compares the result with the well-known EM method and method proposed by Eurocode 4. A brief description of the four mentioned methods follows.

2. “Exact” analysis method

In the analysis method denoted as "exact", the basic unknowns are displacements, and the relations between the generalized element deformations and the generalized element forces are integral. However, using the mathematical theory of linear integral operators [1, 2, 6] it is shown that these basic relations can be presented in the same form as for the elastic homogeneous frame element, but using the operator stiffness matrices.

Basic assumptions of the “exact” method are the following. The Bernoulli’s hypothesis of plane sections is adopted and there is no slip at the steel-concrete interface. In general, cross-section consists of concrete, prestressing steel, steel section, and reinforcement. Concrete is considered a linear viscoelastic aging material. Prestressing steel has a relaxation property while steel and reinforcement behave as linear-elastic materials.

As mentioned above, the final system of equations can be written into the well-known form:

$$\left[\widehat{K}'\right]\left[q\right]=\left[S\right], \tag{1}$$

Where $\left[\widehat{K}'\right]$ is the operator stiffness matrix of the structure, $\left[q\right]$ is the vector of displacements and $\left[S\right]$ is the vector that includes external nodal forces and nodal forces due to element loads. It should be noted that the system of equations (1) represents the system of nonhomogeneous integral equations and $\left[\widehat{K}'\right]$ is the operator stiffness matrix. This system can be solved in the closed form only for some analytical forms of the concrete creep functions, i.e. Rate of Creep Method, Hereditary theory [4]. In other cases, the system can be solved numerically.

3. Simplified analysis method

In the simplified analysis method [1], we assume that generalized displacements q_{λ}, (*λ=1,2,…n; n* is the number of unknowns*)*, change linearly with the concrete creep function F^*, i.e.:

$$q_{\lambda}=q_{\lambda 0}1^*+\Delta q_{\lambda}\left(F^*-1^*\right), \tag{2}$$

where t_0 is the age of concrete when first stress and deformation appear, $q_{\lambda 0}=q_{\lambda 0}(t_0,t_0)$ are displacements at time t_0, Δq_{λ} are unknowns that should be determined, 1^* is the Heaviside step function. The unknowns Δq_{λ} are constants for each pair of time arguments (t,t_0). Bažant [7] in his work introduced the same assumption.

With this assumption at hands, it can be shown that integrals that appear in the element stiffness matrices can be written as linear combination of function F^* and three other functions. Details about these derivations can be found in [1]. Consequently, the ultimate system of nonhomogeneous integral equations transforms into the system of nonhomogeneous algebraic equations, with unknowns Δq_{λ}.

For the following numerical example, the results of the "exact" and simplified analysis methods are compared with the commonly applied Effective modulus method (EM method) and method proposed by the Eurocode 4 [8] design guide.

4. Effective modulus method and EC4 method

The Effective modulus method (EM) is based on the following algebraic relation between stress σ_c and strain ε_c for concrete:

$$\sigma_c(t) = E_{c,eff}\left(\varepsilon_c - \varepsilon_{cs}\right), \quad E_{c,eff} = \frac{E_{c0}}{1+\varphi_r}, \tag{3}$$

Where $E_{c,eff}$ is the effective elastic modulus of concrete, φ_r is the reduced creep coefficient, and ε_{cs} is the concrete shrinkage strain. Therefore, according to this method, the creep of concrete is taken into account through the reduction of the concrete modulus of elasticity. The analysis in time t is the same as analysis in time t_0, with the difference that effective modulus should be used instead of the initial elastic modulus of concrete E_{c0}. Because of its simplicity, the analysis is very widely used in practice, and, with slight modifications, is adopted in Eurocode 4.

In Eurocode 4, the algebraic relation of the EM method (equation 3) is adopted, but with the effective elasticity modulus of concrete $E_{c,eff}$ defined as:

$$E_{c,eff} = \frac{E_{cm}}{1+\psi_L \varphi_r}, \tag{4}$$

where E_{cm} is the secant modulus of elasticity of the concrete for short-term loading, and ψ_L is the creep multiplier that depends on the type of loading and has the following values: *1.1* for permanent loading, *0.55* for effects of shrinkage and *1.50* for prestressing by imposed deformations. The expression from equation (4) is based on the expression proposed by Fritz [9].

5. Numerical example

In order to compare the results of four discussed analysis methods, the symmetric continuous beam from Figure 1 is studied. The beam is loaded with uniformly distributed loading q and concentrated forces P that act at points A, B, A' and B'. Geometric and material data of the considered girder are given in Figure 1, while geometrical properties of cross-sections 1-1 and 2-2 are given in Table 1.

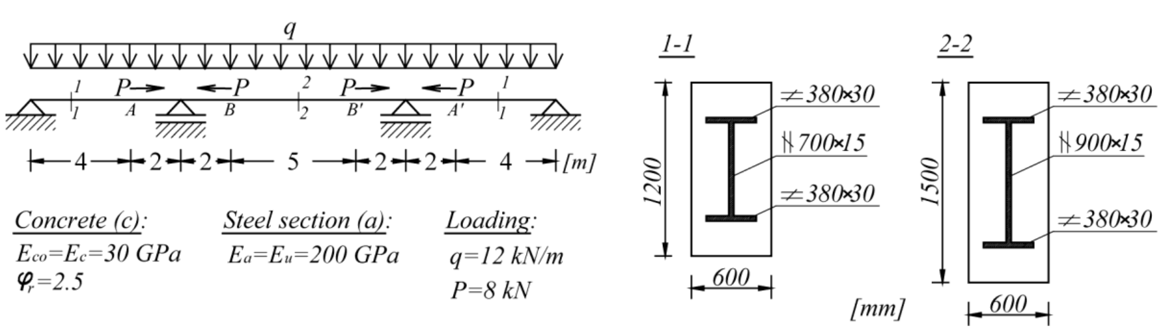

Fig. 1. Continuous composite beam

	Section 1-1	Section 2-2
A_i (m^2)	0.136305	0.165855

J_i (m^2)	$1.591 \cdot 10^{-2}$	$3.028 \cdot 10^{-2}$

Table 1. Geometrical properties of cross-sections *1-1* and *2-2*

The beam is symmetric and only half of it can be analyzed. Therefore, there are two unknown generalized displacements: horizontal displacement u and rotation φ (Figure 2).

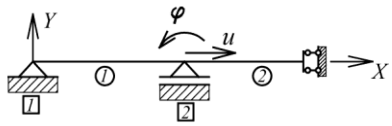

Fig. 2. Symmetric part of the composite beam and unknown generalized displacements

These two unknowns are solutions of the following system of two integral equations:

$$\begin{aligned} &(\widehat{N}'_{gk} + \widehat{N}'_{is})u = \frac{P}{9}1^* \\ &(\widehat{D}'_{gk} + \widehat{E}'_{is})\varphi = q\frac{l_1^2}{8}1^* - q\frac{l_2^2}{12}1^* \end{aligned}, \qquad (5)$$

Where $\widehat{N}'_{gk}$ is the element (*3,3*) of the operator stiffness matrices for the element *1* (l_1*=6m*) and $\widehat{N}'_{is}$ is the element (*1,1*) of the operator stiffness matrices for the element *2*, of length l_2*=9m*. Similarly, $\widehat{D}'_{gk}$ is the element (*5,5*) of the operator stiffness matrix of the element *1* and $\widehat{E}'_{is}$ is the element (*3,3*) of the operator stiffness matrix of the element *2*. In order to obtain the solution of the "exact" method, the concrete creep function is adopted in accordance with the creep function of the aging theory with the constant concrete modulus of elasticity. In this case, the solution of the “exact” method can be found applying the Laplace transformations on the system of equations (5). In this case, the concrete creep function is:

$$F^* = 1^* + \varphi_r\,, \qquad (6)$$

and the corresponding concrete relaxation function obtains a solution to an integral equation [1,2] is:

$$R^* = e^{-\varphi_r}\,, \qquad (7)$$

where φ_r is the reduced concrete creep coefficient.

The solution to the "simplified" method obtains a solution to the following system of equations:

$$\begin{aligned} &(\widehat{N}'_{gk} + \widehat{N}'_{is})(u_0 1^* + \Delta u(F^* - 1)) = \frac{P}{9}1^* \\ &(\widehat{D}'_{gk} + \widehat{E}'_{is})(\varphi_0 1^* + \Delta\varphi(F^* - 1)) = q\frac{l_1^2}{8}1^* - q\frac{l_2^2}{12}1^* \end{aligned}, \qquad (8)$$

that transforms into the algebraic system of equations.

In the solution given in Figure 3 and Table 2, the following function for the reduced concrete creep coefficient φ_r is adopted (t_0*=28* dana):

$$\varphi_r(t,t_0) = 2.5\left(\frac{t-t_0}{863+t-t_0}\right)^{0.3}, \tag{9}$$

This function is determined in accordance with the EC2 [10]. Figure 3 shows the solution for the unknowns *u* and φ over time. Solutions for the horizontal force at support *1* (see Figure 2) and for the bending moment at support *2*, for $t=t_0$ and $t\rightarrow\infty$ are given in Table 2.

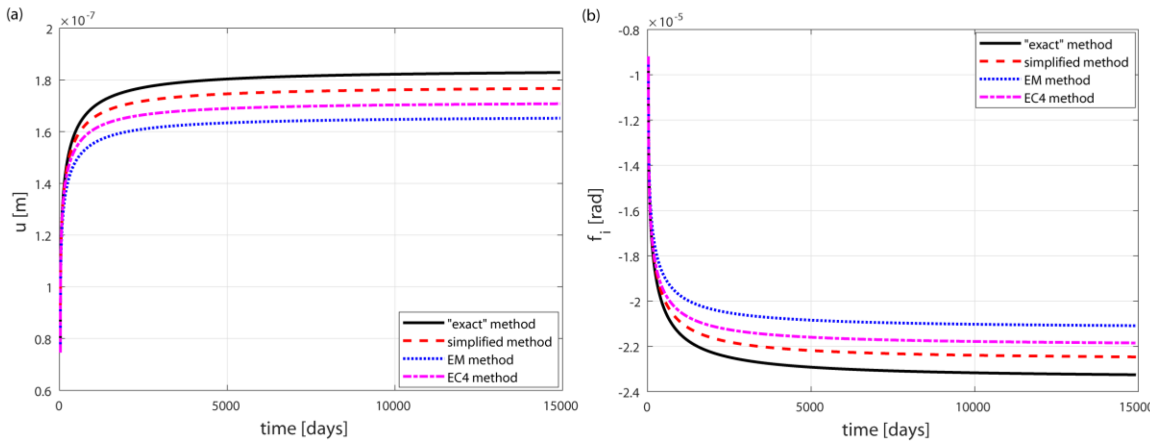

Fig. 3. Comparison of generalized displacements *u* and φ over time for four analysis methods

Results	$t=t_0$ (all methods)	"exact"	simplified	EM	EC4
H_1 [kN]	-3.0056	-3.0157	-3.0151	-3.0145	-3.0145
M_5 [kNm]	68.6266	68.9501	68.9152	68.9002	68.9117

Table 2. The solution for horizontal force H_1 and moment M_5 at time t_0 and $t\rightarrow\infty$

As can be seen, the simplified analysis method gives solution closest to the solution of the "exact" analysis method, while, as expected, results of the EM method are the furthest from the "exact" solution. Results of the EC4 method have improved accuracy in comparison with the EM method.

6. Conclusions

The paper briefly describes the following four analysis methods suitable for visco-elastic analysis of composite steel-concrete beams: "exact" method, simplified method, EM method, and EC4 method. The results obtained by these methods are compared on one numerical example and it was shown that the simplified method gives solution very close to the solution of the "exact" analysis method. This method is much simpler than the "exact" method since requiring the system of nonhomogeneous algebraic equations to be solved instead of a system of nonhomogeneous integral equations. From this reason, it is more suitable for practical application. On the other side, EC4 method is slightly more accurate than the EM method but less accurate than the simplified analysis method.

Acknowledgments

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