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PROBABILISTIČKA ANALIZA POČETNOG LOMA KOMPOZITNIH LAMINATA

Rezime:

U ovom radu sprovedena je analiza inicijacije loma laminatne kompozitne ploče, koristeći slojeviti konačni element zasnovan na Redijevoj slojevitoj teoriji ploča. Sračunato je granično opterećenje na osnovu nekoliko različitih kriterijuma loma koji su implementirani u proračunski model. U radu je izvršena probabilistička analiza loma, u kojoj su materijalne karakteristike i čvrstoća lamine usvojene kao slučajno promenljive veličine. Cilj analize, zasnovane na Monte Karlo metodi, je sračunavanje statističkih parametara graničnog opterećenja kompozitnog laminata. Na osnovu sprovedene analize izvedeni su odgovarajući zaključci.

Ključne reči: početni lom, slojevita teorija, probabilistička analiza, laminat, kompozit

PROBABILISTIC FIRST-PLY FAILURE ANALYSIS OF COMPOSITE LAMINATES

Summary:

The paper deals with the first-ply failure analysis of laminated composite plates using a layered finite element model based on the Full layerwise theory of Reddy (FLWT). Several macroscopic failure criteria have been incorporated into computational model and used to compute the limit load. Since almost no parameter in failure analysis is deterministic, probability prediction of limit load, considering the lamina elastic material properties and strength as random variables is developed. The aim of analysis, based on Monte Carlo Method, is to calculate statistical parameters of the limit load of composite laminate. Appropriate conclusions have been derived based on the conducted analysis.

Key words: first-ply failure, full layerwise theory, probabilistic analysis, laminate, composite

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1. INTRODUCTION

Due to their outstanding strength and stiffness, low maintenance costs and corrosion resistance properties, laminated composite materials have been widely used in the construction of mechanical, aerospace, marine and automotive structures which, in general, require high reliability levels. Failure of a structural element occurs when it cannot perform its intended function. Due to their complex kinematics, laminar composite may fail by several scenarios: fiber yielding, matrix yielding, fiber breakage, delamination of layer, or fracture. It is difficult to incorporate these modes of failure into design of composite structures [1]. A simpler way is to use empirical criteria, similar to the failure criteria used in steel design, but customized for composites. The most common criteria to predict failure of a single ply, as discussed by Soni [2], are maximum stress criterion, maximum strain criterion and quadratic polynomial criteria such as the Tsai-Wu [3], Hoffman [4] and the Tsai-Hill criteria [5]. The maximum stress or strain criteria are defined as those having no interactions between the stress or strain components and they are therefore called *independent failure mode criteria*. The failure surfaces for these criteria are rectangular in stress and strain space, respectively. Quadratic polynomial criteria involve interactions between stress and strain components and they are in fact polynomials based on curve-fitting data from composite material tests. The failure surface for the quadratic polynomial criteria is of ellipsoidal shape.

Determination of first-ply failure (FPF) load is essential in understanding the failure process as well as the reliability of composite laminates. The first series of analytical solutions for the FPF load were presented by Turvey [6, 7], for both symmetric and anti-symmetric simply supported composite laminates based on the classical lamination theory [8]. Reddy and his associates [9, 10] used the finite element method, which is formulated on the basis of the first order shear deformation theory [11], to calculate the linear and nonlinear FPF load of composite laminates subjected to transverse and in-plane (tensile) loading. Kam et al. investigated the FPF probability considering the elastic properties of the material, the fiber orientation and the lamina thickness as random variables [12, 13].

In the case when equivalent-single-layer (ESL) theories are used, interlaminar stress fields are not represented accurately. However, the interlaminar stress fields play an important role on the prediction of FPF load in thick composite laminates subjected to transverse loads [14], so the use of refined laminate theories is recommended [15]. The computational model based on the Full layerwise plate theory (FLWT) [16] is applied, because it is capable to represent the complete 3D stress state of composite laminates with savings in computational time.

In the paper, a layered finite element model based on the FLWT [17] is presented for FPF analysis of laminated composite plates. The model is implemented using object-oriented MATLAB [18] code, while the GUI for pre- and post-processing is developed using GiD [19]. Several quadratic polynomial criteria have been incorporated into computational model and used to compute the FPF load. The presented approach is validated against the available data in the literature. Because of significance of failure criteria in a structural design, for a better confidence in the predicted strength of the laminar composite, it is of great importance to undertake probabilistic studies. Probability prediction of FPF load, considering the lamina elastic material properties and strength as random variables is conducted. Analysis based on Monte Carlo simulation has been performed and appropriate conclusions have been derived.

2. A REVIEW OF POLYNOMIAL FAILURE CRITERIA

The most general polynomial failure criterion for composite materials is the tensor polynomial criterion proposed by Tsai [20]. All other polynomial failure criteria are degenerate cases of this criterion. In index notation the tensor polynomial failure criterion is expressed as:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_i + F_{ijk}\sigma_i\sigma_i\sigma_k + ... \geq 1 \tag{1}$$

where:

σ_i are the stress tensor components in the material coordinates,

F_i, F_{ij} and F_{ijk} are the components of the strength tensors. All components are referred to the material principal axes 1, 2 and 3 of the single lamina.

One problem in applying the tensor polynomial criterion is the determination of the parameter F_{12}. The value of F_{12} is not unique and can vary from a negative value to a positive one. Different quadratic polynomial criteria differ in the way polynomial constants are determined [20].

1.1. THE TSAI-WU CRITERION

The Tsai-Wu criterion is given by:

$$\begin{aligned} &F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 + \\ &+F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \geq 1 \end{aligned} \tag{2}$$

where:

$$\begin{aligned} &F_1 = \frac{1}{X_T} - \frac{1}{X_C}, \quad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C}, \quad F_3 = \frac{1}{Z_T} - \frac{1}{Z_C}, \quad F_{11} = \frac{1}{X_T X_C}, \quad F_{22} = \frac{1}{Y_T Y_C}, \\ &F_{33} = \frac{1}{Z_T Z_C}, \quad F_{44} = \frac{1}{R^2}, \quad F_{55} = \frac{1}{S^2}, \quad F_{66} = \frac{1}{T^2}, \quad F_{12} = -\frac{1}{2}\sqrt{\frac{1}{X_T X_C Y_T Y_C}}, \\ &F_{13} = -\frac{1}{2}\sqrt{\frac{1}{X_T X_C Z_T Z_C}}, \quad F_{23} = -\frac{1}{2}\sqrt{\frac{1}{Y_T Y_C Z_T Z_C}}, \end{aligned} \tag{3}$$

where:

X, *Y* and *Z* are the longitudinal strengths in the 1, 2 and 3 directions,

subscripts *T* and *C* represent tensile and compressive quantities,

R, *S* and *T* represent shear strengths in the 23, 13 and 12 planes, respectively.

Directions 1, 2 and 3 represent: the fiber direction, the direction transverse to the fiber but in the plane of the laminate, and the direction transverse to the fiber and to the laminate, respectively. The coefficients F_1, F_2, F_3 correspond to the linear stress terms and F_{11}, F_{12}, F_{33}, F_{44}, F_{55}, F_{66} correspond to the quadratic stress terms. F_{12}, F_{13}, F_{23} are the coefficients which take into account the interaction effect of various normal stress components.

Terms associated with σ_4, σ_5 and σ_6 which are F_4, F_5, and F_6 are taken to be zero, since shear strengths are the same for positive and negative shear stress. It is also assumed that there is no interaction between shear stresses and normal stresses, thus F_{16}, F_{26} etc. become zero.

1.2. HOFFMAN'S CRITERION

Hoffman's criterion is a special case of the tensor polynomial criterion for the following choice of the parameters F_i and F_{ij}:

$$F_{12} = -\frac{1}{2}\left(\frac{1}{X_T X_C} + \frac{1}{Y_T Y_C} - \frac{1}{Z_T Z_C}\right), \quad F_{13} = -\frac{1}{2}\left(\frac{1}{X_T X_C} + \frac{1}{Z_T Z_C} - \frac{1}{Y_T Y_C}\right),$$
$$F_{23} = -\frac{1}{2}\left(\frac{1}{Y_T Y_C} + \frac{1}{Z_T Z_C} - \frac{1}{X_T X_C}\right) \tag{4}$$

Other strength parameters are the same as in Tsai-Wu criterion explained before.

1.3. THE TSAI-HILL CRITERION

In Hill's criterion, the stress terms do not appear as linear terms, therefore, the F_1, F_2 and F_3 terms are zero. The values of X, Y, Z are taken as either X_T, Y_T, Z_T, or as X_C, Y_C, Z_C, depending upon the sign of σ_1, σ_2 and σ_3, respectively. Strength tensors for this criterion are:

$$F_{11} = \frac{1}{X^2}, \quad F_{22} = \frac{1}{Y^2}, \quad F_{33} = \frac{1}{Z^2}, \quad F_{12} = -\frac{1}{2}\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right),$$
$$F_{13} = -\frac{1}{2}\left(\frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}\right), \quad F_{23} = -\frac{1}{2}\left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right). \tag{5}$$

3. THEORETICAL BASIS OF THE FINITE ELEMENT MODEL BASED ON THE FULL LAYERWISE PLATE THEORY

In the paper, a laminated composite plate made of n orthotropic layers is considered (Figure 1, left). The total plate thickness is denoted as h, while the thickness of the k^{th} lamina is denoted as h_k. The plate is supported along the portion Γ_u of the boundary Γ and loaded with loadings $q_t(x,y)$ and $q_b(x,y)$ acting to either top or the bottom surface of the plate (S_t or S_b).

In the FLWT, piece-wise linear variation of all three displacement components is imposed, leading to the 3-D stress description of all material layers. The displacement field (u, v, w) of an arbitrary point (x,y,z) of the laminate is given as:

$$u(x,y,z) = \sum_{I=1}^{N} U^I(x,y)\Phi^I(z), \quad v(x,y,z) = \sum_{I=1}^{N} V^I(x,y)\Phi^I(z),$$
$$w(x,y,z) = \sum_{I=1}^{N} W^I(x,y)\Phi^I(z) \tag{6}$$

In Eq. 6, N is the number of numerical layers, $U^I(x,y)$, $V^I(x,y)$ and $W^I(x,y)$ are the displacement components in the I^{th} numerical layer of the plate in directions x, y and z, respectively, $\Phi^I(z)$ are selected to be linear layerwise continuous functions of the z-coordinate, and they are given in [11].

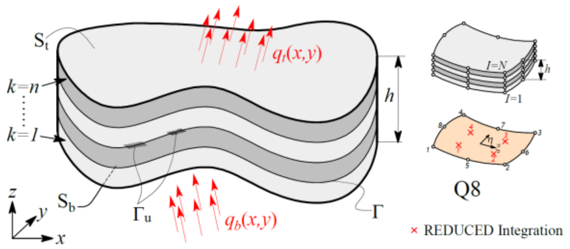

Figure 1 – left: Laminated composite plate with n material layers and N numerical interfaces; right: Quadratic serendipity Q8 layered element with linear layerwise interpolation through the thickness and corresponding Gauss quadrature points for the reduced integration.

The linear strain field associated with the previously shown displacement field can be found in [17]. To reduce the 3D model to the 2D one, the z-coordinate is eliminated by the explicit integration of stress components multiplied with the corresponding functions $\Phi^I(z)$, introducing the constitutive relations of the laminate which can be found in [17]. The system of 3N Euler-Lagrange governing equations of motion of the FLWT can then be derived using the Hamilton's principle (strong form), and they are given in [17].

The finite element discretization is derived by introducing an assumed interpolation of the displacement field into the weak form of the FLWT. All displacement components are interpolated as:

$$U^I(x,y)=\sum_{j=1}^{m}U_j^I\psi_j(x,y),\quad V^I(x,y)=\sum_{j=1}^{m}V_j^I\psi_j(x,y),$$
$$W^I(x,y)=\sum_{j=1}^{m}W_j^I\psi_j(x,y) \tag{7}$$

where:

m is the number of nodes per 2-D element,

U_j^I, V_j^I, W_j^I are the nodal values of displacements U^I, V^I and W^I in the j^{th} element node representing the behaviour of the laminated composite plate in the I^{th} numerical interface,

$\psi_j(x,y)$ are 2D Lagrange interpolation polynomials associated with the j^{th} element node.

The matrix form of the FE model is obtained as:

$$\left[K^{IJ}\right]\left\{\Delta^I\right\}=\left\{F^I\right\} \tag{8}$$

where:

$[K^{IJ}]$ is the element stiffness matrix,

$\{\Delta^I\}$ is the element displacement vector,

$\{F^J\}$ is the element force vector.

$[K^{IJ}]$ is computed from the weak form using 2D Gauss-Legendre quadrature quadrilateral domains. The layered finite elements require only C0 continuity of the generalized displacements along element boundaries, because only translational displacement components are adopted as the nodal degrees of freedom. Quadratic serendipity Q8 layered quadrilateral element has been considered (Figure 1, right). To avoid shear locking, reduced integration is used when evaluating the element stiffness matrix.

After the derivation of characteristic element matrices, the assembly procedure is done in a usual manner. Once the nodal displacements are obtained the stresses are evaluated from the well-known lamina 3D constitutive equation in the Gauss points at the top (t) and bottom (b) interfaces of the considered lamina and they are given in [17]. The stresses are calculated both in the laminate (*xyz*) and the local lamina (123) coordinate systems. This is crucial in the failure analysis, since the failure criteria described earlier require the stresses in the lamina coordinate system.

Since the interlaminar stresses calculated in this way does not satisfy continuous distribution through the laminate thickness, they are post-processed by assuming the quadratic distribution within each layer for every stress component. The procedure for post-computation stresses is in detail described [16].

4. PROCEDURE FOR FIRST-PLY FAILURE

The first-ply failure analysis is based on the assumption that a given ply would fail if the failure index (left-hand side of equation (2)) at any point within the ply reaches a value of 1. The procedure of first-ply failure load calculation requires solving the stress problem for an initial load. The lamina stresses in the lamina coordinate system are then used in a chosen failure criterion to calculate the maximum failure index Φ and check whether the laminate has failed or not. The laminate failure is checked by comparing the absolute of (Φ-1) against δ, where δ is a predetermined value of maximum tolerable error of the failure index (1% in the present study). If the absolute value of (Φ-1) is less than δ, then the first ply within the laminate has failed. If not, the initial load is increased or decreased by a predetermined percentage (10% in the present study) of the initial load and the procedure is repeated until the laminate fails.

The maximum failure index is determined by carrying out a sequential search in the following way: the failure index of the considered element is calculated in every Gauss point at the bottom and top interfaces of each lamina and the maximum value is stored as well as the element number, Gaussian point number, lamina number and interface location. The search is continued until all layered finite elements are searched for the maximum failure index.

5. NUMERICAL EXAMPLES

First example deals with the first-ply analysis of rectangular clamped laminated composite plate of dimensions $a \times b$ = 0.2286 × 0.127m. The plate consists of 3 orthotropic material layers in angle-ply stacking sequence (45/-45/45) and it is loaded by transverse distribution of constant pressure at the top surface of the plate, $q_t(x,y) = q_t$. Each layer of thickness 0.127mm is modeled as a T300/5208 graphite/epoxy unidirectional lamina, with the material properties given in Table 1.

Table 1 – Material properties of T300/5208 graphite/epoxy material

Properties	Values	Properties	Values
E_1	132.5 GPa	X_T	1515 MPa
$E_2 = E_3$	10.8 GPa	X_C	1697 MPa
$G_{12} = G_{13}$	5.7 GPa	$Y_T = Z_T$	43.8 MPa
G_{23}	3.4 GPa	$Y_C = Z_C$	43.8 MPa
$\nu_{12} = \nu_{13}$	0.24	R	67.6 MPa
ν_{23}	0.49	$S = T$	86.9 MPa

The first-ply load and failure location are evaluated using Tsai-Wu, Hoffman and Tsai-Hill failure criteria, respectively. Dimensionless first-ply failure load has been considered:

$$\bar{q}_t = \frac{q_t}{E_2}\left(\frac{a}{h}\right)^4 \tag{9}$$

The present study is performed using 9x5 mesh of quadratic Q8 layered quadrilateral element with reduced integration, Figure 2. A comparison is made against the results obtained in [10], using first-order shear deformation theory. The results are elaborated in Table 2. The average relative differences Δ = (result – reference) / reference [%] are also calculated and given in Table 2:

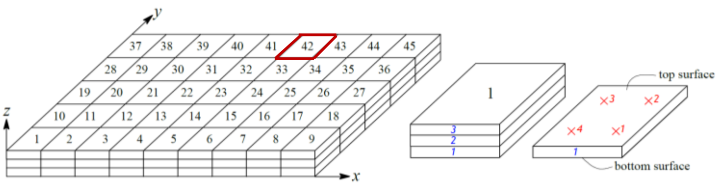

Figure 2: Detail of finite element mesh for laminated composite plate (9 x 5 mesh of quadratic Q8 layered quadrilateral elements).

The results presented in Table 2 indicate that the proposed model predict almost the same FPF load as Mindlin plate model of the angle-ply laminated composite plate [10]. Failure location is the same for all failure criteria and both numerical models Failure occurs in 3rd Gauss point of the 42nd element, at the bottom surface of the 1st layer (see Figure 2).

After the proposed model has been validated and its accuracy was demonstrated through the comparison against the FSDT, probabilistic study accounting for the uncertain nature of mechanical parameters has been conducted. Material parameters considered as uncertain are elastic moduli E_1, E_2 and strengths X_T, X_C, Y_T, Y_C and T. All uncertain parameters follow normal distribution, with the mean values given in Table 1, while the coefficient of variation (COV) is assumed to be 10%. Remaining material parameters are considered as deterministic. Latin Hypercube sampling technique has been applied to generate input values of material variables, used to perform 3,000 Monte Carlo simulations for each considered failure criterion.

Table 2 – Dimensionless first-ply failure load considering different failure criteria and different numerical models

Source	Failure criteria		
	Tsai-Wu	Hoffman	Tsai-Hill
Present	80206.6	72240.6	72947.9
FSDT [10]	79809.2	75597.4	76509.7
Δ [%]	0.50	4.44	4.66

The results in terms of matching distributions, mean values and COVs of FPF load are elaborated in Table 3. Corresponding probability density functions (PDF) are presented in Figure 3. Normal distributed input parameters resulted in three different distribution types of the FPF load, for three failure criteria, respectively. Lognormal, gamma and Gumbel distributions correspond to the Tsai-Wu, Hoffman's and Tsai-Hill failure criteria, respectively. Difference between corresponding COVs is significant. FPF load calculated by Tsai-Wu criterion showed the smallest variation compared to other two criteria. With COVs of 31.1% and 25.3%, Hoffman's and Tsai-Hill criteria revealed great unreliability in calculating FPF load of the out-of-plane loaded laminated composite plate.

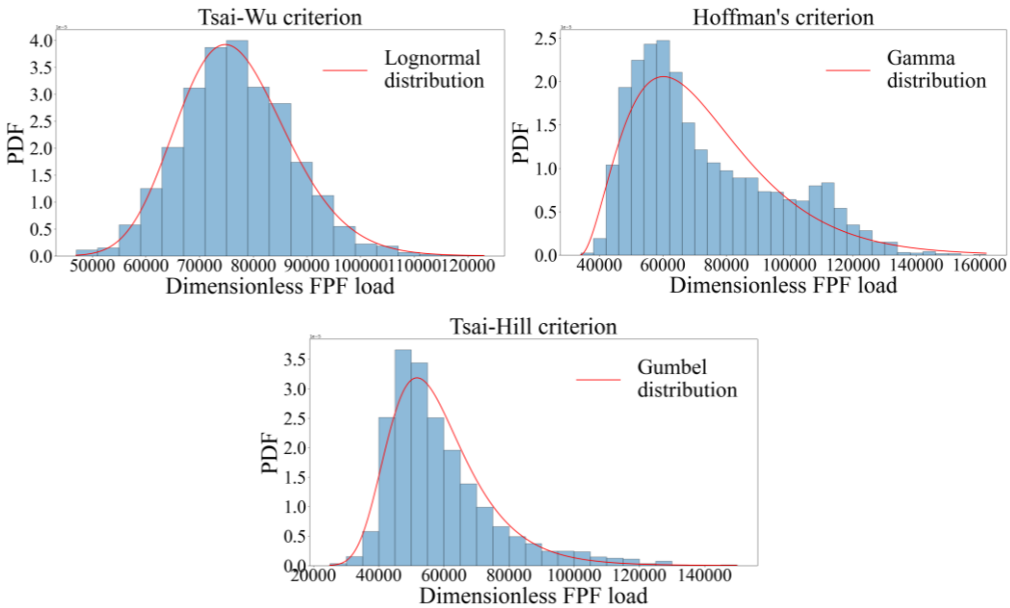

Figure 3 – FPF load histograms and distributions considering different failure criteria

Table 3 – Distribution types, dimensionless mean values and COVs of first-ply failure load of laminated composite plate, considering different failure criteria

Failure criteria	Distribution type	Mean value	COV [%]
Tsai-Wu	Lognormal	76867.6	13.4
Hoffman	Gamma	73357.9	31.1
Tsai-Hill	Gumbel	58449.5	25.3

6. CONCLUSIONS

In the paper, a layered finite element model based on the FLWT is re-called and used for the FPF analysis of laminated composite plate. The computational model is implemented using the original object-oriented MATLAB code, while the GUI for pre- and post-processing is developed using GiD. Several quadratic polynomial criteria, such as the Tsai-Wu, Hoffman's and the Tsai-Hill's, have been incorporated into the computational model and used to compute the FPF load. The presented approach is validated against the available numerical data in the literature, confirming the high accuracy of the presented procedure.

Normal distributions and the COVs of 10% for seven considered material parameters resulted in three different distributions of FPF load for three failure criteria. The FPF load calculated by Tsai-Wu criterion showed the greatest mean value and the lowest COV. Hoffman's and Tsai-Hill criteria showed smaller mean values. Comparing only mean values of FPF load determined from these three criteria, Tsai-Hill criterion could be considered as the most conservative when composite laminated plate is designed. However, the great COV of the corresponding distribution indicates that the calculated limit load is less reliable compared to the least conservative limit load calculated by Tsai-Wu criterion. Significant dissipation of the results when Hoffman's and Tsai-Hill criteria are used causes high unreliability in predicting FPF load. More experimental results in terms of FPF load are required so that the most convenient failure criterion for composite laminated plates design is determined.

Future work includes the implementation of some failure criteria for cross-laminated timber CLT panels in the presented framework. The progressive failure analysis will be conducted using the above method.

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