PREGLED SAVREMENIH SEIZMIČKIH ANALIZA I NAČINA UVOĐENJA PRIGUŠENJA U NJIMA

AN OVERVIEW OF MODERN SEISMIC ANALYSES WITH DIFFERENT WAYS OF DAMPING INTRODUCTION

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PREGLEDNI RAD
REVIEW PAPER
UDK:624.042.7
doi:10.5937/grmk1701003C

1 UVOD

U poslednje dve decenije, razvoj metoda za analizu konstrukcija u uslovima dejstva zemljotresa doživeo je naglu ekspanziju. Formiran je niz mogućnosti za rešavanje uobičajenih i kompleksnih problema, kako u svakodnevnoj inženjerskoj praksi, tako i u naučnim istraživanjima. Međutim, ekspanzijom velikog broja ovih metoda pojavio i se niz pitanja među kojima su: *koju metodu, kada i za koji tip konstrukcije je treba primeniti*? Na ova pitanja je delimično dat odgovor u naučnim publikacijama, ali postoje još mnoga pitanja na koja treba odgovoriti putem obimnih istraživanja i komparativnih studija, kako bi se sprovela sistematizacija, definisali algoritmi i dala uputstva za izbor optimalnog tipa metode za analizu konstrukcija u uslovima dejstva zemljotresa. U EN 1998-1:2004 [23] date su samo osnovne preporuke, dok su u propisima ATC 40 [6], FEMA 440 [26], FEMA 750P [27], FEMA P-58-1 [28] i FEMA P-58-2 [29] data dosta detaljnija uputstva kako i gde primeniti koji tip metode analize konstrukcija u uslovima dejstva zemljotresa.

Razvojem savremene opreme, laboratorija za testiranje elemenata, konstruktivnih celina, modela i *in-situ* testiranja realnih konstrukcija omogućeno je kvalitetnije sagledavanje ponašanja i povećan je nivo sigurnosti novoprojektovanih konstrukcija na dejstvo zemljotresa. S druge strane, razvoj savremenih numeričkih metoda i implementacija u softverska rešenja, uz podršku hardver-

1 INTRODUCTION

In the last two decades, development of methods for analysis of structures exposed to earthquake actions saw a rapid expansion. An array of alternatives for solving common and complex problems was formed, both in everyday engineering practice, and in scientific research. However, the expansion of a large number of methods raised a number of questions such as: *Which method, when and for what type of structure should be implemented?* These queries are answered in part through scientific publications, but there is still a large number of questions which should be answered through extensive research and comparative studies, in order to conduct systematization, define algorithms and provide instructions for choice of an optimal type of a method for analysis of structures exposed to earthquake actions. In EN 1998-1:2004 [23] only basic recommendations were given, while in the regulations ATC 40 [6], FEMA 440 [26], FEMA 750P [27], FEMA P-58-1 [28] and FEMA P-58-2 [29] far more detailed instructions were provided of how, where and which type of method should be implemented.

Improvement of contemporary equipment, laboratories for element testing, structural parts, models and *in-situ* testing of actual structures facilitated a better quality of behaviour analysis and safety level of newly designed structures to earthquake actions was increased. On the other hand, development of contemporary numerical

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skih resursa čiji se kapacitet konstantno povećava, omogućava simulaciju ponašanja konstrukcija na veoma visokom nivou kvaliteta. Generalno se može konstatovati da je razvoj metoda analize konstrukcija za uslove dejstva zemljotresa u direktnoj korelaciji s nizom faktora, od kojih se izdvajaju: razvoj i unapređivanje instrumenata za kontinualni monitoring dejstva zemljotresa u realnom vremenu, arhiviranje, digitalizacija i razvoj baza podataka zemljotresa koje su dostupne putem interneta, razvoj metoda za obradu i procesiranje seizmičkih signala, unapređivanje eksperimentalnih istraživanja na modelima i realnim konstrukcijama, razvoj računarske mehanike i numeričkih metoda, razvoj tehnika paralelnog procesiranja u visokosofisticiranim naučno-istraživačkim centrima, primena novih metoda i materijala u sanaciji konstrukcija, razvoj novih materijala za izgradnju konstrukcija, unapređivanje postojećih i razvoj novih konstruktivnih sistema, razvoj hibridnih metoda analize konstrukcija, uvođenje multidisciplinarnosti u razmatranje problema, razmena iskustava na globalnom nivou putem predavanja, skupova, kongresa, radionica i publikacija. Najkompleksnija i najobimnija istraživanja sprovode se u visokosofisticiranim naučnim centrima, od kojih se izdvajaju vodeći američki centri: *Earthquake Engineering Research Institute* (EERI), *Mid-America Earthquake Center* (MAE), *Multidisciplinary Center for Earthquake Engineering Research* (MCEER), *Pacific Earthquake Engineering Research Center* (PEER), *The John A. Blume Earthquake Engineering Center*, *California Institute of Technology* (CALTECH), *Network for Earthquake Engineering Simulations* (NEES) i tako dalje. U ovim centrima se razvijaju nova teorijska razmatranja i numeričke analize, sprovode eksperimentalna istraživanja i hibridne simulacije, pri čemu se studioznim pristupom mogu pouzdano proceniti, dodatno unaprediti ili odbaciti postojeće ili čak razviti nove metode analiza konstrukcija za uslove dejstva zemljotresa. S obzirom na to što je u poslednje dve decenije težište istraživanja u oblasti zemljotresnog inženjerstva na analizi performansi konstrukcija prema *Performance-Based Earthquake Engineering* (PBEE) metodologiji, to je veliki broj analiza i razvijen u okviru ove metodologije.

Proračun konstrukcija inženjerskih i arhitektonskih objekata sprovodi se u nekoliko faza, od analize ključnih parametara preko modeliranja konstrukcije, pa sve do selekcije analiza kojima se određuju svojstveni oblici, periodi vibracija, sile u presecima štapova i deformacije. Prilikom kreiranja numeričkog modela konstrukcije definišu se geometrijske i materijalne karakteristike štapova, opterećenje i prigušenje sistema. Na osnovu geometrijskih i materijalnih karakteristika štapova i opterećenja formiraju se matrice krutosti i masa štapova, a zatim i matrice krutosti i masa kompletnog sistema. Proračun geometrijskih karakteristika štapa određuje se iz dimenzija i oblika poprečnog preseka štapa i promene geometrije duž štapa, dok se selekcija materijalnih karakteristika štapa sprovodi prema mehaničkim karakteristikama materijala od koga je formiran štap. Proračun masa na sistemu sprovodi se konvertovanjem opterećenja u mase ili direktnim apliciranjem masa na sistemu. Selekcija parametra prigušenja sistema sprovodi se uzimajući u obzir tip materijala od koga je formiran štap, jedan deo ili kompletna konstrukcija. Standardni postupak uvođenja prigušenja u sistemu zasniva se

methods and their implementation in software solutions supported by hardware resources whose capacity is continuously increasing, facilitates high quality simulation of structural behaviour. In general, it can be concluded that the development of methods for analysis of structures exposed to earthquake actions is directly correlated to a number of factors, the principal ones being: development and improvement of instruments for continuous monitoring of earthquake actions in real time, archiving, digitalization and development of earthquake data bases which are accessible on the internet, development of methods for processing seismic signals, improvement of experimental research on models and actual structures, development of computer mechanics and numerical methods, development of parallel processing techniques in highly sophisticated scientific-research centres, implementation of new methods and materials in rehabilitation of structures, improvement of the existing and development of new structural systems, development of hybrid methods of structural analysis, introduction of multidisciplinary approach in problem analysis, exchange of experiences on a global level through lectures, meetings, congresses, workshops and publications. The most complex and most extensive research is conducted in highly sophisticated scientific centres, the following centres being the leading ones: *Earthquake Engineering Research Institute* (EERI), *Mid-America Earthquake Center* (MAE), *Multidisciplinary Center for Earthquake Engineering Research* (MCEER), *Pacific Earthquake Engineering Research Center* (PEER), *The John A. Blume Earthquake Engineering Center*, *California Institute of Technology* (CALTECH), *Network for Earthquake Engineering Simulations* (NEES) etc. In these centres new theoretical approaches and numerical analyses are developed as well as experimental research and hybrid simulations are conducted, whereby a meticulous approach can reliably evaluate, additionally improve or reject the existing methods or even develop new methods for analysis of structures exposed to earthquake actions. Considering that in the last two decades the focus of research in the area of earthquake engineering is on the analysis of structural performance according to *Performance-Based Earthquake Engineering* (PBEE) methodology, a large number of analyses is therefore developed within the framework of this methodology.

Engineering and architectural structures are calculated in several stages, from the analysis of key parameters via structural modelling, through the selection of analyses to determine the eigenforms, periods of vibration, forces in bar sections and deformations. The purpose of numerical modelling of the structure is to define the elements' geometric and material properties and the systems' load and damping. These properties and loads make the basis for forming the bars' stiffness and mass matrices, and subsequently the stiffness and mass matrices of the entire system. The method of calculation of bar's geometrical properties is determined based on dimensions and cross-sectional shape of the bar and changes in geometry along the bar, while the material properties of the bar are selected based on the mechanical characteristics of the material from which it is made. The system's mass is calculated by converting loads to masses or by applying masses directly to the system. The choice of system's damping na definisanju kvantitativne vrednosti koeficijenta relativnog prigušenja ili preko drugih koeficijenata. Međutim, prigušenje se, pored direktnog definisanja u analizi konstrukcija, može uvesti i indirektno.

Analiza i određivanje prigušenja u konstrukcijama sprovodi se eksperimentalnim, analitičkim i numeričkim istraživanjima. Veliki broj eksperimentalnih istraživanja se zasniva na određivanju prigušenja kroz slobodne prigušene oscilacije, a u funkciji logaritamskog dekrementa. Ovakvo prigušenje je u domenu linearno-elastičnog ponašanja, jer se sistem samo izvede iz ravnotežnog položaja i pusti da osciluje. Međutim, ukoliko je pomeranje sistema izazvano dejstvom snažne pobude da osciluje, kao što je dejstvo zemljotresa ili vetra velikog intenziteta, tada se mora razmatrati i histerezisno prigušenje, a koje nastaje usled razvoja nelinearnih deformacija. U analitičkim istraživanjima se rezultati dobijeni eksperimentalnim istraživanjima uvode primenom različitih izraza. Ovi izrazi se, u najvećem broju slučajeva, baziraju na primeni određenih koeficijenata kojim se multipliciraju matrice ili ostali elementi izraza, tako da se prigušenje uvodi eksplicitno ili implicitno. Numerička istraživanja koriste osnovu analitičkih rešenja, pa po analogiji kao inpute za uvođenje prigušenja u sistem koriste niz koeficijenata. Međutim, softveri u kojima je implementirano nelinearno ponašanje sistema omogućavaju da se histerezisno prigušenje određuje preko energije disipacije kroz cikluse nelinearnog ponašanja (histerezisne petlje). S druge strane, u pojedinim softverima je moguće analizirati i slobodne prigušene oscilacije izvodeći sistem iz ravnotežnog položaja i prateći njegov odgovor u vremenskom domenu. Prigušenje se u analizi konstrukcija, najčešće, uvodi kao deo kritičnog prigušenja čije su vrednosti u funkciji tipa materijala, a nezavisno od mase i krutosti sistema [12]. Sa druge strane, primenom ekvivalentnog koeficijenta relativnog prigušenja može se razmatrati prigušenje na različitim tipovima materijala, uvodeći ga u formi kompozitnog prigušenja [1]. Takođe, primenom jedinstvenog ekvivalentnog koeficijenta relativnog prigušenja moguće je uzeti u obzir i viskozno i histerezisno prigušenje u nelinearnoj analizi konstrukcija [48]. Na vrednost koeficijenta relativnog prigušenja utiču kvalitet materijala, amplitude vibracija, period vibracija, svojstveni oblici, tip veza i konfiguracija konstrukcije [20].

Metodologija nelinearne analize oštećenja objekata izloženih incidentnim, a posebno seizmičkim dejstvima, prikazana je u radu [19]. Predloženi postupak je zasnovan na povezanom nizu nelinearnih analiza kojima se prvo simuliraju kolapsi pojedinačnih stubova prizemlja, sa odgovarajuće izabranim scenarijima, posle čega se vrše nelinearne statičke *pushover* analize za bidirekcijsko seizmičko dejstvo. Matrice krutosti konstrukcije na kraju prethodne analize koriste se kao početne matrice krutosti u narednim analizama. Na kraju analize ciljnog pomeranja izvršena je, primenom metode spektra kapaciteta, procena stanja posmatrane zgrade na osnovu određenih globalnih i međuspratnih driftova i odgovarajućeg koeficijenta oštećenja.

U [46] su razmatrani različiti tipovi prigušenja koji se uvode u analizu konstrukcija, kao što su: materijalno/inercijalno, granično/konstruktivno i fluidno/viskozno prigušenje. Konstruktivni sistemi su razmatrani kao kontinualni (viskozno, *Kelvin-Voigt*, vremenski histerezisno i prostorno histerezisno prigušenje) i diskretni (viskozno,

parameter depends on the type of materials of the element/structure. Quantitative value of coefficient of relative damping or other coefficients are determined in the standard procedure. However, in addition to be directly defined, damping may be also introduced into the structural analysis indirectly.

Structural dampings are determined based on experimental, analytical and numerical research. In many experiments damping is determined from free damped oscillations as a function of logarithmic decrement. This damping is in the domain of linear-elastic behaviour. If the system's displacement is induced by a strong excitation that makes it oscillate (effects of earthquake or strong wind), hysteretic damping should also be taken into considered, which is the consequence of the development of nonlinear strains. Experimental results have enabled introduction of analytical expressions based on the use of specific matrix multipliers, so that damping is introduced either explicitly or implicitly. Numerical studies also use analytical solutions so that damping is introduced into the system through a series of coefficients. Software products which also incorporate nonlinear system behaviour enable hysteretic damping to be determined based on energy dissipation through cycles of nonlinear behaviour (hysteretic loops). Some software solutions enable free damped vibrations to be analyzed by simulating the system's out-of-balance state and monitoring its response in the time domain. Damping has been most commonly introduced into structural analysis as part of critical damping whose values depend on the type of material, and not on the mass and stiffness of the system [12]. By applying the equivalent relative damping coefficient, damping can be analyzed in different types of materials, being introduced in the form of composite damping [1]. Using a unitary equivalent relative damping coefficient in nonlinear structural analysis, both viscous and hysteretic damping can be taken into account [48]. In addition to quality of materials, the value of relative damping coefficient is affected by vibration amplitudes and periods, eigenforms of vibration, types of links and structure's configuration [20].

The methodology of nonlinear analysis of damage to objects exposed to incident, and particularly seismic effects, is shown in [19]. The proposed procedure is based on a related set of nonlinear analyses, which first simulate the collapses of individual ground-floor pillars, with appropriately selected scenarios, followed by nonlinear static pushover analyses of bidirectional seismic action. Structural stiffness matrices at the end of the previous analysis are used as the initial stiffness matrices in subsequent analyses. Finally, the analyses of target displacement were carried out using the Capacity Spectrum Method (CSM), and the assessment of state of the building under consideration is performed on the basis of global and floor drifts and appropriate coefficient of damage.

Paper [46] considers different types of damping introduction into structural analysis: material/inertial, ultimate/structural and fluid/viscous damping. Construction systems are considered as continuous (viscous, *Kelvin-Voigt*, time hysteretic and space hysteretic damping) and discrete (viscous, non-viscous and frequency dependent damping). Generalized approach neviskozno i frekventno zavisno prigušenje). Generalizovani pristup u analizi prigušenja sistema kroz viskozno i neviskozno prigušenje prikazan je u studiji [2], dok su u [47] razmatrani faktori redukcije pomeranja i ukupne smičuće sile u funkciji prigušenja.

U radu [49] posmatrani su različiti aspekti seizmičkih dejstava u odnosu na „standardne” aspekte analize ponašanja konstrukcija izloženih seizmičkim dejstvima. Naime, u radu [49] težište je na arhitektonskom i kulturološkom aspektu zgrada koje se svrstavaju u zaštićene objekte, odnosno u nacionalnu kulturnu baštinu. Standardni zahtevi u seizmčkim propisima, kao što su, npr. EN 1998-1:2004, ali i naši stari seizmički propisi, da su dozvoljena oštećenja, ali bez kolapsa prilikom najjačeg zemljotresa, nisu dovoljno prihvatljivi za značajne objekte koji pripadaju kulturnoj baštini. U tom smislu, u radu [49] predložena je detaljnija klasifikacija zaštićenih zgrada u tri kategorije, zavisno od stepena značaja kulturne baštine. U skladu s tim predložene su i odgovarajuće dopune i izmene seizmičkih propisa, kako bi se što bolje zaštitila kulturna dobra ne samo od kolapsa, što je nezamislivo samo po sebi, već i od većih oštećenja.

Cilj istraživanja prikazanog u ovom radu jeste da se sistematizuju metode za analizu konstrukcija u uslovima dejstva zemljotresa i definišu algoritmi modeliranja i načina uvođenja prigušenja u njima u kapacitativnom, vremenskom i frekventnom domenu.

2 OPŠTA SISTEMATIZACIJA SEIZMIČKIH ANALIZA I GENERALNI TRETMAN PRIGUŠENJA U NJIMA

U odnosu na realne fizičke modele objekata, matematički modeli konstrukcija predstavljaju idealizovane modele ponašanja s manjim ili većim stepenom aproksimacije. Analiza propagacije talasa kroz tlo usled dejstva zemljotresa, interakcija konstrukcija–tlo, numeričko modeliranje i analiza konstrukcija izloženih dejstvu zemljotresa konstantno se unapređuju razvojem računarske mehanike. U svakodnevnoj inženjerskoj praksi primenjuju se linearno-elastični modeli ponašanja konstrukcija za analizu statičkih i dinamičkih uticaja. Analize koje pripadaju ovoj grupi su:

- linearna statička analiza (LSA – *Linear Static Analysis*);
- linearna dinamička analiza (LDA – *Linear Dynamic Analysis*);

odnosno:

- ekvivalentna statička analiza (ESA – *Equivalent Static Analysis*);
- spektralna – modalna analiza (SMA –*Spectral – Modal Analysis*).

Uobičajeni postupak primene linearnih proračunskih modela za statičku ili dinamičku analizu ne daje uvid u realno ponašanje zgrada izloženih dejstvu zemljotresa, jer ne uzima u obzir pojavu i razvoj nelinearnih deformacija u nosećoj konstrukciji. Savremene metode za analizu konstrukcija u uslovima dejstva zemljotresa zasnivaju se na primeni nelinearnog ponašanja, uzimajući u obzir razvoj i geometrijske i materijalne nelinearnosti. Analize koje pripadaju ovoj grupi su:

- nelinearna statička analiza (NSA – *Nonlinear Static Analysis*);
- nelinearna dinamička analiza (NDA – *Nonlinear*

to the analysis of the system's damping through viscous and non-viscous damping is shown in the case [2], while [47] considers the factors of reduction in displacement and total shear force as a function of damping.

In [49] various aspects of seismic actions are considered in relation to the "standard" aspects of analysis of behavioural of structures exposed to seismic actions. Here, the focus is on the architectural and cultural aspects of buildings that belong to the group of protected objects, i.e. represent a national cultural heritage. Standard requirements in seismic regulations (EN 1998-1:2004, for example), as well as our old seismic regulations, which allowed damage to occur under the action of the strongest earthquake but no collapse, are not eligible for important objects that belong to the cultural heritage. In this sense, [49] proposes a more detailed classification of protected buildings into three categories, depending on the degree of importance of cultural heritage. In accordance with this, appropriate amendments and changes to seismic regulations are proposed in order to better protect the cultural heritage not only against collapse, which is unthinkable in itself, but also against major damage.

The aim of the research presented in this paper is to systematize methods of analysis of structures in conditions of earthquake action and define modelling algorithms and ways of introducing damping in them in the capacitive, time and frequency domain.

2 GENERAL SYSTEMATIZATION OF SEISMIC ANALYSES AND GENERAL TREATMENT OF DAMPING

In comparison to the actual physical models of structures, mathematical structural models represent idealized behaviour models with a certain extent of approximation. Analysis of wave propagation through the soil due to earthquake actions, soil-structure interaction, numerical modelling and analysis of structures exposed to earthquake action are continuously being improved along with the development of computer mechanics. The linear-elastic models of structural behaviour for analysis of static and dynamic actions are implemented in the everyday engineering practice. Analyses that belong to this group are:

- Linear Static Analysis (LSA),
- Linear Dynamic Analysis (LDA),
- with respect to:
- Equivalent Static Analysis (ESA),
- Spectral - Modal Analysis (SMA).

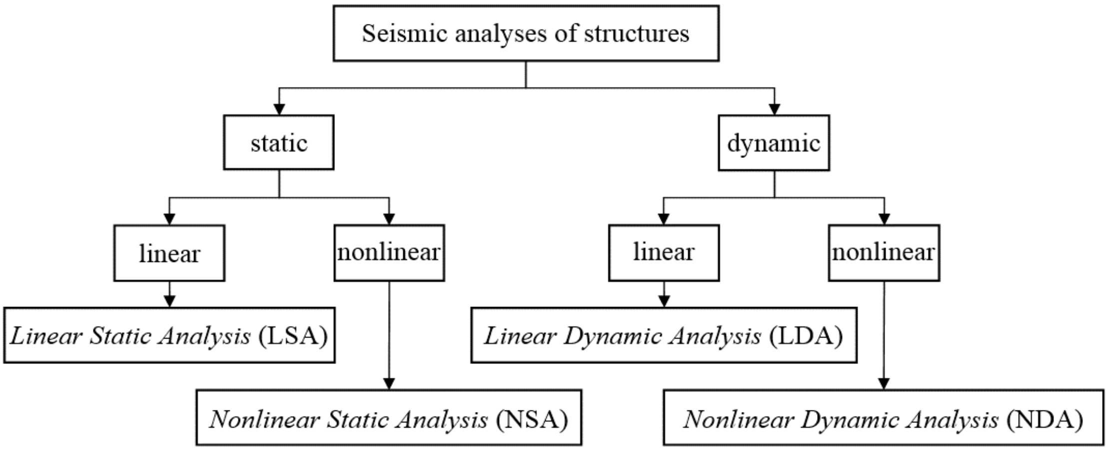
The usual procedure of implementation of linear calculation models for static or dynamic analysis lacks insight in the actual behaviour of structures exposed to earthquake actions, because it fails to consider emergence and development of nonlinear deformations in the bearing structure. Contemporary methods for analysis of structures exposed to earthquake actions are based on implementation of nonlinear behaviour, taking into consideration development and geometrical and material nonlinearities. Analyses that belong to this group are:

- Nonlinear Static Analysis (NSA),
- Nonlinear Dynamic Analysis (NDA).

Dynamic Analysis).

Na slici 1 je prikazan dijagram toka opšte sistematizacije seizmičkih analiza konstrukcija.

Figure 1 shows the flowchart of general systematization of seismic analyses of structures.



Slika 1. Dijagram toka opšte sistematizacije seizmičkih analiza konstrukcija
Figure 1. Flowchart of general systematization of seismic analyses of structures

Prethodno sistematizovane statičke i dinamičke seizmičke analize konstrukcija proračunavaju se primenom neke od metoda za matematičko-numeričko modeliranje i simulaciju ponašanja konstrukcija. Najveću primenu u rešavanju problema analize konstrukcija prema performansama (PBSD – *Performance-Based Seismic Design*) pronašle su:

- metoda konačnih elemenata (FEM – Finite Element Method);
- metoda graničnih elemenata (BEM – Boundary Element Method);

a takođe značajan doprinos u rešavanju problema kolapsa konstrukcija usled dejstva zemljotresa postignut je razvojem:

- metode diskretnih elemenata (DEM – *Discrete Element Method*);
- proširene metode konačnih elemenata (XFEM – *eXtended Finite Element Method*);
- metode primenjenih elemenata (AEM – *Applied Element Method*).

S druge strane, postoji niz seizmičkih metoda koje koriste rešenja NSA ili NDA i kombinuju ih s drugim naučnim disciplinama, tako da se problem razmatra multidisciplinarno u PBEE. Sistematizacija ovih metoda je takođe prikazana u radu.

U procesu modeliranja konstrukcije i pripreme seizmičke analize, prema kojoj će se sprovesti proračun konstrukcije, prigušenje je moguće uvesti preko: prigušenja materijala, prigušenja koje potiče od elemenata veze i prigušenja koje se direktno definiše u analizi. Na slici 2 je prikazan dijagram toka generalnog tretmana prigušenja u seizmičkoj analizi konstrukcija. Prigušenje materijala (*material damping*) uvodi se pri definisanju tipa materijala i može se aplicirati za određenu grupu linijskih, površinskih ili prostornih konačnih elemenata. Ovakav princip uvođenja prigušenja u analizu veoma je povoljan, s obzirom na to što je isto moguće definisati za konstrukcije koje se sastoje iz segmenata različitog tipa materijala, kao na primer:

- jedan deo noseće konstrukcije formira se od

Previously classified static and dynamic analyses of structures are calculated by implementing some of the methods for mathematical-numerical modelling and simulation of structural behaviour. The following methods have found the greatest application in solving problems with *Performance-Based Seismic Design* (PBSD):

- Finite Element Method (FEM),
- Boundary Element Method (BEM),

a considerable contribution to solving the problem of structural collapse due to earthquake actions was achieved by the development of

- Discrete Element Method (DEM),
- eXtended Finite Element Method (XFEM),
- Applied Element Method (AEM).

On the other hand, there is a number of seismic analyses which employ solutions of NSA or NDA and combine them with other scientific disciplines so that the problem is considered multidisciplinary in PBEE. Systematization of these analyses is also presented in the paper.

In the process of modelling the structure and preparing seismic analysis based on which the structure will be calculated, damping can be introduced via material damping, link element damping and damping which is directly defined in the analysis. Figure 2 shows the flowchart of general approach to damping in seismic analysis of structures. Material damping is introduced with the definition of the type of material and can be applied to a particular group of line, surface or spatial finite elements. This principle of damping introduction into the analysis is very advantageous, given that enables damping to be also defined for structures consisting of segments made of different types of materials:

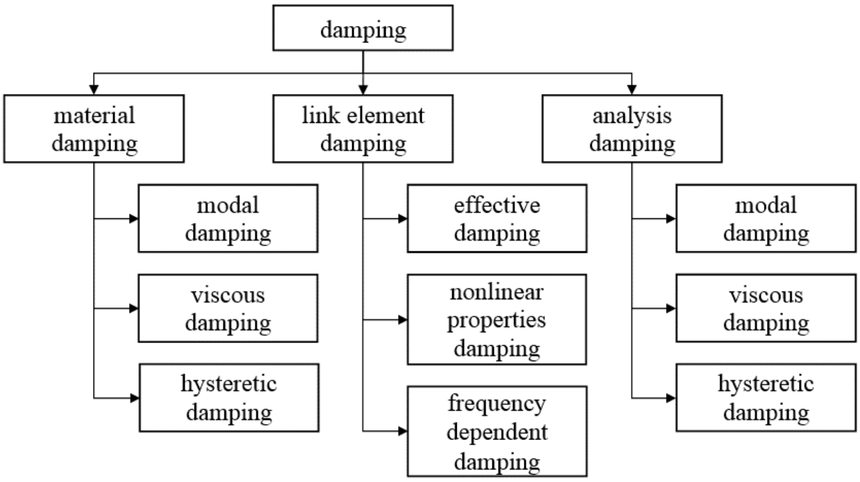
- part of the support structure is made of concrete, and part of steel or wood without introduction of coupling, betona, a drugi deo od čelika ili drveta bez uvođenja sprezanja;

- spregnute konstrukcije beton–čelik, beton–drvo i slično;
- modeliranje konstrukcije sa ispunom, pri čemu je ispuna od materijala koji se razlikuje od noseće konstrukcije;
- problemi interakcije konstrukcija–tlo (SSI - *soil-structure interaction*), pri čemu se posebno može definisati prigušenje za noseću konstrukciju, a posebno za tlo.

Primenom ovog prigušenja može se uvesti i uticaj radijacijskog prigušenja kod tla, tako što bi se tlo modeliralo prostornim (*solid*) konačnim elementima i za njih bi se definisale odgovarajuće mehaničke karakteristike i prigušenje.

- composite structures of concrete - steel, concrete - wood and the like,
- modelling structure with infill, where the infill material is different from that of the support structure,
- problems of soil-structure interaction (SSI), where damping can be defined separately for the supporting structure, and separately for the soil.

It particularly enables defining damping for the supporting structure, specifically for soil (radiation damping), whereby soil is modelled by using spatial solid finite elements, for which the appropriate mechanical properties and damping will be defined.



Slika 2. Dijagram toka generalnog tretmana prigušenja u seizmičkoj analizi konstrukcija [30]
Figure 2. Flowchart of general approach to damping in seismic analysis of structures [30]

U zavisnosti od tipa analize, za koju se definiše prigušenje materijala, generalna podela prigušenja može se sprovesti na: modalno (*modal damping*), viskozno (*viscous damping*) i histerezisno prigušenje (*hysteretic damping*). Modalno prigušenje uvodi se kod SMA i modalne LDA i NDA. Viskozno prigušenje uvodi se kod LDA i NDA za koje se sprovodi numerička integracija, dok se histerezisno prigušenje uvodi kod analize stalnog – postojanog stanja (SSA – *Steady - State Analysis*) i analize snage spektralne gustine (PSDA – *Power Spectral Density Analysis*). U zavisnosti od tipa elementa veze (*link element*), prigušenje se može uvesti kao: efektivno prigušenje (*effective damping*), prigušenje kod nelinearnog ponašanja i prigušenje kod frekventno zavisnih elemenata veze. Efektivno prigušenje se uvodi kod: SMA, LDA i NDA (modalna i numerička integracija), SSA i PSDA. Prigušenje kod frekventno zavisnih elemenata veze uvodi se kod SSA i PSDA. U zavisnosti od tipa analize, generalna podela prigušenja se može sprovesti na: modalno, viskozno i histerezisno prigušenje. Svako ovo prigušenje može se uvesti primenom različitih postupaka koji su prikazani u daljem delu teksta.

Depending on the type of analysis for which material damping is being defined, damping can be generally divided into modal, viscous, and hysteretic damping. Modal damping is introduced into SMA and modal LDA/NDA. Viscous damping is introduced into LDA/NDA which requires numerical integration to be carried out, while hysteretic damping is introduced into *Steady-State Analysis* (SSA) and *Power Spectral Density Analysis* (PSDA). Depending on the type of the link element, damping can be introduced as: effective damping, damping in nonlinear behaviour and damping of frequency dependent link elements. Effective damping is introduced into SMA, LDA (modal and numerical integration), SSA and PSDA. In frequency dependent link elements damping is introduced into SSA and PSDA. Depending on the type of analysis, damping can be generally divided to: modal, viscous and hysteretic damping. Each of the damping can be introduced by applying different procedures presented later in this paper.

2.1 Prigušenje materijala

Prigušenje materijala, u formi modalnog prigušenja, uvodi se primenom koeficijenta relativnog prigušenja ξ_m za različite tipove materijala, a koji predstavlja odnos realnog prigušenja i kritičnog prigušenja i za koji se može pisati:

2.1 Material damping

Material damping, in the form of modal damping, is introduced using relative damping coefficient ξ_m for different types of materials, which represents the ratio of actual and critical damping, wherein:

$$\xi_{m,i} \neq \xi_{m,j} \neq \xi_{m,k} \ldots \neq \xi_{m,n} \tag{1}$$

pri čemu se indeksirano *i* odnosi na *i*-ti materijal koji se koristi u analizi. Ovo prigušenje je poznato i kao kompozitno modalno prigušenje, a njegove vrednosti se nalaze u granicama $0 \leq \xi_m \leq 1$. Prigušenje materijala, u formi viskoznog (proporcionalnog) prigušenja, uvodi se primenom faktora participacije mase i krutosti sistema, tako da se proračun matrice prigušenja sprovodi prema [52]:

where the indexed *i* refers to the *i*-th material used in the analysis. This damping is also known as composite modal damping, and its values are within the limits of $0 \leq \xi_m \leq 1$. Material damping, in the form of a viscous (proportional) damping is introduced by applying the factors of participation of the system's mass and stiffness, so that the damping matrix is calculated as follows [52]:

$$[C] = \alpha[M] + \beta[K], \tag{2}$$

$$\alpha = 4\pi \frac{T_1\xi_1 - T_2\xi_2}{T_1^2 - T_2^2}, \qquad \beta = \frac{1}{\pi} T_1 T_2 \frac{T_1\xi_2 - T_2\xi_1}{T_1^2 - T_2^2}, \tag{3}$$

gde su α i β faktori participacije matrice masa i matrice krutosti u matrici prigušenja sistema, T_1 i T_2 periodi vibracija za prvi i drugi svojstveni oblik, ξ_1 i ξ_2 koeficijenti relativnog prigušenja za prvi i drugi svojstveni oblik. Veza između koeficijenata relativnog prigušenja za prvi i drugi svojstveni oblik i faktora participacije matrice masa i matrice krutosti u matrici prigušenja sistema glasi:

where α and β are factors of participation of mass and stiffness matrices in the system's damping matrix, T_1 and T_2 are periods of vibration for the first and second eigenform, ξ_1 and ξ_2 are relative damping coefficients for the first and second eigenform. The relation between the relative damping coefficient for the first and second eigenform and the factor of participation of mass matrix and stiffness matrix in the system's damping matrix is as follows:

$$\xi_1 = \frac{\alpha}{2\omega_1} + \frac{\beta\omega_1}{2}, \qquad \xi_2 = \frac{\alpha}{2\omega_2} + \frac{\beta\omega_2}{2}, \tag{4}$$

gde su ω_1 i ω_2 ugaone frekvencije za prvi i drugi svojstveni oblik. Ukoliko su koeficijenti relativnog prigušenja jednaki za oba svojstvena oblika vibracija $\xi_1=\xi_2=\xi$, tada izraz (3) postaje:

where ω_1 and ω_2 are angular frequencies for the first and second eigenform. If relative damping coefficients are the same for both eigenforms of vibrations $\xi_1 = \xi_2 = \xi$, then the expression (3) becomes:

$$\alpha = \xi \frac{2\omega_1\omega_2}{\omega_1 + \omega_2}, \qquad \beta = \xi \frac{2}{\omega_1 + \omega_2}. \tag{5}$$

Prigušenje materijala, u formi histerezisnog prigušenja, uvodi se primenom faktora participacije mase i krutosti sistema, analogno principu uvođenja viskoznog prigušenja. Budući da se ovo prigušenje uvodi kod analiza u frekventnom domenu, to se u proračunu primenjuje matrica histerezisnog prigušenja [52]:

Material damping, in the form of hysteretic damping, is introduced by applying the factors of participation of the system's mass and stiffness, analogous to the principle of introduction of viscous damping. Given that this damping is being introduced in analyses in the frequency domain, the calculation uses the hysteretic damping matrix [52]:

$$[D] = \omega[C]. \tag{6}$$

2.2 Prigušenje koje potiče od elemenata veze

Prigušenje koje potiče od elemenata veze, a koji se modeliraju kod linearnih analiza, definiše se preko efektivnog prigušenja c_{eff}. Ovo efektivno prigušenje se uvodi za svaki element veze posebno i za svaku komponentu prigušenja nezavisno (ima ih šest), a njime se može predstaviti, između ostalog, i energija disipacije

2.2 Damping induced by link elements

Damping induced by link elements, which are modelled in linear analyses, is defined through effective damping c_{eff}. This effective damping is introduced individually for each link element and independently for each of the 6 damping components. Besides, it can be used for representing energy dissipation due to nonlinear usled nelinearnog prigušenja i razvoja plastičnih deformacija. Određivanje efektivnog prigušenja sprovodi se analogno određivanju komponenata efektivne krutosti. Ukoliko se element veze definiše s mogućnošću razvoja nelinearnih deformacija, tada se u toku nelinearne analize proračunava disipacija histerezisne energije u elementima veze. S druge strane, postoji mogućnost da se pri nelinearnom ponašanju elemenata veze dodatno uvede prigušenje, a u funkciji tipa samog elementa veze. U slučaju elementa prigušivača (*damper element*), relacija nelinearna sila – pomeranje glasi [52]:

damping and development of plastic strains. Effective damping is determined analogous to determining the components of the effective stiffness. If the link element is defined based on the possibility of developing nonlinear strains, then dissipation of hysteretic energy in link elements is required to be calculated during the nonlinear analysis. On the other hand, in the case of nonlinear behaviour of link elements, damping can be additionally introduced depending on the type of the link element. In the case of damper element, the relation between nonlinear forces and displacements is as follows [52]:

$$f_d = cv^e , \tag{7}$$

gde je f_d sila u elementu prigušivaču, c koeficijent prigušenja ($c=\xi c_c$ – proizvod koeficijenta relativnog prigušenja i koeficijenta kritičnog prigušenja), v brzina deformacije u elementu prigušivaču, e eksponent prigušenja ($0.2 \le e \le 2$). Kod frikcionog izolatora (*friction – pendulum insulator*) prigušenje se uvodi u analizi aksijalne sile f_i:

where f_d is the force in the damper element, c is the damping coefficient ($c=\xi c_c$ - product of the relative and critical damping coefficient), v is the strain rate in the damper element, e is the damping exponent ($0.2 \le e \le 2$). In friction-pendulum insulator, damping is introduced in the axial force analysis f_i:

$$f_i = kd + cv , \tag{8}$$

gde je k krutost izolatora, d pomeranje izolatora, pri čemu se koeficijent relativnog prigušenja ξ može odrediti prema:

where k is insulator stiffness, d is insulator displacement, while relative damping coefficient ξ can be determined from:

$$\xi = \frac{c}{2\sqrt{km}} , \tag{9}$$

gde je m odgovarajuća masa izolatora. U slučaju biaksijalnog frikcionog izolatora (*double - acting friction - pendulum insulator*) ovo prigušenje se uvodi u analizi aksijalne sile preko:

where m is the corresponding insulator mass. In the case of double-acting friction-pendulum insulator, this damping is introduced in axial force analysis through:

$$f_i = cv + \begin{cases} k_c\left(d + \Delta_c\right) & \text{ukoliko je / } \textit{if} \quad \left(d + \Delta_c\right) < 0 \\ k_t\left(d - \Delta_t\right) & \left(d - \Delta_c\right) > 0 , \\ 0 & \text{ostalo / } \textit{other} \end{cases} \tag{10}$$

gde je k_c krutost izolatora pri pritisku, k_t krutost izolatora pri zatezanju, Δ_c zazor otvora (*gap*) pri pritisku, Δ_t zazor otvora pri zatezanju. U slučaju ostalih tipova elemenata veze koji se zasnivaju na histerezisnom ponašanju, kao što su multilinearni plastični (*multilinear plastic*), plastični (*Wen*) i izolator od gume (*rubber insulator*), a koji se primenjuju kod nelinearnih analiza, prigušenje se eksplicitno ne uvodi u proračun, već se u toku analize određuje.

Prigušenje frekventno zavisnih elemenata veze koristi se kod analize u frekventnom domenu, pri čemu frekventno zavisne karateristike predstavljaju kompleksnu impedancu. Realni deo odgovara krutosti, dok imaginarni deo odgovara histerezisnom prigušenju. Frekventno zavisne karakteristike elementa veze sa šest stepeni slobode mogu se prikazati u matričnoj formi (36 elemenata), pri čemu je element matrice impedance [14]:

where k_c is the insulator's compressive stiffness, k_t is the insulator's tensile stiffness, Δ_c is the clearance gap under compression and Δ_t is the clearance gap under tension. In the case of other types of link elements which are based on hysteretic behaviour, such as multi linear plastic, *Wen* and rubber insulators, which can be applied in nonlinear analyses, damping is explicitly left out from calculation, but determined during the analysis.

Damping of frequency dependent link elements is used in analysis in frequency domain, where the complex impedance is represented by frequency dependent properties. The real part corresponds to stiffness, while imaginary part corresponds to hysteretic damping. Frequency dependent properties of link element with six degrees of freedom can be expressed in matrix form (36 elements), wherein the element of impedance matrix [14]:

$$z_i = k_i + ic_i , \tag{11}$$

gde je k_i komponenta krutosti za i-ti stepen slobode, c_i komponenta prigušenja za i-ti stepen slobode.

where k_i is the stiffness component for the i-th degree of freedom, c_i is the damping component for the i-th degree of freedom.

2.3 Prigušenje koje se direktno definiše u analizi

Modalno prigušenje, koje se direktno definiše u analizi, može se uvesti kao: konstantno prigušenje (*constant damping for all modes*), interpolirano prigušenje (*interpolated damping by period or frequency*) i primenom faktora participacije mase i krutosti (*mass and stiffness proportional damping by coefficient*). Konstantno prigušenje se definiše primenom jedinstvenog koeficijenta relativnog prigušenja ξ_c. Ukoliko se u postupku modeliranja konstrukcije definiše samo jedan tip materijala, tada prigušenje koje se uvodi preko materijala ξ_m postaje ekvivalentno prigušenju koje se uvodi kao konstantno prigušenje u analizi ξ_c. Međutim, potrebno je uzeti u obzir da se ovi tipovi prigušenja, različiti po postupku uvođenja, sabiraju, tako da će ukupno prigušenje biti dodatno povećano. Interpolirano prigušenje ξ_i se definiše u funkciji selektovanih perioda vibracija T_i ili frekvencija f_i. Ovde postoji mogućnost da se za određene periode vibracija (frekvencije) posebno definišu koeficijenti relativnog prigušenja, a zatim da se za proračunate periode vibracija (frekvencije) interpolacijom odrede odgovarajući koeficijenti relativnog prigušenja $\xi_{i,i}$:

2.3 Damping that is directly defined in the analysis

Modal damping that is directly defined in the analysis can be introduced as: constant damping for all modes, interpolated damping by period or frequency and applying the factors of mass and stiffness proportional damping by coefficient. Constant damping for all modes is defined by applying a unique relative damping coefficient of ξ_c. If only one type of material is defined in the process of structural modelling, then damping that is introduced through the material ξ_m becomes equivalent to damping which is introduced as a constant damping in the analysis of ξ_c. However, it is necessary to take into account that these differently introduced types of damping are added up, so that overall damping will be further increased. Interpolated damping ξ_i is defined as a function of selected vibration T_i or frequency f_i periods. Here, for certain periods of vibration (frequency) it is possible to separately define relative damping coefficients, and then, using interpolation, to determine the corresponding relative damping coefficients $\xi_{i,i}$ for the calculated periods of vibrations (frequencies).

$$\xi_{i,i} = f(T_i) \vee \xi_{i,i} = f(f_i), \quad i = 1, \ldots n\,. \tag{12}$$

Van definisanog regiona, u kojem je zadato prigušenje, vrednost koeficijenta relativnog prigušenja je konstantna. Uvođenje prigušenja u analizu primenom faktora participacije mase i krutosti sistema sprovodi se: direktnim definisanjem ovih koeficijenata, definisanjem ovih koeficijenata u funkciji perioda vibracija prvog i drugog svojstvenog oblika i definisanjem ovih koeficijenata u funkciji frekvencija prvog i drugog svojstvenog oblika. U određenim softverskim rešenjima postoji mogućnost direktnog definisanja α i β faktora ili da se definišu periodi vibracija prvog i drugog svojstvenog oblika T_1 i T_2 i odgovarajuće vrednosti koeficijenata relativnog prigušenja ξ_1 i ξ_2, a da se zatim sprovede proračun α i β faktora. Takođe, postoji mogućnost da se definišu frekvencije prvog i drugog svojstvenog oblika f_1 i f_2 i odgovarajuće vrednosti koeficijenata relativnog prigušenja ξ_1 i ξ_2, a da se zatim sprovede proračun α i β faktora. Proračun matrice prigušenja se može sprovesti prema [35]:

Outside the defined region, where damping is predefined, the value of the relative damping coefficient is constant. Introduction of damping in the analysis by using mass participation factor and system stiffness is carried out in the following ways: by defining these coefficients directly, defining these coefficients as a function of the period of vibration of the first and second eigenform, and defining these coefficients as a function of frequencies of the first and second eigenform. Some software solutions allow defining factors α and β directly or defining periods of vibration of the first and second eigenform T_1 and T_2, and the corresponding values of relative damping coefficients ξ_1 and ξ_2, which is then followed by the calculation of factors α and β. It is also possible to define the frequencies of the first and second eigenform f_1 and f_2 and the corresponding values of relative damping coefficients ξ_1 and ξ_2, and then to perform the calculation for factors α and β. Damping matrix can be calculated using the following formula [35]:

$$[C] = \alpha[I] + \beta[\Omega^2], \tag{13}$$

gde je $[\Omega^2]$ matrica kvadrata svojstvenih vrednosti sistema, $[I]$ jedinična matrica.

Viskozno prigušenje, koje se direktno definiše u analizi, može se uvesti: primenom faktora participacije mase i krutosti α i β, u funkciji perioda vibracija prvog i drugog svojstvenog oblika T_1 i T_2 (*specify damping by period*) i u funkciji frekvencija prvog i drugog svojstvenog oblika f_1 i f_2 (*specify damping by frequency*). U slučaju uvođenja faktora participacije mase i krutosti α i β proračun matrice prigušenja $[C]$ sprovodi se prema izrazu (2), dok je u preostala dva postupka potrebno poznavati i periode vibracija T_1 i T_2 ili frekvencije f_1 i f_2 i odgovarajuće koeficijente relativnog prigušenja ξ_1 i ξ_2 da bi se proračunala matrica prigušenja $[C]$. Uvođenje prigušenja primenom različitih koeficijenata relativnog prigušenja za prva dva svojstvena oblika (frekvencije) ima niz prednosti, u odnosu na princip korišćenja jedinstvenog koeficijenta relativnog prigušenja.

where $[\Omega^2]$ is the matrix of squares of the system's eigenvalues and $[I]$ is the unit matrix.

Viscous damping, which is directly defined in the analysis, can be introduced by: using factors of mass and stiffness participation (α and β) as a function of vibration periods of the first and second eigenform T_1 and T_2 (specify damping by a period) and as a function of frequencies of the first and second eigenform f_1 and f_2 (specify dumping by frequency). In the case of introducing the mass and stiffness participation factors α and β, the damping matrix [C] is calculated according to the expression (2), while in the remaining two procedures it is required to identify periods of vibration T_1 and T_2 or frequencies f_1 and f_2, and corresponding relative damping coefficients ξ_1 and ξ_2 to calculate the damping matrix $[C]$. Introducing damping by using different relative damping coefficients for the first two eigenforms (frequency) has a number of advantages

Histerezisno prigušenje, koje se direktno definiše u analizi, može se uvesti kao konstantno prigušenje za sve frekvencije (*constant damping for all frequencies*) i interpolirano prigušenje po frekvencijama (*interpolated damping by frequency*). Konstantno prigušenje za sve frekvencije se definiše preko faktora participacije mase i krutosti α i β, tako da se proračun matrice prigušenja [C] sprovodi prema izrazu (2). Interpolirano prigušenje po frekvencijama se uvodi u proračun preko frekvencija f_i i odgovarajućih faktora participacije mase i krutosti α i β. Zatim se za proračunate frekvencije interpolacijom odrede odgovarajući koeficijenti relativnog prigušenja $\xi_{h,i}$.

3 LINEARNA STATIČKA ANALIZA (LSA)

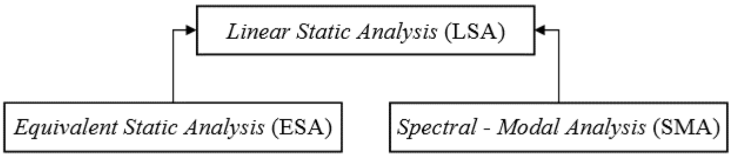
Linearna statička analiza (LSA – *Linear Static Analysis*) koristi se u svakodnevnoj inženjerskoj praksi za proračun konstrukcija na seizmičko dejstvo prema propisima. Proračun se sprovodi tako što se primenom ekvivalentne statičke analize (ESA – *Equivalent Static Analysis*) ili spektralne – modalne analize (SMA – *Spectral - Modal Analysis*) odrede lateralne seizmičke sile, koje se apliciraju na konstrukciju. Zatim se primenom LSA po FEM ili sličnim metodama sprovede proračun, a nakon toga dimenzionisanje konstruktivnih elemenata. Na slici 3 prikazan je dijagram toka proračuna primenom LSA u interakciji sa ESA i SMA.

over the principle of using a unique relative damping coefficient.

Hysteretic damping, which is directly defined in the analysis, can be introduced as constant damping for all frequencies and interpolated damping by frequency. Constant damping for all frequencies is defined through the factors of mass and stiffness participation (α and β), so that the damping matrix [C] can be calculated using expression (2).Interpolated damping across the frequencies is introduced into calculation over the frequencies f_i and corresponding mass and stiffness participation factors α and β. Then the corresponding relative damping coefficients $\xi_{h,i}$ are determined for calculated frequencies using interpolation.

3 LINEAR STATIC ANALYSIS (LSA)

LSA is used in everyday engineering practice for calculating the structures against seismic actions in accordance with the regulations. First, lateral seismic forces which are applied to the structure are determined using *Equivalent Static Analysis* (ESA) or *Spectral-Modal Analysis* (SMA). Then, the calculation is conducted using the LSA to FEM or similar methods, after which dimensioning of structural elements is carried out. Figure 3 shows a flowchart of calculation using LSA in interaction with ESA and SMA.



Slika 3. Dijagram toka proračuna primenom LSA u interakciji sa ESA i SMA
Figure 3: Flowchart of calculation using LSA in interaction with ESA and SMA

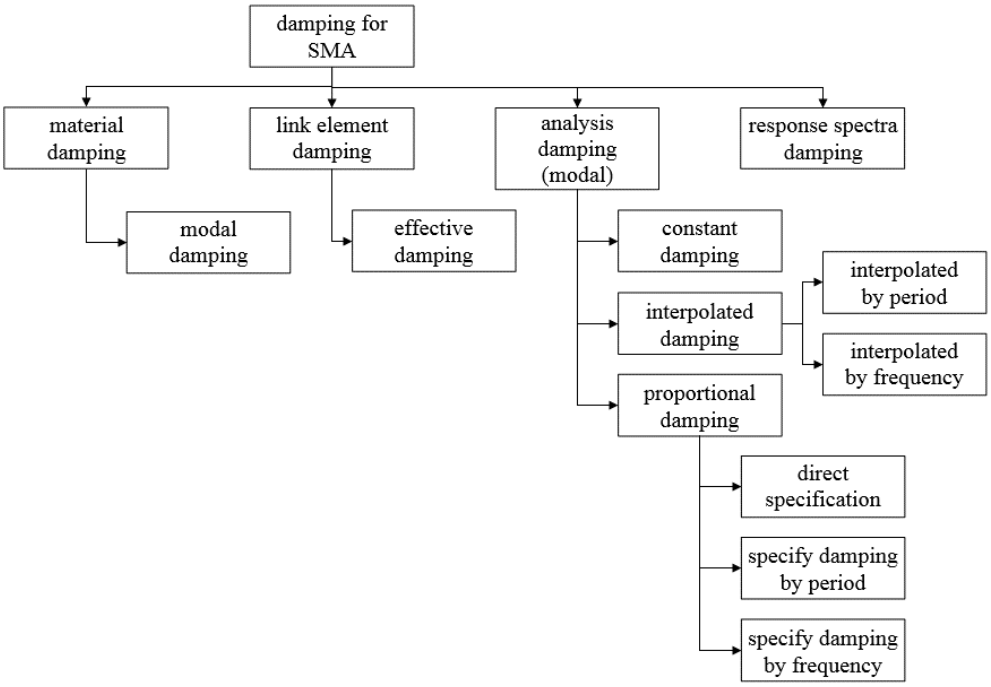
Uvođenje prigušenja u SMA moguće je sprovesti primenom: prigušenja materijala, prigušenja elemenata veze i prigušenja u analizi. Na slici 4 je prikazan dijagram toka uvođenja prigušenja kod SMA.

Prigušenje materijala uvodi se kao modalno prigušenje, dok se prigušenje elemenata veze uvodi kao efektivno prigušenje. Prigušenje koje se direktno definiše u analizi uvodi se kao: konstantno prigušenje, interpolirano prigušenje i primenom faktora participacije mase i krutosti, pri čemu se ovo poslednje prigušenje može uvesti primenom: faktora participacije mase i krutosti α i β, u funkciji perioda vibracija prvog i drugog svojstvenog oblika T_1 i T_2 i u funkciji frekvencija prvog i drugog svojstvenog oblika f_1 i f_2. S druge strane, prilikom generisanja spektra odgovora prigušenje se uvodi preko koeficijenta relativnog prigušenja ξ_{rs}. Međutim, ukupno prigušenje u SMA definiše se preko kumulativnog koeficijenta relativnog prigušenja ξ, tako da se kriva spektra odgovora koriguje prema [43]:

Damping can be introduced into LSA by using: material damping, link element damping and analysis damping. Figure 4 shows the flowchart of introducing damping into LSA.

Material damping is introduced as a modal damping, while the link element damping is introduced as effective damping. The damping which is directly defined in the analysis is introduced as: constant damping, interpolated damping, using the factors of mass and stiffness participation, whereby the latter can be introduced by using the factors of mass and stiffness participation (α and β) as a function of the vibration period of the first and second eigenform T_1 and T_2, and as a function of frequencies f_1 and f_2. On the other hand, when generating the response spectrum, damping is introduced through the relative damping coefficient ξ_{rs}. However, overall damping in the SMA is defined through the cumulative relative damping coefficient ξ, so that the response spectrum curve is corrected according to [43]:

$$S_a = S_{a,rs}\frac{2.31-0.41\cdot log\,\xi}{2.31-0.41\cdot log\,\xi_{rs}}, \tag{14}$$



Slika 4. Dijagram toka uvođenja prigušenja kod SMA [30]
Figure 4. Flowchart of introducing damping into LSA [30]

gde je S_a korigovana spektralna akceleracija koja odgovara prigušenju ξ, $S_{a,rs}$ inicijalna spektralna akceleracija koja odgovara prigušenju ξ_{rs}. Ukoliko su vrednosti koeficijenta relativnih prigušenja jednake $\xi_{rs}=\xi$, tada nema dodatne korekcije spektralnih akceleracija i u analizu se uvodi spektar odgovora koji je generisan za koeficijent relativnog prigušenja ξ_{rs}. Diferencijalne jednačine kretanja sistema sa više stepeni slobode mogu da se transformišu u nezavisne jednačine dobijajući izraz [52]:

where S_a is the corrected spectral acceleration corresponding to damping ξ, and $S_{a,rs}$ is the initial spectral acceleration corresponding to damping ξ_{rs}. If values of relative damping coefficients are the same $\xi_{rs}=\xi$, then no further corrections of spectral accelerations are needed and the response spectrum which is generated for relative damping coefficient ξ_{rs} introduced in the analysis. Differential equations of movement of the system with several degrees of freedom can be transformed in independent equations, leading to the following expression [52]:

$$\{\ddot{Y}\}+[C]\{\dot{Y}\}+[\Omega^2]\{Y\}=[\Phi]^{\mathrm{T}}\{F\}, \tag{15}$$

gde je $\{Y\}$ vektor modalnih koordinata, $[\Phi]$ modalna matrica čije su kolone svojstveni oblici, $\{F\}$ vektor opterećenja sistema. Zbog dijagonalne strukture matrice prigušenja, jednačine su nezavisne (15) i imaju oblik:

where $\{Y\}$ is the vector of modal coordinates, $[\Phi]$ is the modal matrix whose columns are eigenforms, and {F} is the load vector of the system. Due to the diagonal structure of the damping matrix, equations (15) are independent and have the following form:

$$\ddot{y}_i(t)+2\xi_i\omega_i\dot{y}_i(t)+\omega_i^2y_i(t)=r(t), \qquad r(t)=\{\Phi\}_i^{\mathrm{T}}\{F\}, \tag{16}$$

gde je $\{\Phi\}_i$ kolona *i* matrice $[\Phi]$. Prilikom proračuna SMA formira se matrica prigušenja prema izrazu (15), odnosno definišu se koeficijenti relativnog prigušenja prema izrazu (16).

where $\{\Phi\}_i$ is the *i* column of matrix $[\Phi]$. When calculating the SMA, the damping matrix forms according to expression (15), and relative damping coefficients are defined according to expression (16).

4 NELINEARNA STATIČKA ANALIZA (NSA)

Nelinearna statička analiza (NSA – *Nonlinear Static Analysis*) sprovodi se u kapacitativnom domenu, a poznatija je kao *pushover* analiza ili *Nonlinear Static Pushover Analysis* (NSPA). Na abscisi i ordinati kapacitativnog domena predstavljaju se parametri inženjerskog zahteva (EDP – *engineering demand*

4 NONLINEAR STATIC ANALYSIS (NSA)

NSA is conducted in capacitive domain, and it is more known as pushover analysis or *Nonlinear Static Pushover Analysis* (NSPA). On the abscise and ordinate of the capacitive domain, engineering demand parameters (EDP) are displayed, which are actually structural response parameters. *Target Displacement* *parameters*), a što su zapravo parametri odgovora konstrukcije. Kao dopuna konačnog rešenja koje se dobija NSPA, sprovodi se i analiza ciljnog pomeranja (TDA – *target displacement analysis*). NSPA se sprovodi na realnom sistemu s više stepeni slobode (MDOF – *multi degree of freedom*), dok se TDA sprovodi za sistem s jednim stepenom slobode (SDOF – *single degree of freedom*) ili se direktno proračun sprovodi na osnovu realizovane *pushover* krive. Razvoj koncepta NSPA i TDA zgrada, za uslove seizmičkog dejstva, iniciran je pre više od dve decenije, a zvanične implementacije su usledile u ATC 40 [6], EN 1998-1:2004 [23], FEMA 356 [25] i FEMA 440 [26] propise. Danas postoji širok spektar NSPA i TDA. Kod određenih analiza se direktno sprovodi proračun ciljnog pomeranja kroz NSPA (integrisano rešenje), dok se kod određenih analiza ovo sprovodi nezavisno (sukcesivno rešenje). U ovom drugom slučaju je moguće kombinovati rešenja NSPA i TDA primenom različitih pristupa. Takođe, bitan faktor koji se može uzeti u obzir pri klasifikaciji ovih analiza jeste tip lateralnog seizmičkog opterećenja. Dakle, izdvajaju se tri ključna faktora koji determinišu razlike u ovim analizama: tip NSPA, tip TDA i tip lateralnog seizmičkog opterećenja. Sistematizacija NSPA prikazana je bez detaljnijeg klasifikovanja ovih analiza, s tim što se za ove analize koriste različiti tipovi inkrementalno-iterativnih algoritama. Analize koje pripadaju ovoj grupi su [31]:

- nelinearna statička konvencionalna *pushover* analiza (NSCPA – *Nonlinear Static Conventional Pushover Analysis*);
- nelinearna statička adaptivna *pushover* analiza (NSAPA – *Nonlinear Static Adaptive Pushover Analysis*);
- modalna *pushover* analiza (MPA – *Modal Pushover Analysis*);
- multimodalna *pushover* procedura (MMPP – *Multi-Mode Pushover Procedure*);
- metod modalnih kombinacija (MMC – *Method of Modal Combinations*);
- inkrementalna analiza spektra odgovora (IRSA – *Incremental Response Spectrum Analysis*);
- projektovanje konstrukcija prema performansama plastifikacije (PBPD – *Performance-Based Plastic Design*);
- nelinearna statička *pushover* analiza zasnovana na analizi mehanizama loma (NSPA-DMBD – *Nonlinear Static Pushover Analysis - Damage Mechanisms-Based Design*).

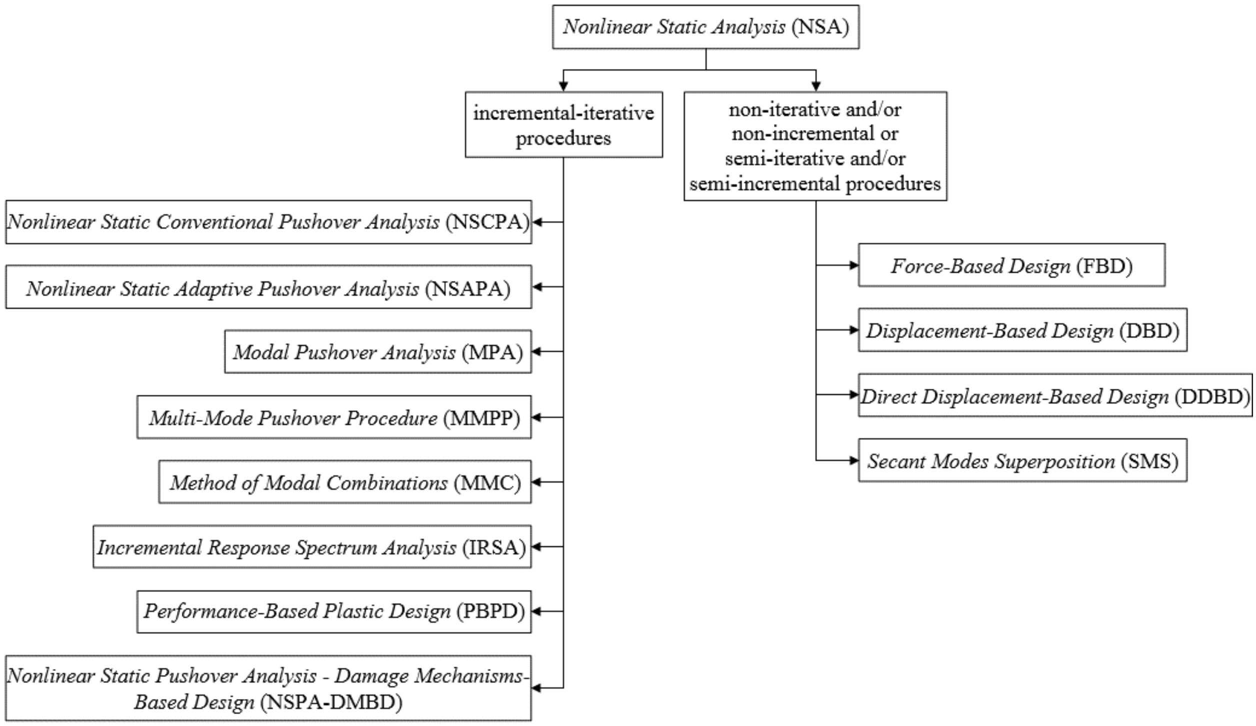
Na slici 5 je prikazana podela NSA prema postupku proračuna. NSCPA se zasniva na konstantnom zadržavanju raspodele lateralnog seizmičkog opterećenja kroz sve faze inkrementalno-iterativne analize, odnosno od inicijalnog linearnog to finalnog kolapsnog stanja konstukcije [4]. NSAPA se zasniva na korekciji lateralnog seizmičkog opterećenja po inkrementima, uzimajući u obzir promenu perioda vibracija konstrukcije i spektralnu amplifikaciju seizmičkih sila prema spektru odgovora ubrzanja ili korekciju pomeranja prema spektru pomeranja [3]. Kontrola inkrementalnog koncepta za NSCPA i NSAPA moguća je preko sila (FBA – *Force-Based Analysis*) ili preko pomeranja (DBA – *Displacement-Based Analysis*). U zavisnosti od toga kako se sprovodi korekcija lateralnih apliciranih sila, moguće su opcije: totalna (TU), inkrementalna (IU) i hibridna (HU) korekcija. U zavisnosti od primenjene

Analysis(TDA) is conducted as a complement of the final solution obtained by NSPA. NSPA is conducted on an actual multi degree of freedom (MDOF) system, while the TDA is conducted on a single degree of freedom system (SDOF) or calculation is directly conducted based on the realized pushover curve. Development of the concept of NSPA and TDA of the buildings designed for seismic areas was initiate more than two decades ago, and official implementations were effected in ATC 40 [6], EN 1998-1:2004 [23], FEMA 356 [25] and FEMA 440 [26] codes. Nowadays, there is a wide range of NSPA and TDA. In case of certain analyses, calculation of target displacement is directly conducted through NSPA (integrated solution), while in other analyses, this is conducted independently (successive solution). In the second case, it is possible to combine solutions of NSPA and TDA by implementing various approaches. Another important factor which can be taken into consideration in classification of these analyses is type of lateral seismic load. Therefore, three key factors which determine differences in these analyses stand prominent: NSPA type, TDA type and lateral seismic load type. Systematization of NSPA is presented without further detailed classification of these analyses, regarding that for these analyses different types of incremental-iterative algorithms are used. Analyses belonging to this group are [31]:

- Nonlinear Static Conventional Pushover Analysis (NSCPA),
- Nonlinear Static Adaptive Pushover Analysis (NSAPA),
- Modal Pushover Analysis (MPA),
- Multi-Mode Pushover Procedure (MMPP),
- Method of Modal Combinations (MMC),
- Incremental Response Spectrum Analysis (IRSA),
- Performance-Based Plastic Design (PBPD),
- Nonlinear Static Pushover Analysis - Damage Mechanisms-Based Design (NSPA-DMBD).

Figure 5 shows the flowchart of NSA according to calculation procedure. NSCPA is based on the continuous retention of distribution of lateral seismic load through all the phases of incremental-iterative analysis, i.e. from initial linear to final collapse state of the structure [4]. NSAPA is based on the correction of lateral seismic load by increments, taking into consideration variation of periods of structural vibrations and spectral amplification of seismic forces according to the acceleration response spectrum or correction of displacement according to the displacement response spectrum [3].Control of incremental concept for NSCPA and NSAPA is possible via forces as *Force-Based Analysis* (FBA) or via displacements as *Displacement-Based Analysis* (DBA). Depending on how correction of lateral applied forces is conducted, the following options are possible: total (TU), incremental (IU) and hybrid (HU) correction. Depending on the applied control and correction, the results with various degree of accuracy are obtained, where application of incremental displacement concept is especially emphasized.

kontrole i korekcije, dobijaju se rezultati s manjim ili većim stepenom tačnosti, gde se posebno naglašava primena inkrementalnog koncepta pomeranja.



Slika 5. Podela NSA prema postupku proračuna [31]
Figure 5. Flowchart of NSA according to the calculation procedure[31]

Kod MPA se *pushover* krive mogu razviti po svojstvenim oblicima ili se kombinovati i dobiti konačna rešenja za veći broj svojstvenih oblika transformacijom u bilinearne krive ekvivalentnog sistema s jednim stepenom slobode, radi proračuna ciljnog pomeranja i parametara odgovora [13]. MMPP [40] i MMC [34], takođe, koriste različite principe za kombinacije uticaja svojstvenih oblika u ukupnom odgovoru sistema izraženo preko *pushover* krivih, gde se, pored standardnih, izdvajaju kombinacije direktnih superpozicija, efektivna modalna superpozicija i slično. IRSA u osnovi koristi SMA i pravilo jednakosti pomeranja, s tim što se ukupan odgovor sistema dobija primenom *pushover* krive [8]. U matematičkom smislu ova analiza se može razmatrati kao adaptivna multimodalna *pushover* analiza, u kojoj se simultano izvršavaju MPA za svaki svojstveni oblik, za odgovarajuće skalirano modalno pomeranje praćeno odgovarajućim pravilom za kombinovanje svojstvenih oblika. Prema PBPD se, za performansna stanja na nivou cele zgrade, koristi unapred odabrani drift ciljnog pomeranja i mehanizam plastifikacije pri tečenju [38]. Projektna ukupna smičuća sila u osnovi objekta, za odabrani nivo seizmičkog hazarda, dobija se iz proračuna odnosa količine ukupnog rada potrebnog da se konstrukcija dovede do nivoa ciljnog pomeranja i odgovarajuće zahtevane energije ekvivalentnog SDOF sistema. NSPA–DMBD nastala je povezivanjem NSPA, metode programiranog ponašanja (CDM – *Capacity Design Method*) i analize mehanizama loma (DMBD – *Damage Mechanisms-Based Design*) [17]. NSPA–

In MPA, pushover curves can be evolved according to eigenforms or they can be combined and final solutions for a large number of eigenforms can be obtained by transformation into bilinear curves of the SDOF, for the purpose of calculation of target displacement and response parameters [13]. MMPP [40] and MMC [34], too, utilize different principles for combinations of actions of eigenforms in the total response of the system, expressed via pushover, where, in addition to the standard ones, combinations of direct superpositions, effective modal superposition and similar stand prominent. IRSA basically uses SMA and the rule of equivalent displacement, whereby the total response of the system is obtained through implementation of the pushover curve [8]. In mathematical sense, this analysis can be considered as adaptive multimodal pushover analysis, in which modal pushover analyses are simultaneously performed for each eigenform for corresponding scaled modal displacement followed by the corresponding rule for combining of eigenforms. According to PBPD method, for performance states at the level of the entire building, a drift of target displacement chosen in advance, and yield plastic mechanism are used [38]. Design of the total shearing force at the ground level of the structure, for the chosen level of seismic hazard, is obtained from the calculation of the amount of total work required to bring the structure to the target displacement level and corresponding required energy of equivalent SDOF system. NSPA-DMBD method came into being by bringing together NSPA, Capacity Design Method (CDM) and Damage

DMBD pripada grupi metoda iterativno-interaktivnog dimenzionisanja (IID – *Iterative-Interactive Design*), s obzirom na to što se postupak analize mehanizma loma sistema sprovodi iterativno, a dimenzionisanje proverava nakon dostignute granične dilatacije.

NSA analize koje se zasnivaju na neiterativnim i/ili neinkrementalnim postupcima ili primenjuju poluiterativne i/ili poluinkrementalne postupke jesu:

- projektovanje prema silama (FBD - *Force-Based Design*);
- projektovanje prema pomeranju (DBD – *Displacement-Based Design*);
- projektovanje prema pomeranju bez iteracija (DDBD – *Direct Displacement-Based Design*);
- metoda sekantne superpozicije (SMS – *Secant Modes Superposition*).

Ove analize koriste i izraze formulisane iz velikog broja numeričkih testova, eksperimentalnih istraživanja i statističkih obrada podataka, primenom regresionih analiza, tako da u literaturi postoji velik broj gotovih rešenja, algoritama i analitičkih postupaka. Primenom ovih analiza moguće je još u fazi konceptualnog projektovanja konstrukcija obuhvatiti njihovo nelinearno ponašanje, ne ulazeći u detaljnije aspekte numeričkog modeliranja i kompleksne numeričke proračune. Fundamentalna razlika između FBD i DBD jeste što se kod prvih rešenje dobija polazeći od sila, a kod drugih od pomeranja. DDBD koristi direktan pristup za dobijanje konačnog rešenja, pri čemu se, putem analitičkih postupaka, odgovor sistema dobija kroz elastoplastične modele ponašanja, uspostavljajući relaciju između prigušenja – duktilnosti i pomeranja – perioda vibracija [45]. SMS je razvijena radi dobijanja brzog i dovoljno pouzdanog nelinearnog odgovora sistema za dejstvo zemljotresa, ne uzimajući u obzir direktno NSPA i NDA, ali bazirajući se na sekantnoj krutosti i indeksima odgovora sistema [44]. Rešenje se dobija direktno, za razliku od metoda kod kojih se rešenje dobija po principu *korak po korak*.

TDA, kao što je već rečeno, predstavlja drugi deo NSA analize. Do sada je razvijen veći broj ovih analiza, među kojima su se, za potrebe naučnih istraživanja i stručnih projekata, ustalile:

- metoda spektra kapaciteta (CSM – *Capacity Spectrum Method*);
- neiterativna metoda spektra kapaciteta (NICSM – *Non-Iterative Capacity Spectrum Method*);
- poboljšana metoda spektra kapaciteta (ICSM – *Improved Capacity Spectrum Method*);
- adaptivna metoda spektra kapaciteta (ACSM – *Adaptive Capacity Spectrum Method*);
- metoda koeficijenata pomeranja (DCM – *Displacement Coefficient Method*);
- iterativna metoda koeficijenata pomeranja (IDCM – *Iterative Displacement Coefficient Method*);
- metoda ekvivalentne linearizacije (ELM – *Equivalent Linearization Method*);
- metoda modifikacije pomeranja (DMM – *Displacement Modification Method*);
- N2 metoda (N2 *Method*);
- inkrementalna N2 metoda (IN2 – *Incremental N2 Method*);
- metoda spektra granice tečenja (YPS – *Yield Point Spectra*).

Mechanisms-Based Design (DMBD) [17].NSPA-DMBD method belongs to the group of Iterative-Interactive Design (IID) methods, regarding that the procedure of analysis of system failure mechanism is conducted iteratively, and dimensioning is verified when the ultimate strains have been reached.

NSA analyses based on the non-iterative and/or non-incremental procedures or implementing semi-iterative and/or semi-incremental procedures are:

- Force-Based Design (FBD),
- Displacement-Based Design (DBD),
- Direct Displacement-Based Design (DDBD),
- Secant Modes Superposition (SMS).

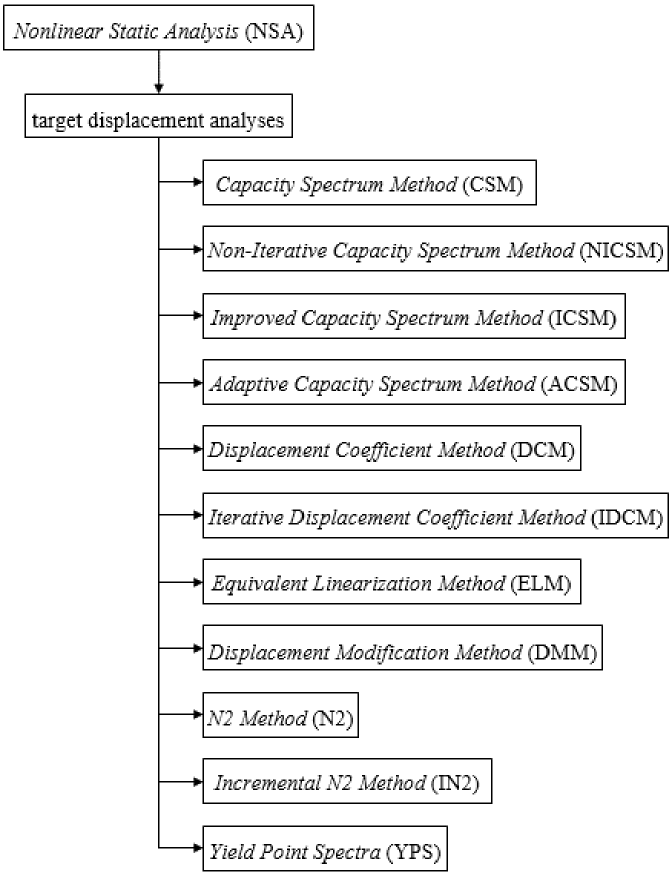
These analyses utilize expressions formulated from a large number of numerical tests, experimental research and statistic data processing, through implementation of regression analyses, so that in literature there is lots of ready-made solutions, algorithm and analytical procedures. By implementing these analyses, it is possible as early as in the phase of conceptual design of structures to include its nonlinear behaviour, without venturing into the more detailed aspects of numerical modelling and complex numerical calculations. Fundamental difference between FBD and DBD analyses is that in the former ones, the solution is obtained using forces as an initial parameter, and in latter ones the displacement parameter is used. DDBD analyses use a direct approach for obtaining the final solution, whereby, through analytical procedures, the response of the system is obtained via elastoplastic behaviour models, by establishing a relation between the damping - ductility and displacement - period of vibrations [43]. SMS analysis is developed with the purpose of obtaining a rapid and sufficiently reliable nonlinear response of the system to earthquake actions, without directly taking into account NSPA and NDA, but basing itself on the secant stiffness and indices of system response [44]. Solution is obtained directly in contrast to the methods where the solution is found *step by step.*

It was presented that TDA represents a second part of NSA analysis. Until now, a large number of these analyses were developed for the purposes of scientific research and professional designs, among which the following are the most common ones:

- Capacity Spectrum Method (CSM),
- Non-Iterative Capacity Spectrum Method (NICSM),
- Improved Capacity Spectrum Method (ICSM),
- Adaptive Capacity Spectrum Method (ACSM),
- Displacement Coefficient Method (DCM),
- Iterative Displacement Coefficient Method (IDCM),
- Equivalent Linearization Method (ELM),
- Displacement Modification Method (DMM),
- N2 Method,
- Incremental N2 Method (IN2),
- Yield Point Spectra (YPS).

Na slici 6 je prikazana podela NSA – TDA prema postupku proračuna.

Figure 6 shows the flowchart of NSA - TDA according to the calculation procedure.



Slika 6. Podela NSA -TDA prema postupku proračuna [31]
Figure 6. Flowchart of NSA - TDA according to the calculation procedure[31]

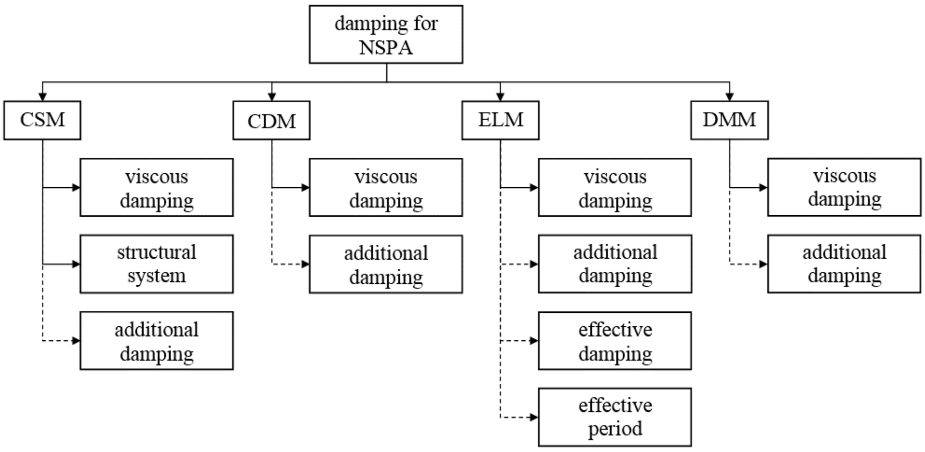
CSM pripada grupi analiza kojom se sprovodi samo TDA iz odnosa krive kapaciteta, krive seizmičkog zahteva i spektra odgovora [6], [32]. Razvijeno je nekoliko tipova CSM koje koriste spektar odgovora u formatu spektralno ubrzanje - spektralno pomeranje (ADRS - *acceleration-displacement response spectra*), pri čemu je postupak određivanja nivoa ciljnog pomeranja iterativan. Ova metoda je implementirana u ATC 40 propise [6]. Kod NICSM se direktno određuje nivo ciljnog pomeranja, bez iteracija, bazirajući se na rešenjima ekvivalentnih linearnih metoda [55]. Takođe, ovoj grupi pripadaju ICSM [54], [33] i ACSM [11], [10] koje su zapravo poboljšane verzije postojeće CSM i koje primenjuju statistički optimizovane linearizovane parametre i adaptivne algoritme za određivanje nivoa ciljnog pomeranja. Primenom DCM sprovodi se samo TDA, koristeći princip multiplikacije grupe koeficijenata kojima se uzima u obzir uticaj različitih faktora ponašanja konstrukcija. Ova metoda je implementirana u FEMA 356 propise [25]. U IDCM implementiran je dvostruki iterativni algoritam koji se sukcesivno sprovodi, a rešenje nivoa ciljnog pomeranja se, između ostalog, pretražuje i po *pushover* krivi [16]. IDCM u osnovi koristi matematičku formulaciju DCM, s tim što je kroz iterativni algoritam znatno unapređeno rešenje dobijanja ciljnog pomeranja. ELM je zapravo novija generacija CSM

CSM belongs to a group of analyses which conduct only TDA from the relations of capacity curve, seismic demand curve and response spectrum [6], [32]. Several types of CSM methods were developed, which use response spectrum in the format spectral acceleration - spectral displacement (ADRS), whereby the procedure of determining target displacement level is iterative. This method is implemented in ATC 40 codes [6]. NICSM directly determines the level of target displacement, without iterations, basing on the solutions of equivalent linear methods [55]. This group also includes ICSM [54], [33] and ACSM [11], [10] which are actually improved versions of the existing CSM and which implement statistically optimized linearized parameters and adaptive algorithms for determination of target displacement level. By implementing DCM only TDA is conducted, employing the principle of multiplication of a group of coefficients which takes into account influence of various factors of structural behaviour. This method is implemented in FEMA 356 codes [25]. In IDCM, successively conducted double iterative algorithm is implemented and the solution of target displacement level is, among other things, searched for using a pushover curve [16]. IDCM basically used mathematical formulation of DCM, whereby, through an iterative algorithm, the solution of target displacement is implementirana u FEMA 440 propise [26], gde se, umesto spektra odgovora u ADRS formatu, koristi modifikovan spektar odgovora (MADRS - *modified acceleration-displacement response spectra*). DMM je, takođe, novija generacija DCM, gde su eliminisani određeni koeficijenti koji participiraju u proračunu, a dodatno su unapređeni delovi proračuna koji se odnose na histerezisne modele ponašanja konstrukcija. Ova metoda je implementirana u FEMA 440 propise [26]. TDA prema N2, implementirana u EN 1998-1:2004 [23] propis, proračunava se uzimajući u obzir neelastični spektar odgovora u funkciji koeficijenta duktilnosti [24]. Proširenje N2 predstavljeno je u formi IN2, kod koje je, osim prezentacije EDP parametara na abscisi i ordinati, moguće koristiti mere intenziteta (IM - *intensity measure*) na ordinati [21]. Na taj način, primenom IN2 može se direktno sprovesti komparacija rešenja sa inkrementalnom dinamičkom analizom (IDA - *Incremental Dynamic Analysis*). Nova spektralna prezentacija seizmičkog zahteva prikazana je YPS metodom, gde je zadržana osnova CSM i NSPA [5]. YPS metoda se može koristiti za projektovanje novih i ojačanje postojećih konstrukcija za odgovarajuće zahtevane nivoe krutosti i nosivosti, uz dodatno ograničenje globalne duktilnosti i drifta.

Kod NSA, odnosno NSPA prigušenje se ne uvodi pre proračuna, već se naknadno definiše nakon proračuna konstrukcije u TDA. Na slici 7 je prikazan dijagram toka uvođenja prigušenja kod NSPA.

considerably improved. ELM is actually a new generation of CSM implemented in FEMA 440 codes [24], where instead of a response spectrum in ADRS format, modified response spectrum is utilized in the format spectral acceleration - spectral displacement (MADRS). DMM is, also, a newer generation of DCM, where certain coefficients participating in calculation were eliminated, while parts of calculation related to hysteretic models of structural behaviour were additionally improved. This method was implemented in FEMA 440 codes [26]. TDA according to N2 method, implemented in EN 1998-1:2004 [23] code is determined by taking into consideration the inelastic response spectrum in function of ductility coefficient [24]. Extension of N2 method is presented in the form of IN2 method, where, except of presentation of EDP parameters on abscissa and ordinate, it is possible to use intensity measure (IM) on ordinate [21]. In this way IN2 method can directly compare solutions with *Incremental Dynamic Analysis* (IDA). New spectral presentation of seismic demand is presented by YPS method, in which the basis of CSM and NSPA was retained [5]. YPS method can be used for designing new and strengthening existing structures for the required levels of stiffness and bearing capacity, with the additional limitation of global ductility and drift.

In NSA or NSPA damping is unlikely to be introduced before calculation; instead, it is subsequently defined after the structure is calculated in TDA. Figure 7 shows the flowchart of introducing damping into NSPA.



Slika 7. Dijagram toka uvođenja prigušenja kod NSPA [30]
Figure 7. Flowchart of introducing damping into NSPA [30]

Postupak uvođenja prigušenja sprovodi se preko jednog globalnog koeficijenta kojim se može uzeti u obzir i viskozno i histerezisno prigušenje. U zavisnosti od tipa analize ciljnog pomeranja moguće su opcije:

- CSM:

Prigušenje se uvodi preko globalnog koeficijenta prigušenja kao osnovno (*inherent*) i dodatno (*additional*) prigušenje, ali se može dodatno uticati i preko tipa konstruktivnog sistema. Za nivo ciljnog pomeranja d_t, koji se određuje kroz iteracije, ukupno efektivno prigušenje u sistemu ξ_{eff} dobija se prema [6]:

Damping is introduced through a global coefficient which can take into account both viscous and hysteretic damping. Depending on the type of TDA, the following options are possible:

- CSM:

Damping is introduced through a global damping coefficient as inherent and additional damping, but it can also be affected through the type of structural system. For the level of target displacement d_t, which is determined by iterations, the overall effective damping in the system ξ_{eff} is obtained from [6]:

$$\xi_{eff} = \kappa\xi_h + \xi_v = \frac{63.7\kappa\left(S_{a,y}S_{d,t} + S_{d,y}S_{a,t}\right)}{S_{a,t}S_{d,t}} + \xi_v\,, \tag{17}$$

gde je ξ_v koeficijent relativnog (viskoznog) prigušenja (5%), ξ_h koeficijent histerezisnog prigušenja (prikazan kao ekvivalentno viskozno prigušenje), $S_{a,y}$ spektralna akceleracija na granici tečenja prikazana u ADRS, $S_{a,t}$ spektralna akceleracija za nivo ciljnog pomeranja, $S_{d,y}$ spektralno pomeranje na granici tečenja, $S_{d,t}$ spektralno pomeranje za nivo ciljnog pomeranja, κ koeficijent kojim se uzima u obzir koliko dobro je histerezisni model konstrukcije aproksimiran bilinearnim histerezisnim modelom.

– CDM:

Prigušenje se uvodi preko koeficijenta efektivnog prigušenja, a koji se se koristi pri generisanju spektra odgovora. U suštini, ovo je viskozno prigušenje, dok se histerezisno određuje iz proračuna, mada se može uvesti i dodatno prigušenje preko ovog koeficijenta [25].

– ELM:

Prigušenje se uvodi preko globalnog koeficijenta prigušenja (osnovno i dodatno prigušenje), ali se kao alternativa može definisati efektivno prigušenje, prikazano preko koeficijenta relativnog prigušenja za histerezisni odgovor sistema [26]:

where ξ_v is the relative (viscous) damping coefficient (5%), ξ_h is the hysteretic damping coefficient (shown as equivalent viscous damping), $S_{a,y}$ is spectral acceleration at the yield point shown in the ADRS format, $S_{a,t}$ is spectral acceleration for the level of target displacement, $S_{d,y}$ is spectral displacement at the yield point, $S_{d,t}$ is spectral displacement for the level of target displacement, κ is the coefficient that takes into account how well the structure's hysteretic model is approximated by the bilinear hysteretic model.

– CDM:

Damping is introduced through the effective damping coefficient which is used in generating the response spectrum. In fact, this is a viscous damping, while a hysteretic damping is determined from calculation, but additional damping may also be introduced over this coefficient [25].

– ELM:

Damping is introduced through the global damping coefficient (basic and additional damping), but effective damping can be defined as an alternative solution, shown through the relative damping coefficient for the system's hysteretic response [26]:

$$\begin{array}{ll} 1 < \mu < 4: & \xi_{eff} = A\left(\mu-1\right)^2 + B\left(\mu-1\right)^3 + \xi_h \\ 4 \le \mu \le 6.5: & \xi_{eff} = C + D\left(\mu-1\right) + \xi_h \\ \mu > 6.5: & \xi_{eff} = E\left(\dfrac{F\left(\mu-1\right)-1}{\left(F\left(\mu-1\right)\right)^2}\right)\left(\dfrac{T_{eff}}{T_0}\right)^2 + \xi_h \end{array}\,, \tag{18}$$

gde su vrednosti za *A*, *B*, *C*, *D*, *E*, *F* date u [26], dok se efektivan period vibracija određuje prema:

where values of *A*, *B*, *C*, *D*, *E*, *F* are given in [26], while the effective period of vibrations is determined by:

$$\begin{array}{ll} 1 < \mu < 4: & T_{eff} = \left(G\left(\mu-1\right)^2 + H\left(\mu-1\right)^3 + 1\right)T_0 \\ 4 \le \mu \le 6.5: & T_{eff} = \left(I + J\left(\mu-1\right) + 1\right)T_0 \\ \mu > 6.5: & T_{eff} = \left(K\left(\sqrt{\dfrac{\left(\mu-1\right)}{1+L\left(\mu-2\right)}} - 1\right) + 1\right)T_0 \end{array}\,, \tag{19}$$

gde su vrednosti za *G*, *H*, *I*, *J*, *K*, *L* date u [26], dok je T_0 inicijalan period vibracija nelinearnog sistema.

– DMM:

Prigušenje se uvodi preko koeficijenta efektivnog prigušenja, a koji se se koristi pri generisanju spektra odgovora. U suštini, ovo je viskozno prigušenje, dok se histerezisno određuje iz proračuna, mada se može uvesti i dodatno prigušenje preko ovog koeficijenta [26].

where values for *G*, *H*, *I*, *J*, *K*, *L* are given in [26], while T_0 is the initial period of vibrations of the nonlinear system.

– DMM:

Damping is introduced through the effective damping coefficient, which is used in generating the response spectrum. Essentially, this is a viscous damping, while a hysteretic is determined from calculation, but additional damping may also be introduced over this coefficient [26].

5 LINEARNA DINAMIČKA ANALIZA (LDA)

Linearna dinamička analiza (LDA – *Linear Dynamic Analysis*) sprovodi se u vremenskom domenu, tako što se za ulazni seizmički signal koristi akcelerogram prirodnog ili veštačkog zemljotresa. Na ordinati vremenskog domena se predstavljaju EDP i IM kao

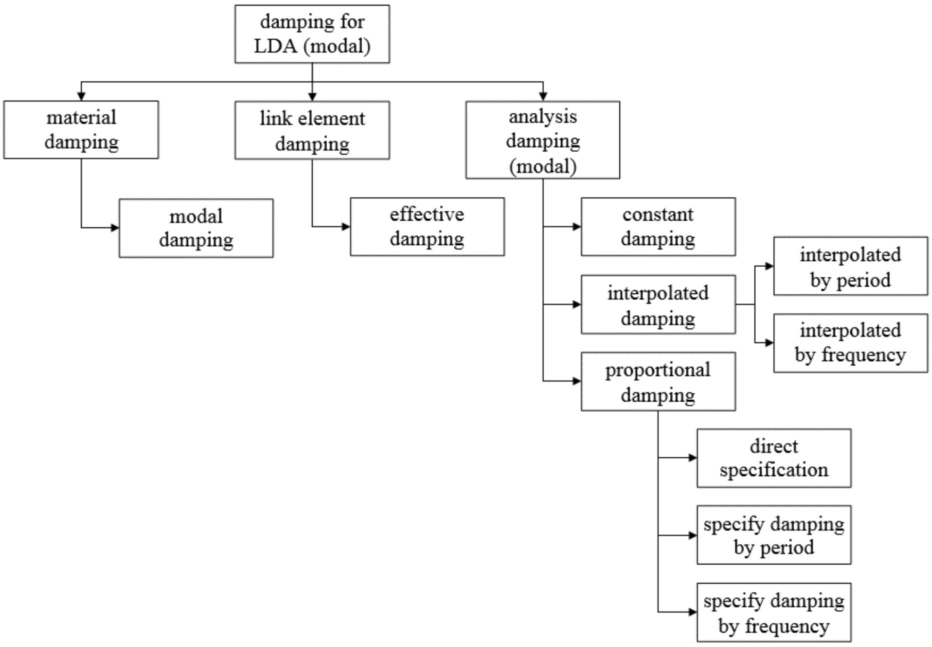
5 LINEAR DYNAMIC ANALYSIS (LDA)

LDA is conducted in time domain with accelerogram of natural or artificial earthquake used as the input seismic signal. The ordinate of time domain presents the EDP and IM as time dependent variables. Using LDA direct solution is obtained for the level of target promenljive u vremenu. Primenom LDA se dobija direktno rešenje za nivo ciljnog pomeranja, tako da je ovo ujedno i TDA. Akcelerogram se skalira i/ili kompatibilizuje prema projektnom spektru odgovora iz propisa ili se koristi reprezentativan spektar odgovora grupe akcelerograma, pa se on naknadno skalira i/ili kompatibilizuje prema projektnom spektru odgovora iz propisa.

Uvođenje prigušenja u modalnu LDA moguće je sprovesti primenom: prigušenja materijala, prigušenja elemenata veze i prigušenja u analizi. Na slici 8 je prikazan dijagram toka uvođenja prigušenja kod modalne LDA. Prigušenje materijala se uvodi kao modalno prigušenje, dok se prigušenje elemenata veze uvodi kao efektivno prigušenje. Prigušenje koje se direktno definiše u analizi uvodi se kao: konstantno prigušenje, interpolirano prigušenje i primenom faktora participacije mase i krutosti, pri čemu se ovo poslednje prigušenje može uvesti primenom: faktora participacije mase i krutosti α i β, u funkciji perioda vibracija prvog i drugog svojstvenog oblika T_1 i T_2 i u funkciji frekvencija prvog i drugog svojstvenog oblika f_1 i f_2.

displacement, so this is TDA in the same time. The accelerogram is scaled and/or made compatible according to the spectrum response under terms of regulations, or a representative response spectrum is used from the accelerogram group, which is later scaled and/or made compatible based on the project response spectrum provided in regulations.

Damping in modal LDA can be introduced by using: material damping, link element damping and damping in the analysis. Figure 8 shows the flowchart of damping introduction into modal LDA. Material damping is introduced as modal damping, while the link element damping is introduced as effective damping. Damping that is directly defined in the analysis is introduced as: constant damping, interpolated damping, and using the factors of mass and stiffness participation, whereby the latter can be introduced using: factors of mass and stiffness (α and β) participation as function of vibration period and frequency.



Slika 8. Dijagram toka uvođenja prigušenja kod modalne LDA [30]
Figure 8. Flowchart of introducing damping into modal LDA [30]

Diferencijalne jednačine kretanja sistema sa više stepeni slobode, kod modalne LDA, formulišu se u matričnom obliku [52]:

Differential equations of movement of systems with several degrees of freedom, such as modal LDA, are formulated in matrix form [52]:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\}, \tag{20}$$

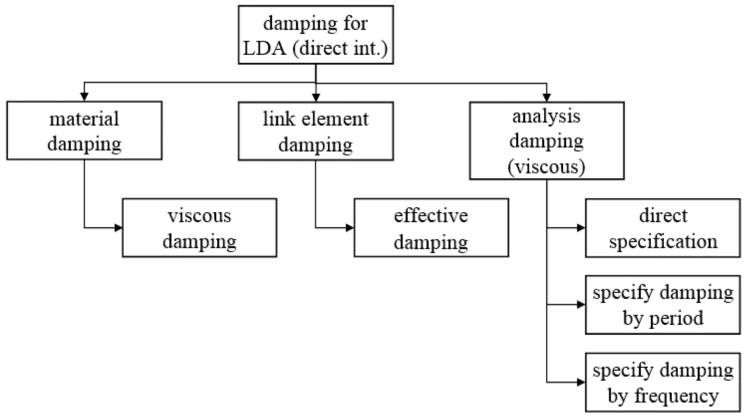
gde je [*M*] matrica masa, [*K*] matrica krutosti, $\{\ddot{u}\}$ vektor ubrzanja, $\{\dot{u}\}$ vektor brzine, {*u*} vektor pomeranja konstrukcije, {*F*} vektor spoljašnjeg opterećenja. Prilikom proračuna modalne LDA formira se matrica prigušenja prema izrazu (20), a u zavisnosti od tipa prigušenja koje je definisano pre izvršenja analize.

Uvođenje prigušenja u LDA (numerička integracija) moguće je sprovesti primenom: prigušenja materijala,

where [*M*] is the mass matrix, [*K*] the stiffness matrix, $\{\ddot{u}\}$ the acceleration vector, $\{\dot{u}\}$ the velocity vector, {*u*} the structural displacement vector, and {*F*} the external load vector. When calculating the modal LDA, the damping matrix is formed according to the expression (20), and depending on the type of damping which is defined prior to execution of the analysis.

In numerical integration LDA damping can be introduced by using: material damping, link element prigušenja elemenata veze i prigušenja u analizi. Na slici 9 je prikazan dijagram toka uvođenja prigušenja kod LDA (numerička integracija).

damping and damping in the analysis. Figure 9 shows the flowchart of damping introduction into numerical integration LDA.



Slika 9. Dijagram toka uvođenja prigušenja kod LDA (numerička integracija) [30]
Figure 9. Flowchart of introducing damping into numerical integration LDA [30]

Prigušenje materijala se uvodi kao viskozno prigušenje, dok se prigušenje elemenata veze uvodi kao efektivno prigušenje. Prigušenje koje se direktno definiše u analizi uvodi se primenom: faktora participacije mase i krutosti α i β, u funkciji perioda vibracija prvog i drugog svojstvenog oblika T_1 i T_2 i u funkciji frekvencija prvog i drugog svojstvenog oblika f_1 i f_2. Diferencijalne jednačine kretanja sistema sa više stepeni slobode, kod LDA (numerička integracija), formulišu se u matričnom obliku [52]:

Material damping is introduced as viscous damping, while the link element damping is introduced as effective damping. Damping that is directly defined in the analysis is introduced by using: the factors mass and stiffness participation (α and β) as a function of the vibration period and a function of frequency. In numerical integration LDA, differential equations of the movement of system with several degrees of freedom are formulated in matrix form [52]:

$$[M]\{\Delta\ddot{u}\}+[C]\{\Delta\dot{u}\}+[K]\{\Delta u\}=\{\Delta F\}, \tag{21}$$

gde je $\{\Delta u^{\cdot\cdot}\}$ vektor inkrementa ubrzanja, $\{\Delta u^{*}\}$ vektor inkrementa brzine, $\{\Delta u\}$ vektor inkrementa pomeranja konstrukcije, $\{\Delta F\}$ inkrement vektora spoljašnjeg opterećenja. Prilikom proračuna LDA (numerička integracija) formira se matrica prigušenja prema izrazu (21), a u zavisnosti od tipa prigušenja koje je definisano pre izvršenja analize.

where $\{\Delta\ddot{u}\}$ is the acceleration increment vector, $\{\Delta\dot{u}\}$ the speed increment vector, $\{\Delta u\}$ the vector of structural displacement increment, and $\{\Delta F\}$ is the vector of external load increment. When calculating the numerical integration LDA, a damping matrix is formed according to expression (21) as a function of the type of damping which is defined prior to the execution of the analysis.

6 NELINEARNA DINAMIČKA ANALIZA (NDA)

U odnosu na rešenja koja se dobijaju u kapacitativnom domenu primenom NSA, kod NDA rešenja se dobijaju u vremenskom domenu. Proračun nelinearnog odgovora se sprovodi primenom numeričke integracije, pri čemu se najčešće primenjuje *Newmark*-ova metoda prosečnog ubrzanja (AAM – *Average Acceleration Method*) ili metoda linearnog ubrzanja (LAM – *Linear Acceleration Method*), a takođe, primenjuje se i *Wilson*-ov, *Hilber–Hughes–Taylor*-ov i *Chung–Hulbert*-ov postupak. Najtačnije metode za analizu seizmičkog odgovora sistema jesu NDA analize, ukoliko se uzima u obzir potpun razvoj materijalne nelinearnosti, plastičnim zglobovima ili propagacijom neelastičnih deformacija primenom vlakana, i geometrijske nelinearnosti, kada se u analizi uzimaju u obzir velike deformacije i pomeranja. U ove analize se ubrajaju:

- nelinearna dinamička analiza zasnovana na modalnoj i numeričkoj integraciji (NDA – *Nonlinear*

6 NONLINEAR DYNAMIC ANALYSIS(NDA)

In comparison with the solutions obtained in the capacitive domain using NSA, in NDA solutions are obtained in time domain. Nonlinear response calculation is conducted by implementing numerical integration, whereby the most frequently implemented is *Newmark Average Acceleration Method* (AAM) or *Newmark Linear Acceleration Method* (LAM), along with the procedures by *Wilson*, *Hilber-Hughes-Taylor* and *Chung-Hulbert*. The most accurate methods for seismic response analysis are NDA, if considering full development of material nonlinearity through plastic hinges or propagation of inelastic deformations by using fibres, and geometrical nonlinearities when the analysis takes into account large deformations and displacements. These analyses include:

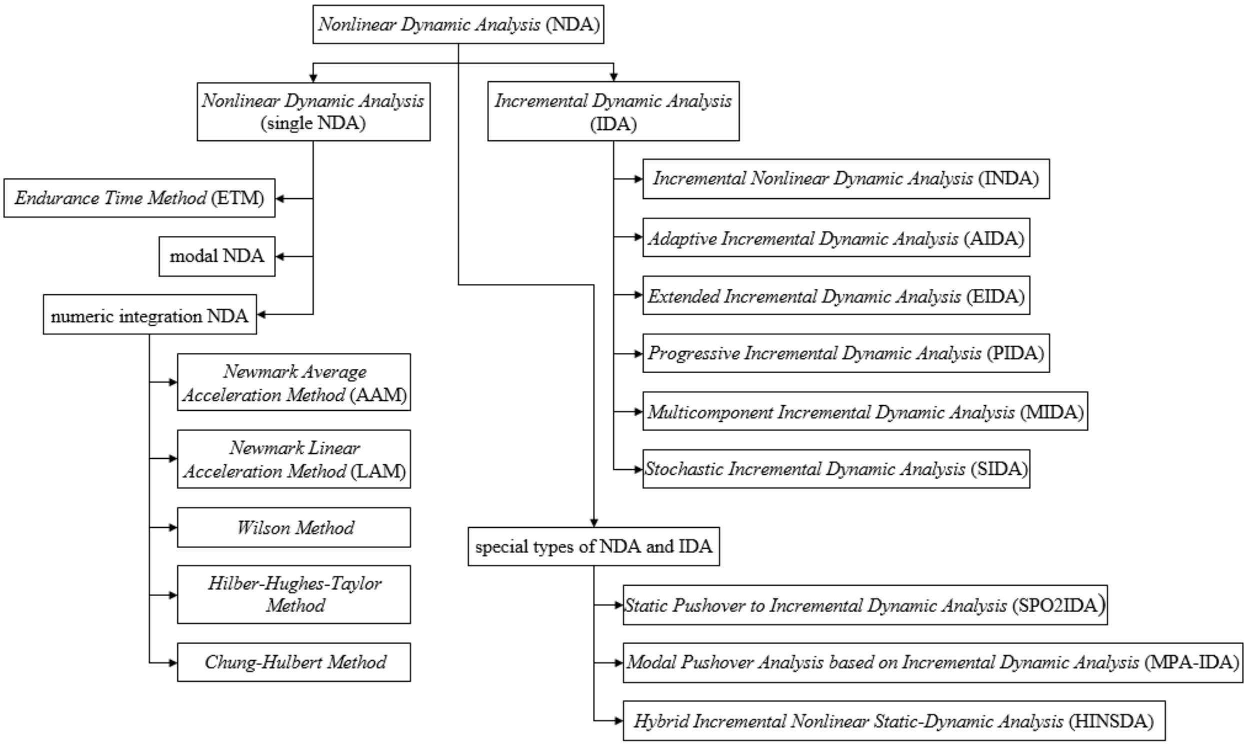
- modal and numerical integration *Nonlinear Dynamic Analysis*(NDA),
- *Endurance Time Method* (ETM).

Dynamic Analysis);

– metod vremena izdržljivosti (ETM – *Endurance Time Method*).

Na slici 10 je prikazana podela NDA prema postupku proračuna [31].

Figure 10 shows the NDA flowchart according to the calculation procedure [31].



Slika 10. Podela NDA prema postupku proračuna [31]
Figure 10. NDA flowchart according to the calculation procedure[31]

Primenom NDA dobija se, kao što je već rečeno, odgovor sistema u vremenskom domenu, ali za samo jedan nivo seizmičkog zahteva. S druge strane, primenom ETM dobija se odgovor sistema u vremenskom domenu s kontinualnim priraštajem nelinearnih deformacija, od inicijalnog elastičnog do kolapsnog stanja [7]. Specifičnost ove metode ogleda se u primeni posebno konstruisane funkcije pobude – akcelerograma koji je, između ostalog, dodano kompatibilizovan prema spektru odgovora i optimizovan za nelinearan odgovor sistema.

Ukoliko se primeni set NDA sukcesivno povećavajući faktor skaliranja akcelerograma, tada se konačno rešenje može dobiti u kapacitativnom domenu. U tom smislu je veoma povoljno sprovoditi komparaciju rešenja dobijenih NSA i IDA. Faktički, rešenje dobijeno iz seta NDA u vremenskom domenu transformiše se u kapacitativan domen. Ovo se sprovodi tako što se izdvajaju ekstremne i odgovarajuće diskretne vrednosti, koje se zatim interpoliraju splajn funkcijom. Analize koje pripadaju ovoj grupi jesu [31]:

– inkrementalna dinamička analiza (IDA – *Incremental Dynamic Analysis*);

– inkrementalna nelinearna dinamička analiza (INDA – *Incremental Nonlinear Dynamic Analysis*);

– adaptivna inkrementalna dinamička analiza (AIDA – *Adaptive Incremental Dynamic Analysis*);

As already mentioned, a system response in time domain is obtained by implementing NDA, but only for one level of seismic demand. On the other hand, implementation of ETM provides system response in time domain with continuous increase of nonlinear deformations, from initially elastic to collapse state [7]. Singularity of this method reflects in implementation of specially designed excitation function (accelerogram) which is, inter alia, additionally compatible with the response spectrum and optimized for nonlinear system response.

If a set of NDA is implemented while successively increasing scaling factor of the accelerogram, then the final solution is obtained in capacitive domain. Thus, it is favourable to conduct comparison of solutions obtained by NSA and IDA. Actually, solution obtained from a set of NDA in time domain is transformed into capacitive domain. This is performed by singling out extreme and corresponding discrete values which are then interpolated by spline functions. Analyses belonging to this group are [31]:

– Incremental Dynamic Analysis (IDA),

– Incremental Nonlinear Dynamic Analysis (INDA),

– Adaptive Incremental Dynamic Analysis (AIDA),

– Extended Incremental Dynamic Analysis (EIDA),

– Progressive Incremental Dynamic Analysis (PIDA),

- proširena inkrementalna dinamička analiza (EIDA – *Extended Incremental Dynamic Analysis*);
- progresivna inkrementalna dinamička analiza (PIDA – *Progressive Incremental Dynamic Analysis*);
- multikomponentalna inkrementalna dinamička analiza (MIDA – *Multicomponent Incremental Dynamic Analysis*);
- stohastička inkrementalna dinamička analiza (SIDA – *Stochastic Incremental Dynamic Analysis*).

Termin IDA već je ustaljen u naučnim istraživanjima [51], dok je termin INDA prvi put uveden u [18] i ove analize se odnose na set NDA kod kojih se akcelerogram sukcesivno skalira, pri čemu je konstrukcija modelirana tako da najbolje opisuje realan fizički model konstrukcije i gde je uveden razvoj potpune materijalne i geometrijske nelinearnosti. AIDA se zasniva na adaptivnoj promeni selekcije zapisa ubrzanja tla pri različitim intenzitetima kretanja tla [39], dok se kod EIDA uvode neizvesnosti: zavisne od modela konstrukcije (*epistemic uncertainty*) i zavisne od seizmičkog hazarda i selekcije zapisa ubrzanja tla (*aleatoric uncertainty*) [22]. Neizvesnosti zavisne od modela konstrukcije određuju se primenom *Latin Hypercube Sampling* (LHS) metode. PIDA je razvijena kako bi se skratilo vreme potrebno za sprovođenje obimnih IDA, a da se zadrži nivo kvaliteta rešenja [9]. Takođe, slično PIDA, razvijene su MIDA i SIDA, s tim što se prvom analizom može razmatrati nelinearan odgovor sistema za različite uglove dejstva zemljotresa [37], a drugom analizom se stohastičkim modelovanjem, između ostalog, primenom *Point Estimation Method* (PEM) dobija rešenje u domenu kapaciteta [53].

Posebni tipovi NDA koji dobijaju rešenja u kombinaciji s drugim metodama jesu:

- statička *pushover* analiza zasnovana na inkrementalnoj dinamičkoj analizi (SPO2IDA – *Static Pushover to Incremental Dynamic Analysis*);
- modalna *pushover* analiza zasnovana na inkrementalnoj dinamičkoj analizi (MPA-IDA – *Modal Pushover Analysis based on Incremental Dynamic Analysis*);
- hibridna inkrementalna nelinearna statička-dinamička analiza (HINSDA – *Hybrid Incremental Nonlinear Static-Dynamic Analysis*).

SPO2IDA je razvijena u okviru istraživanja [50], a bazira se na primeni NSPA i niza regresionih analiza kojima se simulira IDA odgovora sistema. Na taj način se dobija odgovor sistema u kapacitativnom domenu, pri čemu se na abscisi koriste EDP parametri, a na ordinati IM mere. Kod IDA–MPA seizmički odgovor sistema se određuje iz NDA analize SDOF sistema, koji je ekvivalentan MDOF sistemu [42]. Radi dobijanja bržeg i dovoljno pouzdanog rešenja, u odnosu na INDA, razvijena je potpuno nova procedura nazvana hibridna nelinearna statička-dinamička analiza (HNSDA – *Hybrid Nonlinear Static-Dynamic Analysis*) [18]. U HNSDA analizi se koristi nelinearan odgovor MDOF sistema iz NSPA za proračun na korigovanom SDOF sistemu primenom NDA. Ukoliko se nelinearan odgovor sistema razmatra u kapacitativnom domenu, tada ova analiza postaje hibridna inkrementalna nelinearna statička-dinamička analiza (HINSDA).

Ključni aspekt kod TDA za NDA jeste procesiranje akcelerograma prema teoriji obrade signala. Na slici 11 je prikazana podela NDA – TDA prema postupku

- Multicomponent Incremental Dynamic Analysis (MIDA),
- Stochastic Incremental Dynamic Analysis (SIDA).

The term IDA is already well-established in scientific research [51], while the term INDA was for the first time introduced in [18] and these analyses refer to a set of NDA in which an accelerogram is successively scaled, whereby the structure is modelled to provide the best possible actual physical model of a structure and in which development of complete material and geometric nonlinearity was introduced. AIDA is based on the adaptive variation of selection of ground motion records at different intensities of ground motion [39], while EIDA introduces into the calculation epistemic (depending on the structure model) and aleatoric (depending on the seismic hazard and selection of ground motion records) uncertainties [22]. Epistemic uncertainty is determined by implementing *Latin Hypercube Sampling* (LHS) method. PIDA was developed with an aim of shortening the time necessary for performing of extensive IDA, while retaining the quality level of the solution [9]. Also, similar to PIDA, MIDA and SIDA were developed, whereby the former analysis can analyze a nonlinear system response for different angles of earthquake actions [37], and the latter analysis, through stochastic modelling inter alia, by implementing *Point Estimation Method* (PEM) a solution in capacitive domain is obtained [53].

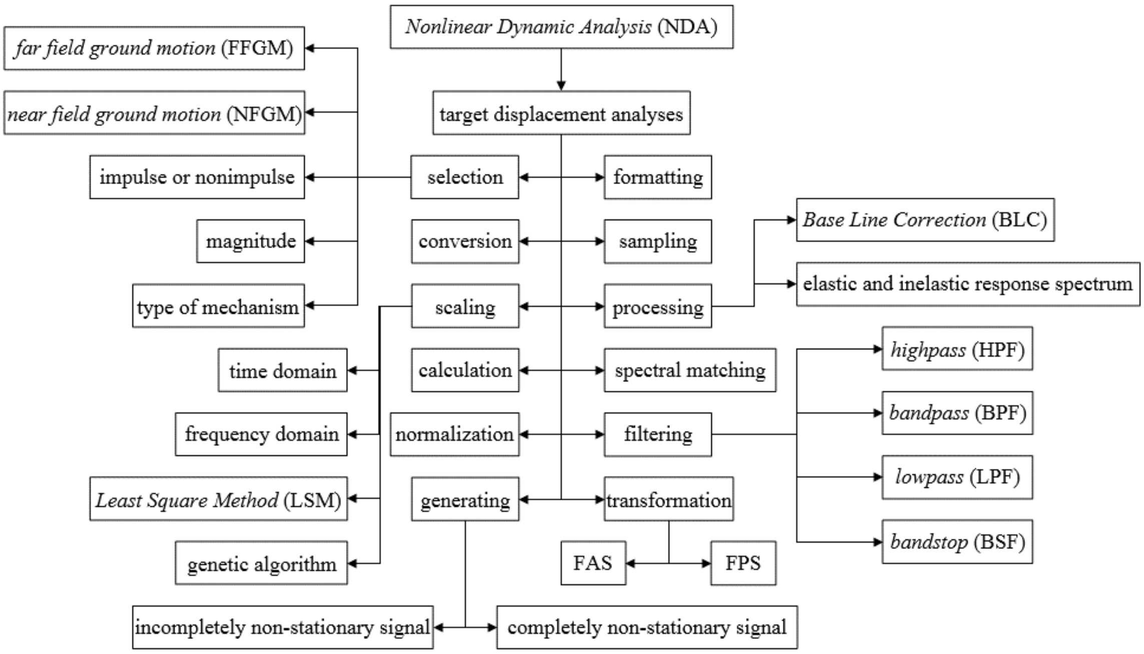
Special types of NDA which obtain solutions in combination with other methods are:

- Static Pushover to Incremental Dynamic Analysis (SPO2IDA),
- Modal Pushover Analysis based on Incremental Dynamic Analysis (MPA-IDA),
- Hybrid Incremental Nonlinear Static-Dynamic Analysis (HINSDA).

SPO2IDA method is developed in the framework of research [50], and it is based on implementation of NSPA and a number of regression analyses which simulate IDA system response. The obtained system response is located in a capacitive domain, whereby EDP parameters are used on abscissa and IM parameters on ordinate. In the case of IDA-MPA, seismic system response is determined from NDA of SDOF system, which is equivalent to MDOF system [42]. In order to obtain a more rapid and sufficiently reliable solution, in comparison with INDA, a completely new procedure called *Hybrid Nonlinear Static-Dynamic Analysis* (HNSDA) was developed [18]. Nonlinear response to MDOF system is used in HNSDA from NSPA intended for calculation on the corrected SDOF system by implementing NDA. If nonlinear system response is considered in a capacitive domain, then this analysis becomes *Hybrid Incremental Nonlinear Static-Dynamic Analysis* (HINSDA).

The key aspect for TDA, for NDA, is processing of an accelerogram according to signal processing theory. Figure 11 shows the NDA - TDA flowchart according to the calculation procedure [31].

proračuna [31].



Slika 11. Podela NDA - TDA prema postupku proračuna [31]
Figure 11. NDA - TDA flowchart according to the calculation procedure[31]

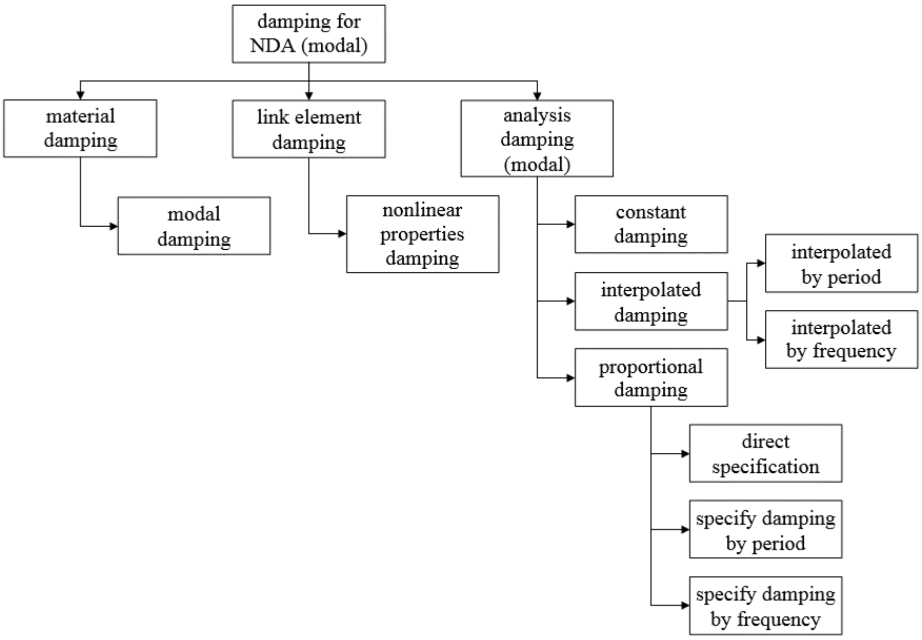
Postupak procesiranja akcelerograma obuhvata analizu, interpretaciju i prezentaciju akcelerograma kroz faze: selekcija, formatiranje, konvertovanje, semplovanje, skaliranje, kalkulacija, procesiranje, kompatibilizacija (*spectral matching*), normalizacija, filtriranje, generisanje i transformacija [15]. Ove procedure se izvršavaju u vremenskom, frekventnom, frekventno-vremenskom i kapacitativnom domenu. Selekcija je procedura odabira određenog tipa zemljotresa ili grupe zemljotresa prema unapred zadatim kriterijumima, kao što je selekcija prema kriterijumima da li su zemljotresi udaljeni (FFGM – *far field ground motion*) ili bliski (NFGM – *near field ground motion*), impulsni ili neimpulsni zemljotresi, prema magnitudi, tipu mehanizma, udaljenosti od mesta iniciranja propagacije seizmičkih talasa, brzini smičućih talasa u tlu za gornjih 30m dubine, hipocentralnom rastojanju ili prema nekom drugom kriterijumu. Formatiranje je procedura transformacije oblika zapisa akcelerograma iz baze zemljotresa i prilagođavanje softveru za analizu konstrukcija, dok je konvertovanje procedura transformacije jednih jedinica mere u druge. Skaliranje je skup procedura kojima se direktno ili indirektno multipliciraju vrednosti ubrzanja akcelerograma prema određenim kriterijumima. Skaliranje akcelerograma se sprovodi primenom nekoliko procedura, od kojih se izdvajaju: skaliranje akcelerograma u vremenskom domenu, skaliranje akcelerograma u frekventnom domenu, skaliranje preko spektra odgovora primenom metode najmanjih kvadrata (LSM – *Least Square Method*), skaliranje preko spektra odgovora primenom genetičkog algoritma, kompatibilizacija (*spectral matching*) i slične procedure. Kalkulacija je skup procedura kojima se određuju bazni parametri akcelerograma, kao što su mere intenziteta (IM), dok je procesiranje skup procedura koje mogu biti različitog karaktera, kao što je korekcija bazne linije (BLC – *Base*

The accelerogram processing procedure includes analysis, interpretation and presentation of accelerogram through the phases: selection, formatting, conversion, sampling, scaling, calculation, processing, spectral matching, normalization, filtering, generating and transformation [15]. These procedures are executed in time, frequency, frequency-time and capacitive domain. Selection is a procedure of choosing certain type of earthquakes or group of earthquakes according to the criteria set in advance, such as the selection according to the criteria whether earthquakes are far field ground motion (FFGM) or near field ground motion (NFGM), impulse or non-impulse ones, according to their magnitude, type of mechanism, distance from the location of initiation of propagation of seismic waves, velocity of shear waves in the ground in the top 30m of depth, hypo central distance or according to some other criteria. Formatting is the procedure of transformation of accelerogram record from the earthquake database and adaptation for the software for structural analysis, while conversion is the procedure of transformation of a measurement unit into the other one. Scaling is a set of procedures which directly or indirectly multiply values of acceleration of the accelerogram according to certain criteria. Scaling of accelerograms is performed by implementing several procedures, the following ones standing prominent: scaling of accelerograms in time domain, scaling of accelerograms in frequency domain, scaling through response spectrum implementing *Least Square Method* (LSM), scaling through the response spectrum by implementing genetic algorithm, spectral matching and similar procedures. Calculation is a set of procedures which determine basic parameters of accelerogram, such as intensity measures (IM), while processing is a set of procedures which can have different character such as *Base Line Correction* (BLC), *Line Correction*), konstrukcija elastičnog i neelastičnog spektra odgovora i slične procedure. Kompatibilizacija je procedura kreiranja reprezentativnog (kompatibilnog) akcelerograma na osnovu jednog realnog ili grupe akcelerograma prema zadatom projektnom spektru odgovora. Normalizacija je procedura uravnoteženja dve komponente zemljotresa kada se koriste akcelerogrami za bidirekciono seizmičko dejstvo, dok je filtriranje procedura primene određenih filtera u cilju eliminacije nebitnih frekvencijskih opsega i zadržavanja bitnih frekvencijskih opsega. Najčešće se koriste visokopropusni (HPF – *highpass*) i pojasnopropusni (BPF – *bandpass*), a takođe i niskopropusni (LPF – *lowpass*) i pojasna brana (BSF – *bandstop*) filter. Generisanje je procedura kreiranja novih akcelerograma, kao što su veštački (*artificial*) ili sintetički (*synthetic*) akcelerogrami na osnovu definisanih procedura u frekventnom domenu. Ovi akcelerogrami se generišu kao nepotpuni nestacionarni ili potpuni nestacionarni akcelerogrami. Transformacija je procedura kojom se određuje frekvencijski sadržaj akcelerograma, odnosno vrednosti amplituda po frekvencijama u frekventnom domenu primenom *Fourier*-ovih transformacija.

Uvođenje prigušenja u modalnu NDA moguće je sprovesti primenom: prigušenja materijala, prigušenja kod nelinearnog ponašanja elemenata veze i prigušenja u analizi. Na slici 12 je prikazan dijagram toka uvođenja prigušenja kod modalne NDA. Prigušenje materijala se uvodi kao modalno prigušenje, dok se prigušenje elemenata veze uvodi uzimajući u obzir predefinisane parametre za nelinearno prigušenje i razvoj histerezisnog ponašanja. Prigušenje koje se direktno definiše u analizi uvodi se identično kao kod modalne LDA: konstantno prigušenje, interpolirano prigušenje i primenom faktora participacije mase i krutosti.

structure of elastic and inelastic response spectrum and similar procedures. Spectral matching is a procedure of creation of representative (compatible) accelerogram on the basis of one real or group of accelerograms according to the given design response spectrum. Normalization is the procedure of balancing two earthquake components when accelerograms for bidirectional seismic action are used, while filtering is the procedure of implementation of certain filters with the purpose of elimination of unimportant frequency range and retaining important frequency range. Highpass (HPF) and bandpass (BPF), as well as lowpass (LPF) and bandstop (BSF) filters are used most frequently. Generation is the procedure of creation of new accelerograms such as artificial or synthetic accelerograms, based on the defined procedures in frequency domain. These accelerograms are generated as incompletely non-stationary or completely non-stationary accelerograms. Transformation is the procedure used for determining the frequency content of an accelerogram, i.e. values of amplitudes by frequencies in a frequency domain via implementing *Fourier* transforms.

Damping in modal NDA analysis can be introduced by using: material damping, damping in the nonlinear behaviour of link elements and damping in the analysis. Figure 12 shows the flowchart of damping introduction into modal NDA. Material damping is introduced as modal damping, while damping of link elements is introduced taking into account the predefined parameters for nonlinear damping and the development hysteretic behaviour. Damping that is directly defined in the analysis is introduced in the same way as in modal LDA: constant damping, interpolated damping and using the factors of mass and stiffness participation.



Slika 12. Dijagram toka uvođenja prigušenja kod modalne NDA [30]
Figure 12. Flowchart of introducing damping into modal NDA [30]

Diferencijalne jednačine kretanja sistema s više stepeni slobode, kod nelinearne dinamičke (modalne) analize, formulišu se u matričnom obliku [52]:

Differential equations of movement of system with several degrees of freedom, such as the modal NDA, are formulated in a matrix form [52]:

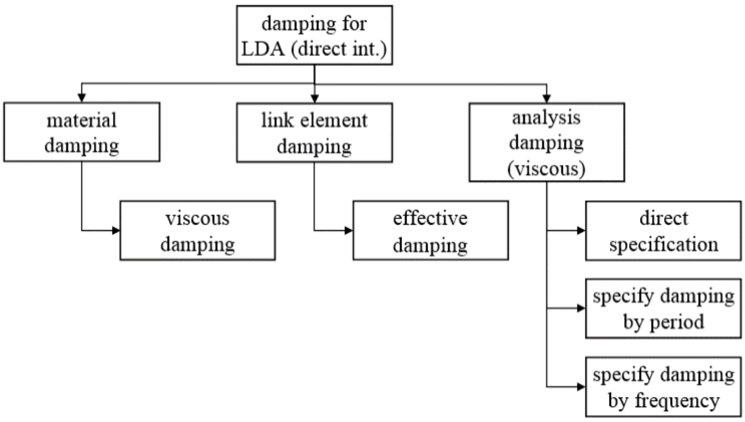
$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}+\{F\}_{NL}=\{F\}, \tag{22}$$

gde je $\{F\}_{NL}$ vektor sila koje potiču od elemenata veza s nelinearnim ponašanjem. Prilikom proračuna nelinearne dinamičke (modalne) analize formira se matrica prigušenja prema izrazu (22), a u zavisnosti od tipa prigušenja koje je definisano pre izvršenja analize. Efikasnost ove analize je u tome što se razdvaja vektor sila koje potiču od elemenata veza s nelinearnim ponašanjem od matrica elastične krutosti i prigušenja.

Uvođenje prigušenja u NDA (numerička integracija) moguće je sprovesti primenom: prigušenja materijala, prigušenja elemenata veze i prigušenja u analizi. Na slici 13 je prikazan dijagram toka uvođenja prigušenja kod NDA (numerička integracija). Prigušenje materijala se uvodi kao viskozno prigušenje, dok se prigušenje elemenata veze uvodi uzimajući u obzir predefinisane parametre za nelinearno prigušenje i razvoj histerezisnog ponašanja. Prigušenje koje se direktno definiše u analizi uvodi se identično kao kod LDA (numerička integracija). Diferencijalne jednačine kretanja sistema sa više stepeni slobode, kod NDA (numerička integracija), formulišu se analogno izrazu (22), s tim što se matrica krutosti formira uzimajući u obzir razvoj geometrijske i materijalne nelinearnosti, a proračunava primenom inkrementalno-iterativnog postupka. Prilikom proračuna NDA (numerička integracija) formira se matrica prigušenja prema izrazu (22), a u zavisnosti od tipa prigušenja koje je definisano pre izvršenja analize.

where $\{F\}_{NL}$ is the force vector coming from joint elements with nonlinear behaviour. During the calculation of modal NDA, a damping matrix is formed according to the expression (22), depending on the type of damping which is defined prior to the execution of the analysis. The efficiency of this analysis is reflected in separating the force vector originating from the link element with nonlinear behaviour from the matrix of elastic stiffness and damping.

In numerical integration NDA damping can be introduced by using: material damping, link element damping and damping in the analysis. Figure 13 shows the flowchart of damping introduction into numerical integration NDA. Material damping is introduced as viscous damping, while the link element damping is introduced taking into account the predefined parameters for nonlinear damping and the development hysteretic behaviour. Damping that is directly defined in the analysis is introduced in the same way as in numerical integration LDA. In numerical integration NDA, differential equations of movement of system with several degrees of freedom are formulated in analogy with expression (22), provided that the stiffness matrix is formed taking into account the development of geometric and material nonlinearity, and it is calculated using the incremental - iterative procedure. When calculating the numerical integration NDA the damping matrix is formed according to the expression (22), and depending on the type of damping which is defined prior to the execution of the analysis.



Slika 13. Dijagram toka uvođenja prigušenja kod NDA (numerička integracija) [30]
Figure 13. Flowchart of introducing damping into numerical integration NDA [30

U slučaju izraženog nelinearnog ponašanja, usled stalnog pada krutosti sistema, nastupa konstantno smanjenje prigušenja, mada to i nema fizičkog opravdanja [14]. Tada je najpovoljnije da se matrica prigušenja formira na početku proračuna, kao proporcionalna početnoj linearnoj matrici krutosti uz zanemarenje člana koji je proporcionalan matrici masa. Objašnjenje za ovo leži u činjenici da su efekti histerezisne disipacije, kod nelinearnih sistema, dominantniji u odnosu na efekte viskoznog prigušenja, a koje je izraženo kod linearnih sistema. Eliminacija člana koji je proporcionalan matrici masa omogućava veće prigušenje viših svojstvenih oblika u odnosu na prigušenje nižih svojstvenih oblika.

Constant reduction of damping occurs in the case of pronounced nonlinear behaviour due to constant reduction of the system stiffness, although it lacks any physical justification [14]. Beginning of calculation is the best time to form the damping matrix as proportional to initial linear stiffness matrix while neglecting the member which is proportional to the mass matrix. This can be explained by the fact that effects of hysteretic dissipation in nonlinear systems are more dominant than the effects of viscous damping, which is more expressed in linear systems. Eliminating the member which is proportional to the mass matrix allows higher eigenforms to be damped more than the lower eigenforms.

7 SEIZMIČKE ANALIZE PREMA PERFORMANCE-BASED EARTHQUAKE ENGINEERING (PBEE)

Performance-Based Earthquake Engineering (PBEE) metodologija je inicirana u poslednjih dvadesetak godina prvo na determinističkom, a zatim i na probabilističkom nivou. PBEE metodologija se zasniva na multidisciplinarnom pristupu putem: računarske mehanike, numeričke metode, dinamike konstrukcija, nelinearne analize, teorije armiranobetonskih konstrukcija, teorije plastičnosti, mehanike loma, interakcije konstrukcija–tlo, zemljotresnog inženjerstva, inženjerske seizmologije, primene savremenih propisa za projektovanje konstrukcija, inženjerske statistike i verovatnoće. Razvoj savremene PBEE metodologije omogućava kompletnije i kompleksnije sagledavanje i tretiranje problema analizom hazarda (*hazard analysis*), analizom konstrukcije (*structural analysis*), analizom oštećenja (*damage analysis*) i analizom štete (*loss analysis*) [36], [41]. Analiza hazarda se predstavlja promenljivom mere intenziteta (IM), kojim se kvantifikuje pomeranje tla, dok se analiza konstrukcije predstavlja primenom inženjerskog parametra zahteva (EDP). Analiza oštećenja se predstavlja promenljivom mere oštećenja (DM), a analiza štete promenljivom odluke (DV). Uspostavljanje veze između IM i EDP sprovodi se preko modela seizmičkog zahteva (*seismic demand model*), a koji se određuje primenom probabilističke analize seizmičkog zahteva (PSDA – *Probabilistic Seismic Demand Analysis*) i INDA analize. Međutim, pre uspostvaljanja veze EDP–IM potrebno je razmotriti IM promenljivu primenom probabilističke analize seizmičkog hazarda (PSHA - *Probabilistic Seismic Hazard Analysis*). Na osnovu određenog IM iz PSHA i EDP iz PSDA, NDA ili čak preko NSPA, uspostavlja se korelacija EDP–IM, najčešće preko spektralnog ubrzanja za IM i globalnog ili međuspratnog drifta za EDP. Model seizmičkog zahteva u PSDA analizi se može predstaviti i preko krivih povredljivosti (*fragility curves*). Uspostavljanje veze između EDP i DM sprovodi se preko modela oštećenja (*damage model*), a koji se određuje primenom probabilističke analize seizmičkog oštećenja (PSDamA – *Probabilistic Seismic Damage Analysis*), INDA ili NSPA, dok se uspostavljanje veze između DM i DV sprovodi preko modela štete (*loss model*), a koji se određuje primenom probabilističke analize seizmičke štete (PSLA – *Probabilistic Seismic Loss Analysis*), INDA ili NSPA.

8 ZAVRŠNE NAPOMENE

Primenom sprovedene sistematizacije seizmičkih analiza može se vrlo efikasno razmotriti koji tip analize se može primeniti u fazama preliminarnih i finalnih analiza za naučna istraživanja i stručne projekte. Autori su napravili sopstvenu sistematizaciju seizmičkih analiza, s tim što pojedine seizmičke analize mogu pripadati i prelaznim kategorijama analiza. Posebno je to slučaj kod onih analiza koje koriste multidisciplinarnu formulaciju problema, pa naučnoj i stručnoj javnosti ostaje da detaljnije razmotre matematičke formulacije svih pojedinačnih seizmičkih analiza.

Na konceptualnom nivou, uvođenje prigušenja u analizu konstrukcija trebalo bi razmatrati u funkciji tipa analize, da li je u pitanju linearna ili nelinearna analiza, odnosno u funkciji tipa domena u kojem se razmatra odgovor sistema, gde postoji mogućnost razmatranja u

7 SEISMIC ANALYSES ACCORDING TO PERFORMANCE-BASED EARTHQUAKE ENGINEERING (PBEE)

PBEE methodology has been initiated in the recent twenty years, firstly on deterministic and then probabilistic level. PBEE methodology is based on multidisciplinary approach through: computer mechanics, numerical methods, structural dynamics, nonlinear analyses, theory of reinforced concrete structures, theory of plasticity, failure mechanics, soil-structure interaction, earthquake engineering, engineering seismology, implementation of contemporary regulations for structural design, engineering statistics and probability. Development of contemporary PBEE methodology facilitates a more complete and complex analysis and treatment of the problem through hazard analysis, structural analysis, damage analysis and loss analysis [36], [41]. Hazard analysis is represented by variable intensity measure (IM), which quantifies ground displacement, while structural analysis is represented by implementation of engineering demand parameter (EDP). Damage analysis is represented by variable damage measure (DM), and loss analysis by variable decision variables (DV). Relation is established between IM and EDP through seismic demand model, which is determined by implementation of *Probabilistic Seismic Demand Analysis* (PSDA) and INDA. However, prior to establishing relation EDP-IM it is necessary to consider IM variable by implementing *Probabilistic Seismic Hazard Analysis* (PSHA). Based on the IM determined from PSHA and on EDP from PSDA, NDA or even via NSPA, a correlation EDP-IM is established, most often via the spectral acceleration for IM and global or inter storey drift for EDP. Model of seismic demand in PSDA analysis can be represented via fragility curves. Establishment of correlation between EDP and DM is conducted via damage model, which is determined by implementation of *Probabilistic Seismic Damage Analysis* (PSDamA), INDA or NSPA, while establishment of correlation between DM and DV is conducted using loss model, and determined by implementation of *Probabilistic Seismic Loss Analysis* (PSLA), INDA or NSPA.

8 CONCLUSION REMARKS

By implementing the conducted systematization of nonlinear seismic analyses, one can efficiently analyze which type of analysis can be implemented in the phases of preliminary and final analyses for scientific research and professional projects. The authors created their own systematization of analyses, considering that certain nonlinear seismic analysis can belong to transitional categories of analyses. It is particularly the case in those analyses which employ multidisciplinary problem formulation, thus a more in-detail consideration of mathematical formulations of all individual nonlinear seismic analyses remains to be performed.

At the conceptual level, damping introduction into structural analysis should be considered as a function of the type of analysis (linear or nonlinear analysis), that is, a function of the domain type in which the system kapacitativnom, vremenskom ili frekventnom domenu. Budući da modeliranje prigušenja u analizi konstrukcija predstavlja jednu od najvećih nepoznanica, to je ovim naučnim istraživanjem ponuđeno inženjerskoj javnosti da se putem dijagrama tokova mogu otkloniti već postojeće nedoumice u vezi s modeliranjem prigušenja. Klasičan pristup kojim se uvodi prigušenje u analizu konstrukcija bazira se na koeficijentu relativnog prigušenja ξ. Međutim, treba znati da nije ekvivalentno kada se uzima u obzir koeficijent relativnog prigušenja i kada se (histerezisno) prigušenje uvodi u analizu primenom faktora participacije mase i krutosti α i β, a koji su određeni za vrednost koeficijenta relativnog prigušenja. Takođe, prilikom uvođenja prigušenja u analizu treba voditi računa o tome da se dodatno ne ignoriše postojanje prigušenja u sistemu, da se ne dupliraju vrednosti prigušenja ili da se uvodi prigušenje istog tipa, a na dva različita načina.

Zahvalnost: Ovaj rad je proistekao iz dela istraživačkog projekta TR 36043, koji je podržan od strane Ministarstva prosvete i nauke Republike Srbije. Zahvaljuje se na ovoj podršci (R. Folić).

9 LITERATURA
REFERENCES

[1] Abdelraheem Farghaly A.: Parametric Study on Equivalent Damping Ratio of Different Composite Structural Building Systems, Steel and Composite Structures, Vol. 14, No. 4, pp. 349–365, 2013.

[2] Adhikari S.: *Damping Models for Structural Vibration*, PhD Dissertation, Cambridge University, Cambridge, UK, 2000.

[3] Antoniou S., Pinho R.: *Development and Verification of a Displacement-Based Adaptive Pushover Procedure*, Journal of Earthquake Engineering, Vol. 8, No. 5, pp. 643–661, 2003.

[4] Antoniou S., Pinho R.: *Advantages and Limitations of Adaptive and Non-Adaptive Force Based Pushover Procedures*, Journal of Earthquake Engineering, Vol. 8, No. 4, pp. 497–522, 2004.

[5] Aschheim M., Black E.: *Yield Point Spectra for Seismic Design and Rehabilitation*, Earthquake Spectra, Vol. 16, No. 2, pp. 317–335, 2000.

[6] ATC 40, *Seismic Evaluation and Retrofit of Concrete Buildings, Vol. 1*, Applied Technology Council, Redwood City, USA, 1996.

[7] Avanaki M., Estekanchi H.: *Collapse Analysis by Endurance Time Method*, International Journal of Optimization in Civil Engineering, Vol. 2, No. 2, pp. 287–299, 2012.

[8] Aydinogly M.: An Incremental Response Spectrum Analysis Procedure Based on Inelastic Spectral Displacements for Multi-Mode Seismic Performance Evaluation, Bulletin of Earthquake Engineering, Vol. 1, No. 1, pp. 3–36, 2003.

[9] Azarbakht A., Dolšek M.: *Progressive Incremental Dynamic Analysis for First-Mode Dominated Structures*, ASCE Journal of Structural Engineering, Vol. 137, 2011.

[10] Bhatt C., Bento R.: The Extended Adaptive Capacity Spectrum Method for the Seismic Assessment of Plan-Asymmetric Buildings, Earthquake Spectra, Vol. 30, No. 2, pp. 683–703, 2014.

[11] Casarotti C., Pinho R.: *An Adaptive Capacity Spectrum Method for Assessment of Bridges Subjected to Earthquake Action*, Bulletin of Earthquake Engineering, Vol. 5, No., pp. 377-390, 2007.

response is analyzed (capacitive, time or frequency domain). Since the modelling of damping is one of the biggest issues in structural analysis, this research suggests to the engineering community a flowchart based solution to eliminate the existing dilemmas about damping modelling. Traditional approach of damping introduction into structural analysis is based on the relative damping coefficient ξ. However, one should know that taking into account the relative damping coefficient is not equivalent to introducing (hysteretic) damping into the analysis using factors of mass and stiffness participation α and β, which were defined for the value of the relative damping coefficient. Also, when introducing damping into the analysis, care should be taken about the existence of damping in the system, not to duplicate the damping values or introduce the same type of damping but in two different ways.

Acknowledgements: The work reported in this paper is a part of the investigation within the research project TR 36043 supported by the Ministry for Education and Science Republic of Serbia. This support is gratefully acknowledged (R. Folić).

[12] Chopra A.: Dynamics of Structures: Theory and Applications to Earthquake Engineering, Prentice-Hall, Englewood Cliffs, USA, 1995.

[13] Chopra A., Goel R.: *A Modal Pushover Analysis Procedure to Estimating Seismic Demands for Buildings*, Pacific Earthquake Engineering Research Center, University of California Berkeley, PEER Report 2001/03, 2001.

[14] *CSI Analysis Reference Manual*, Computers and Structures, Berlekey, USA, 2009.

[15] Ćosić M., Brčić S.: „Metodologija pripreme i obrade akcelerograma za linearne i nelinearne seizmičke analize konstrukcija”, Izgradnja Vol. 66, No. 11–12, pp. 511–526, 2012.

[16] Ćosić M., Brčić S.: *Iterative Displacement Coefficient Method: Mathematical Formulation and Numerical Analyses*, Journal of the Croatian Association of Civil Engineers, Vol. 65, No. 3, pp. 199–211, 2013.

[17] Ćosić M., Brčić S.: *The Development of Controlled Damage Mechanisms-Based Design Method for Nonlinear Static Pushover Analysis*, Facta Universitatis - Series: Architecture and Civil Engineering, Vol. 12, No. 1, pp. 25–40, 2014.

[18] Ćosić M.: „Nelinearna statička i dinamička seizmička analiza okvirnih zgrada prema performansama”, doktorska disertacija, Građevinski fakultet, Univerzitet u Beogradu, Beograd, 2015.

[19] Ćosić M., Folić R.: *Performance Analysis of Damaged Buildings Applying Scenario of Related Non-Linear Analyses and Damage Coefficient*, Building materials and structures, Vol. 58, No. 3, pp. 3–27, 2015.

[20] Di Sarno L, Elnashai A.: *Fundamentals of Earthquake Engineering*, John Wiley & Sons, Chichester, UK, 2008.

[21] Dolšek M., Fajfar P.: *IN2 - A Simple Alternative for IDA*, The 13th World Conference on Earthquake Engineering, Paper No. 3353, Vancouver, Canada, 2004.

[22] Dolšek M.: *Estimation of Seismic Response Parameters Through Extended Incremental Dynamic Analysis*, Computational Methods in Earthquake Engineering, Vol. 21, pp. 285–304, 2010.

[23] EN 1998-1:2004, *Eurocode 8: Design of Structures for Earthquake Resistance – Part 1: General Rules, Seismic Actions and Rules for Buildings*, Brussels, Belgium, 2004.

[24] Fajfar P.: *A Nonlinear Analysis Method for Performance-Based Seismic Design*, Earthquake Spectra, Vol. 16, No. 3, pp. 573–592, 2000.

[25] FEMA 356, *Pre-Standard and Commentary for the Seismic Rehabilitation of Buildings*, American Society of Civil Engineers, Federal Emergency Management Agency, Washington D. C., USA, 2000.

[26] FEMA 440, *Improvement of Nonlinear Static Seismic Analysis Procedures,* Applied Technology Council (ATC-55 Project), Federal Emergency Management Agency, Washington D. C., USA, 2005.

[27] FEMA 750P, *NEHRP Recommended Seismic Provisions for New Buildings and Other Structures*, Building Seismic Safety Council, Federal Emergency Management Agency, Washington D. C., USA, 2009.

[28] FEMA P-58-1, *Seismic Performance Assessment of Buildings, Methodology*, Applied Technology Council, Federal Emergency Management Agency, Washington, USA, 2012.

[29] FEMA P-58-2, *Seismic Performance Assessment of Buildings*, Implementation Guide, Applied Technology Council, Federal Emergency Management Agency, Washington D. C., USA, 2012.

[30] Folić R., Ćosić M., Folić B.: *Damping Models for Flow Chart Based Structural Analysis*, The 15th International Science Conference VSU, Sofia, Bulgaria, pp. 155–164, 2015.

[31] Folić R., Ćosić M.: *Performance-Based Non-linear Seismic Methods of Structures: A Review of Scientific Knowledge in the Last 20 Years*, The 16th International Scientific Conference VSU, Sofia, Bulgaria, pp. 146–156, 2016.

[32] Freeman S.: *Review of the Development of the Capacity Spectrum Method*, ISET Journal of Earthquake Technology, Paper No. 438, Vol. 41, No. 1, pp. 1–13, 2004.

[33] Guyader A., Iwan W.: *An Improved Capacity Spectrum Method Employing Statistically Optimized Linearization Parameters*, The 13th World Conference on Earthquake Engineering, Paper No. 3020, Vancouver, Canada, 2004.

[34] Kalkan E., Kunnath S.: *Method of Modal Combinations for Pushover Analysis of Buildings*, The 13th World Conference on Earthquake Engineering, Paper No. 2713, Vancouver, Canada, 2004.

[35] Kovačević D.: „MKE modeliranje u analizi konstrukcija”, Građevinska knjiga, Beograd, 2006.

[36] Krawinkler H.: *Challenges and Progress in Performance-Based Earthquake Engineering*, International Seminar on Seismic Engineering for Tomorrow - In Honor of Professor Hiroshi Akiyama, Tokyo, Japan, pp. 1–10, 1999.

[37] Lagaros N.: *Multicomponent Incremental Dynamic Analysis Considering Variable Incident Angle*, Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance, Vol. 6, No. 1–2, pp. 77–94, 2010.

[38] Liao W-C.: *Performance-Based Plastic Design of Earthquake Resistant Reinforced Concrete Moment Frames*, Doctoral dissertation, The University of Michigan, Ann Arbor, USA, 2010.

[39] Lin T., Baker J.: *Introducing Adaptive Incremental Dynamic Analysis: A New Tool for Linking Ground Motion Selection and Structural Response Assessment*, Safety, Reliability, Risk and Life-Cycle Performance of Structures & Infrastructures, Taylor & Francis Group, London, UK, pp. 805–811, 2013.

[40] Manoukas G., Avramidis.: *Improved Multimode Pushover Procedure for Asymmetric in Plan Buildings Under Biaxial Seismic Excitation – Application to Tall Buildings*, The Structural Design of Tall and Special Buildings, Vol. 24, No. 6. pp. 397–420, 2014.

[41] Moehle J., Krawinkler H.: *A Framework Methodology for Performance-Based Earthquake Engineering*, The 13th World Conference on Earthquake Engineering, Paper No. 679, Vancouver, Canada, 2004.

[42] Moon K., Han S., Lee T., Seok S.: *Approximate MPA-Based Method for Performing Incremental Dynamic Analysis*, Nonlinear Dynamics, Vol. 67, No. 4, pp. 2865–2888, 2011.

[43] Newmark N., Hall W.: *Earthquake Spectra and Design*, Earthquake Engineering Research Institute (EERI), Berkeley, USA, 1982.

[44] Peloso S., Pavese A.: *Secant Modes Superposition: A Simplified Method for Seismic Assessment of RC Frames*, The 14th World Conference on Earthquake Engineering, Beijing, China, Paper No. 14_05-01-0254, 2008.

[45] Priestley M., Calvi G., Kowalsky M.: *Displacement-Based Seismic Design of Structures*, IUSS Press, Pavia, Italy, 2007.

[46] Puthanpurayil A., Dhakal R., Carr A.: *Modelling of In-Structure Damping: A Review of the State-of-the-Art*, IX Pacific Conference on Earthquake Engineering Building an Earthquake-Resilient Society, Paper no. 091, Auckland, New Zealand, 2011.

[47] Ramirez O., Constantinou M., Whittaker A., Kircher C., Chrysostomoue C.: *Elastic and Inelastic Seismic Response of Buildings with Damping Systems*, Earthquake Spectra, Vol. 18, No. 3, pp. 531–547, 2002.

[48] Smyrou E., Priestley N., Carr A.: *Modelling of Elastic Damping in Nonlinear Time-History Analyses of Cantilever RC Walls*, Bulletin of Earthquake Engineering, Vol. 9, No. 5, pp. 1559–1578, 2011.

[49] Stojnić N., Kuzović D.: *Proposal of Reducing Seismic Damages on Immovable Cultural Properties (Building Structures)*, Building materials and structures, Vol. 59, No. 4, pp. 31-46, 2016.

[50] Vamvatsikos D.: *Seismic Performance, Capacity and Reliability of Structures as Seen Through Incremental Dynamic Analysis*, PhD Dissertation, Stanford University, Stanford, USA, 2002.

[51] Vamvatsikos D., Cornell A.: *Incremental Dynamic Analysis*, Earthquake Engineering and Structural Dynamics, Vol. 31, No. 3, pp. 491–514, 2002.

[52] Wilson E.: *Three-Dimensional Static and Dynamic Analysis of Structures*, Computers and Structures, Inc., 2002.

[53] Yu H., Lu D-G., Song P-Y., Wang G-Y.: *Stochastic Incremental Dynamic Analysis Considering Random System Properties*, The 14th World Conference on Earthquake Engineering, Beijing, China, Paper No. 14_S15-047, 2008.

[54] Yu-Yuan L., Kuo-Chun C.: *An Improved Capacity Spectrum Method for ATC-40*, Earthquake Engineering & Structural Dynamics, Vol. 32, No. 13, pp. 2013-2025, 2003.

[55] Yu-Yuan L., Miranda E.: Non-Iterative Capacity Spectrum Method Based on Equivalent Linearization for Estimating Inelastic Deformation Demands of Buildings, Structural Engineering / Earthquake Engineering, Vol. 21, No 2, pp. 113–119, 2004.

REZIME

PREGLED SAVREMENIH SEIZMIČKIH ANALIZA I NAČINA UVOĐENJA PRIGUŠENJA U NJIMA

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Autori rada su, na osnovu analize velikog broja naučnih radova, dali prikaz sopstvene originalne sistematizacije seizmičkih analiza konstrukcija, a veliki deo njih je razvijen u poslednje dve decenije. Seizmičke analize su klasifikovane generalno u dve (četiri) grupe: linearne i nelinearne statičke analize; i linearne i nelinearne dinamičke analize. Posebno su klasifikovane analize nelinearnog seizmičkog odgovora konstrukcija, a posebno analize ciljnog pomeranja kojim se definiše odnos seizmičkog zahteva i seizmičkog odgovora. S druge strane, klasifikacija je sprovedena i u funkciji da li se nelinearan odgovor sistema dobija primenom inkrementalno-iterativnih procedura ili primenom poluiterativnih i/ili poluinkrementalnih procedura. Nelinearne dinamičke analize su klasifikovane prema konceptu matematičke formulacije, odnosno da li se zasnivaju na samo jednoj dinamičkoj analizi, većem broju dinamičkih analiza ili dobijaju rešenja u kombinaciji s drugim metodama. Primenom sprovedene sistematizacije seizmičkih analiza može se vrlo efikasno razmotriti koji tip analize je optimalan za analizu konstrukcija i koji tip analize je potrebno uzeti u obzir u fazi preliminarnih i finalnih analiza za naučna istraživanja i stručne projekte.

U radu su, takođe, prikazani aspekti modeliranja prigušenja u analizi konstrukcija sistematizacijom tipova prigušenja i formiranim dijagramima tokova, a u zavisnosti od tipa primenjene analize: linearne i nelinearne, statičke i dinamičke. Sistematizacija prigušenja je sprovedena prema načinu uvođenja u proračun i to preko prigušenja materijala, prigušenja elemenata veze i prigušenja koja se direktno uvode u analize, a koje se sprovode u kapacitativnom, vremenskom i frekventnom domenu. Primenom razvijenih dijagrama tokova, u procesu kreiranja numeričkih modela konstrukcija, može se vrlo efikasno razmotriti koji tip prigušenja treba odabrati i na koji način uvesti prigušenje u analizu konstrukcija. Takođe, primenom razvijenih dijagrama tokova mogu se definisati i alternativni pristupi uvođenja prigušenja u analizu konstrukcija.

Ključne reči: seizmičke analize, prigušenje, sistematizacija, performanse, konstrukcije

SUMMARY

AN OVERVIEW OF MODERN SEISMIC ANALYSES WITH DIFFERENT WAYS OF DAMPING INTRODUCTION

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The authors of the paper, on the basis of the analysis of a large number of scientific papers, presented their original systematization of seismic analyses of structures, where a large number of them were developed during the last twenty years. Seismic analyses are generally classified into two (four) groups: *Linear Static Analyses* (LSA) and *Nonlinear Static Analyses* (NSA), and *Linear Dynamic Analyses* (LDA) and *Nonlinear Dynamic Analyses* (NDA). The analyses of nonlinear seismic structural response were classified separately from the *Target Displacement Analyses* (TDA) which defines the relationship of the seismic demand and seismic response. On the other hand, classification was also conducted depending on whether a nonlinear response of the system is obtained by the implementation of incremental-iterative procedures or by implementation of semi-iterative and/or semi-incremental procedures. NDA were classified according to the concept of mathematical formulation, i.e. whether they are based on only one dynamic analysis, several dynamic analyses or they are solved in combination with other methods. By implementing the conducted systematization of seismic analyses, one can efficiently consider which type of analysis is optimal for structural analysis and which type of analysis should be taken into account in the phase of preliminary and final analyses in the course of scientific research and professional projects.

This paper also presents the aspects of damping modelling in structural analysis through the systematization of damping types and flowcharts, depending on the type of analysis applied: linear and nonlinear, static and dynamic. Damping has been systematized based on the way it was introduced into calculations, i.e. over material damping, link element damping and damping directly introduced into the analyses which are conducted in capacitive, time and frequency domains. In the process of creating numerical structural models, the type of damping and the way of its introduction into structural analysis can be very efficiently selected by applying the flow charts developed. By applying the developed flowcharts, alternative approaches to the introduction of damping into structural analysis can also be defined.

Keywords: seismic analyses, damping, systematization, performances, structures