Received: 11 February 2022 | Revised: 14 April 2022 | Accepted: 26 May 2022
DOI: 10.1002/suco.202200132

ARTICLE

Span-to-depth ratio limits for deflection control of reinforced concrete elements

Nenad Pecić | Snežana Mašović | Saša Stošić

Faculty of Civil Engineering, University of Belgrade, Belgrade, Serbia

Correspondence
Snežana Mašović, Faculty of Civil Engineering, University of Belgrade, Bulevar Kralja Aleksandra 73, Belgrade, Serbia.
Email: smasovic@grf.bg.ac.rs

Abstract
Indirect deflection control usually applies the limitation of the span-to-depth ratio. The procedures that have developed in the past decades have mostly been derived for predefined values of some of the relevant parameters. Assigning values to parameters simplifies the application of criteria. Predefined values may not be suitable for all design situations and some of the recently published procedures allow direct selection of relevant parameters. This paper offers new expressions for span-to-depth ratio limits for deflection control. Expressions include cross-section size, area, and position of tensile and compressive reinforcement, mechanical and long-term properties of concrete and deflection limit. The load level is represented by the stress in the tensile reinforcement so that the ratio of live and total load can be arbitrary. The dimensionless form of deflection calculation by numerical integration of the mean curvature according to Eurocode 2 is used to formulate the expressions. The proposed procedure is verified with a large number of numerical simulations for different combinations of inputs. Measured long-term deflections, available from the literature, are also used to test the procedure. The expressions are suitable for both deflection control and conceptual design.

KEYWORDS
deflection control, flexural members, reinforced concrete, slenderness limits

1 | INTRODUCTION

High-strength structural materials and fast construction procedures highlight serviceability issues. Smaller dimensions of concrete cross-sections and higher reinforcement stresses, together with increased effects of concrete creep due to earlier loading, result in reduced effective stiffness and increased deformability. Deflection limitation becomes a relevant design criterion.

Discussion on this paper must be submitted within two months of the print publication. The discussion will then be published in print, along with the authors' closure, if any, approximately nine months after the print publication.

Deflection limits are traditionally based on avoiding two groups of consequences: visual unacceptability and damage or malfunction of finishes. Model Code 2010 (MC 2010[1]) and Eurocode 2 (EN 1992-1-1[2]) apply general limits given in ISO 4356. The deflection under quasi-permanent load should not exceed span/250. The deflection after construction that could damage finishes (incremental deflection) is limited to span/500.

Predicting the deflection of concrete elements is an extremely complex problem due to the influence of many material properties and environmental conditions. The overall reliability of the calculation is strongly affected by the reliability of the prediction of concrete properties and load history. In addition, changes in stiffness due to

TABLE 1 Values l/d (l/h) for solid slabs

	Simple slab	One end continuous	Both ends continuous	Cantilever	Note
ACI 318-19 CSA A23.3-04	$l/20$	$l/24$	$l/28$	$l/10$	Thickness h
BS 8110-1:1997	$l/20$	—	$l/26$	$l/7$	(Effective) depth d

cracking and changes in curvature due to creep and shrinkage of concrete make the calculation very complex.

Deflection verification procedures, recommended in design codes, have different levels of complexity. The methods are based either on the calculation of the deflection itself, or on indirect control.

Indirect deflection control may be sufficiently accurate, provided that the relevant design data correspond to the predefined values of implicit parameters that are removed by the simplifications. Indirect methods generally apply span-to-depth ratio limits. Assigning representative values to almost all variables involved in the calculation of deflection leads to simple criteria that are in the form span/number, as in ACI 318,[3] CSA A23.3[4] or BS 8110-1,[5] Table 1. The area of reinforcement is according to the ultimate limit state (ULS) design.

Moderate differences in values from Table 1 stem from several causes: assumed load composition and load history; various models for the effective stiffness of cracked elements; properties of selected representative materials; reinforcement service stress, which depends on the grade and overall safety factor.

Additional reinforcement compared to that required by ULS design is a useful tool that can reduce the depth shown in Table 1. BS 8110-1[5] provided a modification factor for span-to-depth ratio to account for the stress in tensile reinforcement, which ranged up to two.

Taking into account the different situations related to construction works, an efficient span-to-depth criterion should include more parameters. An overview of several span-to depth criteria and studies comparing different procedures is presented herein.

Rangan[6] proposed a method for deflection control of concrete beams and one-way slabs based on allowable span-to-depth ratios. An approximation for Branson's effective stiffness was used together with the long-term deflection multiplier from ACI 318. This approach, with various modifications, is found in later proposals. Limitations of total and incremental deflection were considered.

Gilbert[7] used computer simulations to test Rangan's procedure and noticed a good fit in the case of low-reinforced girders (beams and one-way slabs). He extended Rangan's expression to two-way slabs and plates by introducing a corrective multiplier.

Scanlon and Choi[8] proposed an iterative procedure to obtain minimum thickness for one-way slabs similar to Rangan's proposal (limits are set for thickness instead of effective depth). Conservative simplification was applied to determine the lower limit of the effective moment of inertia according to ACI (Branson).

Scanlon and Lee[9] proposed a more general span-to-depth (thickness) equation related to one-way elements, two-way slabs and flat plates. The procedure is based on the one previously formulated by Scanlon and Choi.[8] A less conservative value of the ratio effective second moment of area-to-gross moment of inertia is proposed.

Bischoff and Scanlon[10] developed equations for maximum span-to-depth (thickness) ratios using incremental deflection limits. The effective moment of inertia, proposed by Branson, was modified to avoid overestimating the stiffness of lightly reinforced elements according to Bischoff's previous research.[11] The deflection increment was determined based on immediate and long-term deflections with corresponding stiffness, where part of the live load was considered as sustained.

Gardner[12] provided a useful overview of the span-to-depth (thickness) ratio from several codes (ACI318-08, BS8110-97, CSA A23.3-04, AS2600-2009, EN 1992-1-1), including suggestions for improvement from some authors discussing code provisions. The span-to-depth (thickness) requirements from different codes are tabulated for a range of selected design parameters and efficient comparison is possible.

Lee et al.[13] also presented a comparison of indirect deflection control procedures from different codes, including the proposal by Scanlon and Lee.[9] Span-to-thickness limits for slabs were compared in a parametric study that included variation of span and load level. A similar comparison was made for beams. The height corresponding to the limit deflection was also determined by an iterative procedure (deflection was calculated according to ACI 318 and the reinforcement corresponded to the ULS design). The span-to-thickness ratio thus obtained is shown together with the requirements from the considered procedures.

A comprehensive analysis was performed to develop the procedure presented in EN 1992-1-1.[2] Useful conclusions were summarized in the background document[14] (Corres Peiretti et al.). This study performed a detailed analysis of the effects of relevant parameters such as concrete class, cross-section dimensions, reinforcement curtailment, relative humidity, loading history, and load composition. Expressions for the limit value of the span-to-depth ratio, depending on the concrete class and the required tensile reinforcement according to ULS design, were formulated. The expressions given in EN 1992-1-1[2] (and MC 2010[1]) have the quasi-permanent-to-ultimate load ratio and the long-term concrete properties built-in, while the tensile strength of concrete is related to the compressive strength. The ratio of quasi-permanent to ultimate load was set at 0.50. Selected values were also assigned to other relevant parameters.

Creep and shrinkage of concrete are explicitly included in recent proposals.

Pérez Caldentey et al.[15] proposed a procedure for deflection control of flexural concrete elements that takes into account several relevant parameters and allows the use of an arbitrary deflection limit. The effective creep coefficient and the equivalent moment of inertia are used to calculate the total deflection due to quasi-permanent load. The impact of concrete shrinkage is considered by the equivalent first moment of area of the reinforcement. Approximate expressions are given for the coefficients by which the required equivalent cross-sectional characteristics are calculated, assuming that the reinforcement corresponds to the ULS design. The procedure for span-to-depth limit refers to a simple beam, while the general deflection factor takes into account different support conditions. The factor is modified in the part that considers the shrinkage of concrete.

Marí et al.[16] developed span-to-depth limits for deflection control that also take into account a number of relevant parameters. Long-term deflection is obtained by multiplying the initial deflection and the proposed long-term factor. The long-term factor relies on the AAEM method and considers creep and shrinkage. To obtain the initial deflection, a simple expression for the effective moment of inertia of a cracked segment is proposed. The general deflection coefficient is used to account for different support conditions, and the expressions for the equivalent moment of inertia and the long-term factor are extended to cover continuous members and T-shaped sections. The expression for long-term deflection is converted into a limit of span-to-depth ratio. Arbitrary deflection limits can be applied. The slenderness limit is also discussed as a function of the stress level in the tensile reinforcement.

It is convenient that the relevant parameters explicitly participate in the criterion so that values corresponding to the construction process can be applied. Apart from the simplest criteria (span/number, as in Table 1), most procedures require data from the ULS design (mainly reinforcement ratio). Therefore, if they are used for a conceptual design, some form of iterative calculation is usually required.

This paper presents the original span-to-depth criterion for deflection control that takes into account 11 relevant parameters in explicit form (some of them through dimensionless coefficients). The deflection due to quasi-permanent load, calculated by numerical integration of the long-term curvatures, is converted into expressions suitable for limiting the span-to-depth ratio. A comprehensive parametric analysis has been performed to test the compliance of the proposed expressions with the reference values. The procedure is suitable for both deflection control and conceptual design.

2 | PROPOSED METHOD FOR INDIRECT DEFLECTION CONTROL

The span-to-depth ratio (l/d) limits derive from the dimensionless procedure described in Pecić et al.[17] Total deflections under quasi-permanent load are used as reference values for calibration of the proposed method. Deflections were calculated by numerical integration of the mean curvature[1,2]:

$$\kappa = \zeta\kappa_{II} + (1-\zeta)\kappa_I \tag{1}$$

The interpolation coefficient ζ is:

$$\zeta = 1-\beta\left(\frac{M_{cr}}{M}\right)^2 \tag{2}$$

The coefficient β equals 1.0 for short-term and 0.5 for long-term loads. The relevant value of the coefficient ζ comes either from the quasi-permanent (qp) or from the characteristic ($char$) load, and is greater of two:

$$\zeta = \max\left\{\zeta_{qp} = \left[1-0.5\left(\frac{M_{cr}}{M_{qp}}\right)^2\right], \quad \zeta_{char} = \left[1-1.0\left(\frac{M_{cr}}{M_{char}}\right)^2\right]\right\} \tag{3}$$

The construction load can be used instead of the characteristic load, if relevant. It is convenient to use a reduced value of the concrete tensile strength $f_{ct,red}$ to calculate the cracking moment M_{cr}. The cracked part of span identifies from:

$$M \geq M_{cr,red} = W \cdot f_{ct,red} \tag{4}$$

where W is the section modulus of the uncracked section.

The reduced concrete tensile strength $f_{ct,red}$

$$f_{ct,red} = \begin{cases} \sqrt{\beta} \cdot f_{ct} = \sqrt{0.5} \cdot f_{ct} \approx 0.7 \cdot f_{ct} & \text{for } M_{qp} \geq 0.7 M_{char} \\ \dfrac{M_{qp}}{M_{char}} \cdot f_{ct} & \text{for } M_{qp} \leq 0.7 M_{char} \end{cases} \tag{5}$$

allows the application of Equation (3) in Equation (2) in a form:

$$\zeta = 1 - \left(\frac{W \cdot f_{ct,red}}{M_{qp}}\right)^2 = 1 - \left(\frac{M_{cr,red}}{M_{qp}}\right)^2 \tag{6}$$

to calculate the deflection due to quasi-permanent load.

The reduced concrete tensile strength $f_{ct,red}$ is used to account for the tension induced by internal and external restraints and/or the occurrence of short-term loads that are bigger than quasi-permanent load. It has been shown[17] that the average tensile strength of concrete ($f_{ct} = f_{ctm}$) provides satisfactory prediction of deflection, while the use of flexural tensile strength of concrete ($f_{ct,fl}$) underestimates the deflection.

Calculation of the deflection by Equations (1) and (2) can be transformed into a dimensionless procedure.[18] The deflection is expressed by the dimensionless coefficient $\widetilde{u}_t$ and the stress in the tensile reinforcement, Equation (7). If the deflection limit is $span/N$, where N is selected number, than:

$$u_t = \widetilde{u}_t \frac{l^2}{d} \frac{\sigma_s}{E_s} \leq \frac{l}{N} \tag{7}$$

where

$$\frac{l}{d} \leq \frac{1}{N} \frac{E_s}{\sigma_s} \frac{1}{\widetilde{u}_t} \tag{8}$$

Coefficient $\widetilde{u}_t$ from Equation (7) is, in general, a function of seven nondimensional parameters:

$$\widetilde{u}_t = \widetilde{u}_t \left(sys, \rho, C, \delta, k_s, \varphi_{eff}, \varepsilon_{sh}\right) \tag{9}$$

Dimensionless parameter C[17,18]:

$$C = \frac{\alpha f_{ct,red}}{\sigma_s} \tag{10}$$

is used to calculate the mean curvature in dimensionless form.

The effective reinforcement ratio ρ of the tensile reinforcement A_{s1} at the cross section with the maximum bending moment in the span, Equation (11), and the parameter C, Equation (10), describe the magnitude of the load as well as the crack formation:

$$\rho = \frac{E_s}{E_c} \frac{A_{s1}}{bd} = \alpha \frac{A_{s1}}{bd} \tag{11}$$

E_c is the modulus of elasticity of concrete associated with creep coefficient. For a given C, value of the coefficient ρ corresponds to a particular load level. The load at which a crack is formed at the place of the maximum bending moment in the span is determined with the value of $\rho = \rho_C$ ($\rho < \rho_C$ corresponds to the element without cracks in the span, while $\rho \geq \rho_C$ corresponds to the element with cracks). It is assumed that the tensile and compressive reinforcement are constant at a length at which the bending moment does not change sign. According to Corres Peiretti et al.,[14] the curtailment of the bars does not have a large effect on the calculated deflection.

The effective creep coefficient φ_{eff} accounts for multiple long-term loads within a quasi-permanent load:

$$\varphi_{eff} = \frac{\sum_i g_i \varphi(\infty, t_i)}{\sum_i g_i} \tag{12}$$

Parameter sys introduces the effect of the structural system. The basic solution was created for a simple beam ($sys = SB$).

The criterion from Equation (8) for a different structural system (sys) can be obtained from a corresponding simple beam, having the same span, cross section, and maximum bending moment in the span. The simplest method is to apply the elastic deflections ratio. For a simply supported beam subjected to a uniform load, the deflection u is:

$$u(SB) = \frac{5}{48} \times M_D \frac{l^2}{EI} = K(SB) \times M_D \frac{l^2}{EI} \tag{13}$$

where $K(SB) = 5/48$, and M_D is the maximum bending moment in the span. Similarly, for a span with different support conditions (sys):

$$u(sys) = K(sys) \times M_D \frac{l^2}{EI} \tag{14}$$

Provided that the member has the same maximum bending moment in the span M_D as the corresponding simple beam (SB):

$$u(sys) = \frac{K(sys)}{K(SB)} \times u(SB) \tag{15}$$

Equation (9) becomes:

$$\begin{aligned}\widetilde{u}_t &= \widetilde{u}_t\left(sys,\rho,C,\delta,k_s,\varphi_{eff},\varepsilon_{sh}\right)\\ &= \frac{K(sys)}{K(SB)}\times\widetilde{u}_t\left(SB,\rho,C,\delta,k_s,\varphi_{eff},\varepsilon_{sh}\right)\end{aligned}\tag{16}$$

and can be applied in Equation (8).

The following expressions (17) and (18) have been created by parametric analysis of the values of $\widetilde{u}_t$ in Equation (8). The proposed limits of span-to-depth ratio are:

$$\begin{aligned}\left(\frac{l}{d}\right)^{prop}_{\lim} = &\frac{50\left(5.5-\varphi_{eff}\right)}{\sigma_s+100(1-k_s)\varepsilon_{sh}}\times\frac{1}{(1-\delta)^3\rho\left(1-\sqrt[3]{\rho}\right)}\times A(k_s)\\ &\times F_{sys}\times F_N \text{ for } \rho<\rho_C\end{aligned}\tag{17}$$

and

$$\begin{aligned}\left(\frac{l}{d}\right)^{prop}_{\lim} = &\frac{500(3+\delta)}{\sigma_s\left(1+0.2\varphi_{eff}\right)+130\varepsilon_{sh}}\times\left(1+\frac{0.5}{\sqrt{\rho-\rho_C}}\right)\\ &\times B(k_s)\times F_{sys}\times F_N \text{ for } \rho\geq 1.1\rho_C\end{aligned}\tag{18}$$

where l is the span, d is the effective depth of cross section, σ_s is the stress in the tensile reinforcement due to quasi-permanent load in the cross section with maximum bending moment in the span (in MPa); σ_s is calculated as for the cracked section, ρ is the effective reinforcement ratio given by Equation (11), φ_{eff} is the effective creep coefficient given by Equation (12), ε_{sh} is the final shrinkage strain in ‰ ($\varepsilon_{sh}\times 10^3$),

$$\delta=(h-d)/h,\tag{19}$$

$$k_s=A_{s2}/A_{s1}.\tag{20}$$

A_{s2} is the compressive reinforcement in the same cross-section as A_{s1} (cross-section with the maximum bending moment in the span).

Parameter ρ_C can be approximated as (Pecić[18]):

$$\rho_C = C\left(0.22C+\delta^2+0.3\delta+0.17\right)\tag{21}$$

and C is given by Equation (10).

In the interval $\rho_C\leq\rho<1.1\rho_C$, linear interpolation between the value from Equation (17) for $\rho=\rho_C$ and the value from Equation (18) for $1.1\rho_C$ applies. This step is necessary due to the singularity of Equation (18) when ρ is close to ρ_C.

Multipliers $A(k_s)$ and $B(k_s)$:

$$A(k_s)=\frac{3+(1+4.5k_s)\left(1+\varphi_{eff}\right)\rho}{3+\left(1+\varphi_{eff}\right)\rho}\tag{22}$$

$$B(k_s)=\frac{1+2[1+(1.9-5\delta)k_s]\left(1+\varphi_{eff}\right)\rho}{1+2\left(1+\varphi_{eff}\right)\rho}\tag{23}$$

account for the compressive reinforcement ($A(k_s)=B(k_s)=1$ when there is no compressive reinforcement or it is neglected).

Multiplier F_{sys}:

$$F_{sys}=\frac{5/48}{K(sys)}\tag{24}$$

accounts for the structural system. $K(sys)$ is obtained from Equation (14) for arbitrary restraint conditions (e.g., $F_{sys}=1.0$ for simple beam, 1.35 for one end restrained or 1.67 for both ends restrained span).

Multiplier F_N:

$$F_N=\frac{250}{N}\tag{25}$$

accounts for selected deflection limit ($F_N=1$, for $l/250$).

Results presented in the study[14] indicate that limiting the incremental deflection of $l/500$ is a stricter condition, when applicable. Calculation of the incremental deflection is a demanding task and requires additional assumptions on the construction schedule that affect the result. In that case, instead of calculation of the incremental deflection, a pragmatic solution may be to apply a stricter limit for the total deflection, for example $l/300$. That can be simply accomplished with appropriate value of coefficient F_N, without involving any additional approximation.

Relevant parameters $\left(sys,\rho,\ \sigma_s,\varphi_{eff},\varepsilon_{sh},E_c,E_s,f_{ct,red},\delta,k_s,N\right)$ are, explicitly or through dimensionless parameters, included in expressions (17) and (18). Therefore, Equations (17) and (18) are suitable for solving various tasks. Among others, the proposed method enables reliable adoption of the structural depth at an early stage of design, for various design situations, when the calculation of deflection by any procedure is not yet available. This is illustrated by Examples 1a and 2a in Appendix A. Appropriate structural depth ensures successful deflection verification in later design phases, as shown in Examples 1b and 2b.

Analysis of Equation (18) shows that additional tensile reinforcement in the amount of 25% of that required for ULS design (which roughly changes σ_s from 250 to

200 MPa, for quasi-permanent load) reduces the required effective depth by about 10% (see examples in Appendix A). Compressive reinforcement that amounts to 50% of the tensile reinforcement ($k_s = 0.5$) has a similar effect. The effects of additional tensile reinforcement on the possible reduction of the required structural depth are shown in Examples 1b and 2b in Appendix A.

Measured long-term deflections from several published research programs are used in Chapter 4 to evaluate the proposed procedure.

3 | NUMERICAL SIMULATIONS

Reference values of long-term deflections were obtained according to the rigorous method[1,2] by numerical integration of curvatures. The span of the simple beam was divided into 50 equal segments. Values $\widetilde{u}_t$ were calculated for a permanent uniform load.

Reference values of the span-to-depth ratio $(l/d)_{\lim}^{ref}$ were derived from Equation (8) with $N = 250$ and

$$sys = SB,$$

$$\varphi_{eff} = 1.7,\ 2.2\,\text{and}\ 2.7,$$

$$\varepsilon_{sh} = 0.3, 0.45\,\text{and}\,0.6\left(\times 10^{-3}\right),$$

$$\rho\ =\ 0.005 \div 0.150\,(21\,\text{values}),$$

$$\sigma_s =\ 150,\ 200\ \text{and}\ 250\,(\text{MPa}),$$

$$k_s =\ 0.0,\ 0.5\ \text{and}\ 1.0,$$

$$\delta\ =\ 0.1\ \text{and}\,0.2,$$

$$C =\ 0.04,\ 0.06, 0.08\ \text{and}\ 0.10,$$

resulting in $3(\varphi) \times 3(\varepsilon_{sh}) \times 2(\delta) \times 3(A_{s2}/A_{s1}) \times 4(C) \times 3(\sigma_s) \times 21(\rho) = 13608$ test samples.

Corresponding values of the span-to-depth ratio according to the proposed Equations (17) and (18), $(l/d)_{\lim}^{prop}$, were also calculated for comparison. Results are presented in the supplementary document: http://imksus.grf.bg.ac.rs/research/supp_slend_lim.pdf. Certain combinations of input data do not correspond to the actual values of tensile strength of concrete, so that the effective number of samples in the created database is 8937, as explained in the document.

The compliance of the proposed method with the reference procedure is tested by the relative change Δ:

$$\Delta = \frac{(l/d)_{\lim}^{prop} - (l/d)_{\lim}^{ref}}{(l/d)_{\lim}^{ref}} \times 100\%. \tag{26}$$

A negative value of Δ indicates that a particular result from expression (17) or (18) is conservative (on the safe side).

The histogram of the relative change Δ, when there is no compressive reinforcement ($k_s = 0$), is shown in Figure 1 (2979 values from the created database).

The mean value of Δ is -0.0428, and 90.70% of the results are within the interval (-0.10, $+0.05$). The span-to-depth criterion proposed by Equations (17) and (18) is (intentionally) shifted to the safe side, as can be seen from Figure 1.

The histogram of Δ for 8937 values from the database ($k_s = 0, 0.5\ \text{and}\ 1$) is presented in Figure 2.

The mean value of Δ is -0.0392, and 84.85% of the results are within the interval (-0.10, $+0.05$). Larger deviations are observed at load levels that are slightly higher than those that initiate cracking in the span. However, these are cases where prediction of deflection is less reliable and additional safety is desirable.

The proposed method allows the selection of various combinations of input parameters, since they are explicitly represented in Equations (17) and (18). For design purposes, one should define the values of input parameters. The appropriate values of the effective creep coefficient and shrinkage strain depend on the construction schedule and the expected environmental conditions. Parametric analysis was performed to estimate values of these parameters for common situations.

The effective creep coefficient φ_{eff} (Equation (12)) is estimated by combining two load compositions (LC) with two load histories (TH), as shown in Table 2.

Thirty percent of the live load is taken as permanent ($\psi_2 = 0.3$). The effective creep coefficient φ_{eff} and corresponding value of the total shrinkage strain ε_{sh} were calculated for RH = 50% and RH = 80%, and for—values of the notional size of the element h_0 (150, 200, and 300 mm). The expressions from Annex B of EN 1992-1-1[2] and concrete class C30 were used. However, the ratio of quasi-permanent and characteristic load is not predefined in the proposed method, so the selected time histories and load compositions are intended only to estimate the most common range for φ_{eff} and ε_{sh}.

The coefficient φ_{eff} ranged from 1.61 to 2.06 for RH = 80%, (for $h_0 = 300$ mm, LC1, TH2, cement type R, and for $h_0 = 150$ mm, LC2, TH1, cement type S, respectively) and from 2.15 to 2.87, for RH = 50% (the same combination of parameters as for RH = 80%). The final value of shrinkage strain ranged from 0.21‰ to 0.39 ‰

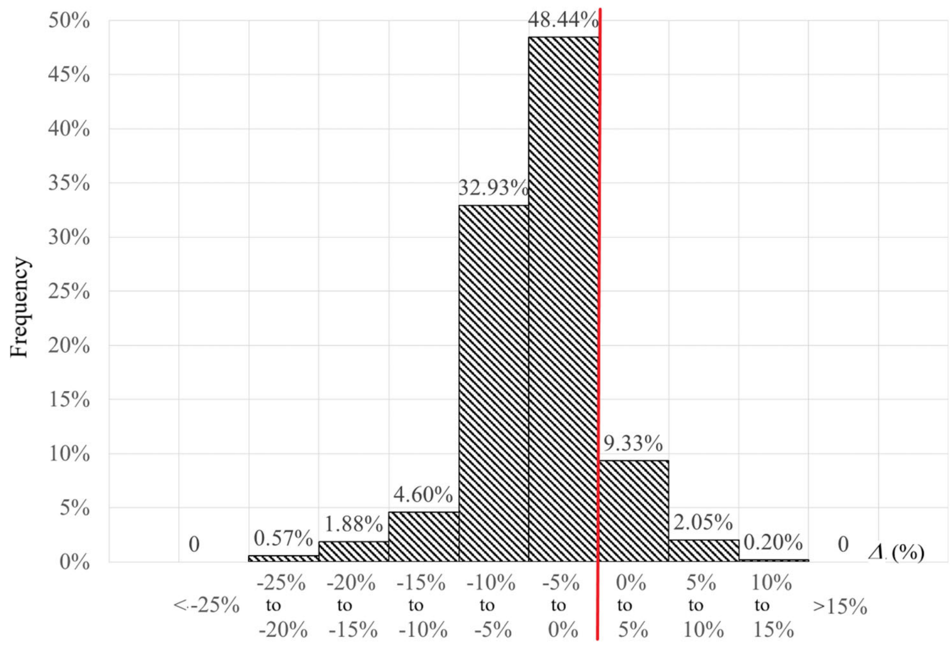

FIGURE 1 Histogram of the relative change Δ: $k_s = 0$ (2979 samples)

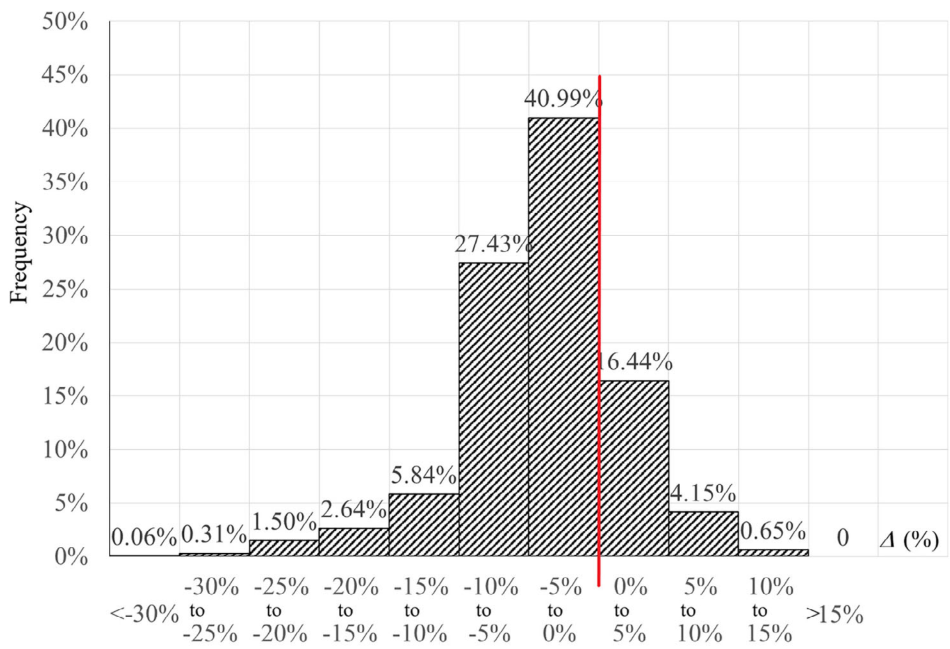

FIGURE 2 Histogram of the relative change Δ: $k_s = 0$, 0.5, and 1 (8937 samples)

TABLE 2 Assumed load compositions (LC) and load histories (TH)

		Self-weight	Superimposed dead load	Live load
% of total load	LC1	45	30	25
	LC2	60	20	20
Load history, days	TH1	10	60	365
	TH2	14	60	180

for RH = 80%, (for $h_0 = 300$ mm, cement type S, and for $h_0 = 150$ mm, cement type R, respectively) and from 0.34‰ to 0.67‰, for RH = 50%.

The estimated representative values are:

$\varphi_{eff} = 1.8$ and $\varepsilon_{sh} = 0.30 \times 10^{-3}$ for humid (outside) conditions (RH = 80%) and

$\varphi_{eff} = 2.5$ and $\varepsilon_{sh} = 0.50 \times 10^{-3}$ for dry (inside) conditions (RH = 50%).

TABLE 3 Coefficients for the expressions (27) and (28)

	A	B	D	G	Comment
Humid conditions	185	30	370	29	$\varphi_{eff} = 1.8$ and $\varepsilon_{sh} = 0.30 \times 10^{-3}$
Dry conditions	150	50	333	43	$\varphi_{eff} = 2.5$ and $\varepsilon_{sh} = 0.50 \times 10^{-3}$

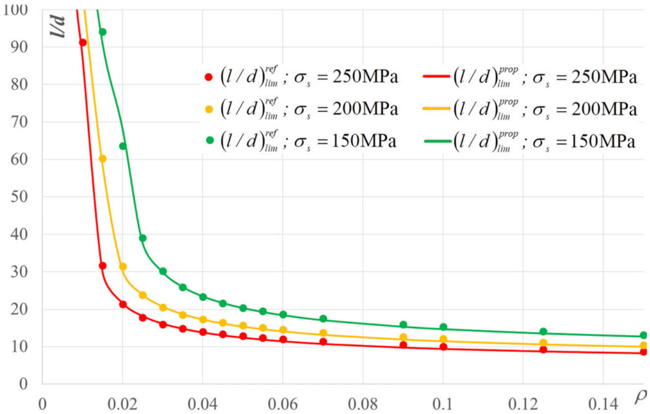

FIGURE 3 Comparison of the proposed method and reference values (dry conditions, $\delta = 0.1$)

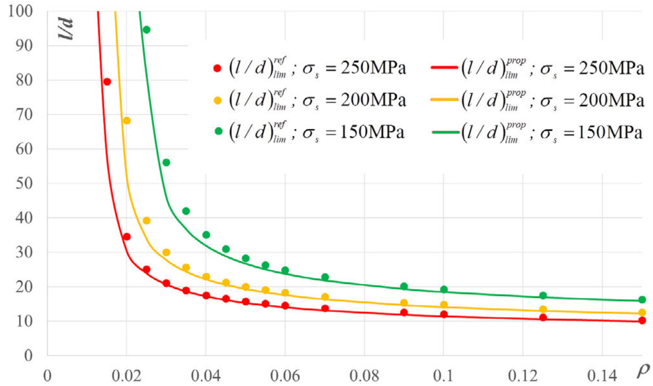

FIGURE 4 Comparison of the proposed method and reference values (humid conditions, $\delta = 0.2$)

TABLE 4 Experimental data—Washa and Fluck[19]

Specimen (1)	b [cm] (2)	d [cm] (3)	h [cm] (4)	A_{s1} [cm^2] (5)	A_{s2} [cm^2] (6)	l [m] (7)	$f_{ct,red}$ [MPa] (8)	$E_c(t_0)$ [GPa] (9)	$\varphi(t, t_0)$ (10)	ε_{sh} [‰] (11)	E_s [GPa] (12)	$u_{t,meas}$ [mm] (13)
A1/A4	20.3	25.7	30.5	8.52	8.52	6.10	2.00	20.38	3.76	0.69	206	23.6
A2/A5	20.3	25.7	30.5	8.52	4.00	6.10	2.00	20.38	3.76	0.69	206	32.3
A3/A6	20.3	25.7	30.5	8.52	0.00	6.10	2.00	20.38	3.76	0.69	206	44.7
B1/B4	15.2	15.7	20.3	4.00	4.00	6.10	1.69	18.76	4.40	0.75	206	51.1
B2/B5	15.2	15.7	20.3	4.00	2.00	6.10	1.69	18.76	4.40	0.75	206	62.0
B3/B6	15.2	15.7	20.3	4.00	0.00	6.10	1.69	18.76	4.40	0.75	206	86.4
C1/C4	30.5	10.2	12.7	5.16	5.16	6.34	1.62	18.45	4.46	0.76	206	80.0
C2/C5	30.5	10.2	12.7	5.16	2.58	6.34	1.62	18.45	4.46	0.76	206	100.6
C3/C6	30.5	10.2	12.7	5.16	0.00	6.34	1.62	18.45	4.46	0.76	206	140.7
D1/D4	30.5	10.2	12.7	5.16	5.16	3.81	1.74	18.17	4.30	0.75	206	27.7
D2/D5	30.5	10.2	12.7	5.16	2.58	3.81	1.74	18.17	4.30	0.75	206	33.8
D3/D6	30.5	10.2	12.7	5.16	0.00	3.81	1.90	18.86	4.11	0.74	206	48.5
E1/E4	30.5	5.87	7.62	2.84	2.84	5.33	1.81	18.48	4.58	0.76	206	124.0
E2/E5	30.5	5.87	7.62	2.84	1.42	5.33	1.81	18.48	4.58	0.76	206	128.8
E3/E6	30.5	5.87	7.62	2.84	0.00	5.33	1.81	18.48	4.58	0.76	206	184.9

With these values, for $N = 250$ and $k_s = 0$, Equations (17) and (18) become:

$$\left(\frac{l}{d}\right)^{prop}_{\lim} = \frac{A}{\sigma_s + B} \times \frac{1}{(1-\delta)^3 \rho \left(1 - \sqrt[3]{\rho}\right)} \times F_{sys} \quad \text{for}\ \rho < \rho_C \tag{27}$$

$$\left(\frac{l}{d}\right)^{prop}_{\lim} = \frac{D \times (3+\delta)}{\sigma_s + G} \times \left(1 + \frac{0.5}{\sqrt{\rho - \rho_C}}\right) \times F_{sys} \quad \text{for}\ \rho \geq 1.1\rho_C \tag{28}$$

The coefficients A, B, D, and G are given in Table 3 for humid and dry conditions.

Equations (27) and (28) represent concise form of Equations (17) and (18) with predefined values of φ_{eff} and ε_{sh}. These short expressions may be suitable for conceptual design.

Figures 3 and 4 show span-to-depth limits $(l/d)_{\lim}^{prop}$ obtained by Equations (27) and (28), together with the reference values $(l/d)_{\lim}^{ref}$, for 3 values of the stress in the tensile reinforcement ($\sigma_s = 250, 200, 150$ MPa). The corresponding parameters C are 0.048, 0.06 and 0.08, respectively ($f_{ct,red}$ is about 2 MPa). The assumed values of δ are 0.1 (Figure 3, dry conditions) and 0.2 (Figure 4, humid conditions), and $F_{sys} = 1$ (simple beam).

Figures 3 and 4 show that the proposed expressions match the reference values very well. The deviations are mostly on the safe side. As previously shown, this conclusion also applies to other combinations of parameters included in the created database.

4 | COMPARISON WITH EXPERIMENTAL RESULTS

The proposed method complies with the reference procedure for checking the deflection (calculation of deflection from curvatures). To assess the overall validity of the method, the results are also compared with the measured deflections under long-term loading. Three published experiments with extensive measurements on both samples and girders were selected.

The required data from each experiment are tabulated below (Tables 4, 6 and 8). The calculated values according to the proposed method are shown in Tables 5, 7 and 9.

The number $N = N_{meas}$ (column 20) is calculated from the measured deflection $u_{t,meas}$ (column 13) and the actual span l (column 7), $N_{meas} = l/u_{t,meas}$. The required effective depth d_{req} (column 21) from expression (18), with $F_N = 250/N_{meas}$ and $F_{sys} = 1$, is compared with the actual depth d_{prov} (column 3) according to criterion Δ (Equation (26), column 22).

Equations (17) and (18) are formulated for elements loaded uniformly by a distributed load. However, they can be successfully applied for other load patterns. The combination of distributed and point loads is common in experiments. The stress in the tensile reinforcement due to the bending moment in the representative cross-section gives the required l/d ratio, Equations (17) and (18).

4.1 | G. W. Washa and P. G. Fluck[19]

This experiment was designed to examine the impact of compressive reinforcement on deflections. In total, 34 beams of rectangular cross-section were tested, of which 30 are presented below. The layout of the beams is shown in Figure 5.

The area of the tensile reinforcement was fairly uniform—about 1.6% of the concrete cross-sectional area ($b \cdot d$), and the stress in all specimens was about 140 MPa. All beams were loaded after 14 days with uniformly distributed load. Deflections were measured over 2½ years

TABLE 5 Calculated data according to the proposed method for the experiment—Washa and Fluck[19]

Specimen (1)	σ_s [MPa] (14)	C (15)	ρ (16)	δ (17)	ρ_C (18)	k_s (19)	$N = l/u_t$ (7)/(13) (20)	d_{req} [cm] (21)	d_{prov} [cm] (3)	Δ [%] (22)	Average Δ [%] (23)
A1/A4	137	0.148	0.165	0.16	0.041	1.00	258	32.2	25.7	−20%	−11.4%
A2/A5	137	0.148	0.165	0.16	0.041	0.50	189	30.1	25.7	−14%	
A3/A6	137	0.148	0.165	0.16	0.040	0.00	136	28.7	25.7	−10%	
B1/B4	139	0.133	0.183	0.23	0.043	1.00	119	18.4	15.7	−15%	
B2/B5	138	0.135	0.183	0.23	0.043	0.50	98	18.1	15.7	−13%	
B3/B6	136	0.137	0.183	0.23	0.044	0.00	71	16.1	15.7	−2%	
C1/C4	137	0.132	0.186	0.20	0.039	1.00	79	12.2	10.2	−17%	
C2/C5	136	0.133	0.186	0.20	0.040	0.50	63	11.9	10.2	−14%	
C3/C6	135	0.134	0.186	0.20	0.040	0.00	45	11.0	10.2	−7%	
D1/D4	138	0.143	0.189	0.20	0.043	1.00	138	12.5	10.2	−19%	
D2/D5	137	0.144	0.189	0.20	0.044	0.50	113	12.6	10.2	−19%	
D3/D6	135	0.154	0.182	0.20	0.047	0.00	79	10.8	10.2	−6%	
E1/E4	142	0.142	0.177	0.23	0.046	1.00	43	5.91	5.87	−1%	
E2/E5	141	0.143	0.177	0.23	0.046	0.50	41	6.77	5.87	−13%	
E3/E6	139	0.145	0.177	0.23	0.047	0.00	29	5.82	5.87	1%	

under uncontrolled ambient conditions. Creep coefficient and shrinkage strain for each particular specimen, shown in Table 4, were calculated from the measured values on the prisms, taking into account the size effect and compressive strength. Expressions according to EN 1992-1-1[2] were used, and a mean value of ambient humidity of 50% was adopted for the calculation. The modulus of elasticity at time of loading was measured. Tensile strength (not measured) at the age of 14 days was also calculated from compressive strength at the age of 28 days. All required data are summarized in Table 4. The measured deflections are shown in column 13.

Required effective depths d_{req} by Equation (18) (column 21), together with necessary input data for Equations (10), (11), (19)–(21) are shown in Table 5. The calculated stress in the tensile reinforcement σ_s is in column (14).

The required effective depths d_{req} are greater than the actual ones (column 3), so Equation (18) is conservative with the respect to this experiment. Also, the larger the compressive reinforcement, the more the result is on the safe side. The relative change, Equation (26), is in column 22. The mean value of Δ (column 23) is −11.4%, which is reasonably good. However, due to the small total height of the specimens, any deviation in the position of the reinforcement significantly affects the result. Expression (18) also shows this fact.

4.2 | J. P. Jaccoud and R. Favre[20]

This experimental program was a reference study for the CEB bilinear model for the calculation of the deflection of reinforced concrete elements. These very well designed tests with moderate scale factor (about two) consisted of four series of one-way and two-way slabs. The layout of the slabs is shown in Figure 6.

The results for 6 slabs from the C series, loaded at 4 load levels (0.3, 0.4, 0.5 and 0.6 of their ultimate load), are presented below. Slabs C12 and C22 were loaded closely above the cracking load (30% of the ultimate load). The remainder of the slabs (C13—40%, C14, C24—50%, and C15—60%) were fully cracked. In addition to self-weight, four-point bending was applied. The environmental conditions were controlled. The average RH was about 60%.

Numerous specimens were tested for mechanical properties. Creep and shrinkage were measured on prisms. Values shown in Table 6 were calculated from accompanying samples, taking into account the size effect and compressive strength. The axial tensile strength, which was suggested to be applied together with the bilinear method, was reported as 50% of the measured flexural strength. All required data are summarized in Table 6. The measured deflections are shown in column 13.

Required effective depths d_{req} are shown in Table 7. The required effective depths d_{req} by Equation (18) are close to the actual ones (column 3). The relative change (column 22) shows a good match with the measured values. The mean value of Δ (column 23) is only 0.7%, which is very good.

4.3 | Gilbert and Nejadi[21]

Twelve simply supported specimens (six beams and six one-way slabs) were moist cured for 14 days and then loaded with sustained loads for 400 days. The beams were loaded by four point bending and the slabs were loaded uniformly with a distributed load. Concrete compressive and tensile strength, modulus of elasticity, as well as deflections at midspan were measured at different times. The variable parameters were tensile reinforcement, load level, and concrete cover. The layout of the test specimens is shown in Figure 7.

Two specimens ("a," "b") were made for each combination of concrete cover and reinforcement. Type "a" slabs (S) were loaded to about 50%, while type "b" slabs were loaded to about 30% of the corresponding ultimate load. Type "a" beams (B) were loaded to about 45%, while type "b" beams were loaded to about 25% of the corresponding ultimate load. The creep coefficient and

TABLE 6 Experimental data—Jaccoud and Favre[20]

Specimen (1)	b [cm] (2)	d [cm] (3)	h [cm] (4)	A_{s1} [cm^2] (5)	A_{s2} [cm^2] (6)	l [m] (7)	$f_{ct\text{,}red}$ [MPa] (8)	$E_c(t_0)$ [GPa] (9)	$\varphi(t, t_0)$ (10)	ε_{sh} [‰] (11)	E_s [GPa] (12)	$u_{t,meas}$ [mm] (13)
C12	75.0	13.0	16.0	5.65	0.57	3.10	2.14	28.3	2.06	0.37	200	7.6
C22	75.0	13.0	16.0	5.65	0.57	3.10	2.02	30.8	2.01	0.27	200	7.0
C13	75.0	13.0	16.0	5.65	0.57	3.10	1.97	29.2	2.00	0.31	200	12.6
C14	75.0	13.0	16.0	5.65	0.57	3.10	2.14	28.3	2.06	0.37	200	17.3
C24	75.0	13.0	16.0	5.65	0.57	3.10	1.79	30.9	2.00	0.31	200	16.7
C15	75.0	13.0	16.0	5.65	0.57	3.10	2.03	28.7	1.83	0.21	200	19.5

TABLE 7 Calculated data according to the proposed method for the experiment—Jaccoud and Favre[20]

Specimen (1)	σ_s [MPa] (14)	C (15)	ρ (16)	δ (17)	ρ_C (18)	k_s (19)	$N = l/u_t$ (7)/(13) (20)	d_{req} [cm] (21)	d_{prov} [cm] (3)	Δ [%] (22)	Average Δ [%] (23)
C12	143	0.106	0.041	0.19	0.030	0.1	408	13.4	13.0	−3%	0.7%
C22	143	0.092	0.038	0.19	0.026	0.1	443	14.2	13.0	−8%	
C13	190	0.071	0.040	0.19	0.020	0.1	246	12.7	13.0	2%	
C14	238	0.064	0.041	0.19	0.017	0.1	179	12.3	13.0	6%	
C24	238	0.049	0.038	0.19	0.013	0.1	186	12.5	13.0	4%	
C15	286	0.049	0.040	0.19	0.013	0.1	159	12.5	13.0	4%	

TABLE 8 Experimental data—Gilbert and Nejadi[21]

Specimen (1)	b [cm] (2)	d [cm] (3)	h [cm] (4)	A_{s1} [cm^2] (5)	A_{s2} [cm^2] (6)	l [m] (7)	$f_{ct,red}$ [MPa] (8)	$E_c(t_0)$ [GPa] (9)	$\varphi(t, t_0)$ (10)	ε_{sh} [‰] (11)	E_s [GPa] (12)	$u_{t,meas}$ [mm] (13)
B1a	25.0	30.0	34.8	4.02	0.00	3.50	1.40	22.82	1.71	0.83	200	12.1
B1b	25.0	30.0	34.8	4.02	0.00	3.50	1.40	22.82	1.71	0.83	200	7.4
B2a	25.0	30.0	33.3	4.02	0.00	3.50	1.40	22.82	1.71	0.83	200	12.4
B2b	25.0	30.0	33.3	4.02	0.00	3.50	1.40	22.82	1.71	0.83	200	7.9
B3a	25.0	30.0	33.3	6.03	0.00	3.50	1.40	22.82	1.71	0.83	200	13.3
B3b	25.0	30.0	33.3	6.03	0.00	3.50	1.40	22.82	1.71	0.83	200	7.9
S1a	40.0	12.9	16.0	2.26	0.00	3.50	1.40	22.82	1.71	0.83	200	25.1
S1b	40.0	12.9	16.0	2.26	0.00	3.50	1.40	22.82	1.71	0.83	200	19.8
S2a	40.0	12.9	16.0	3.39	0.00	3.50	1.40	22.82	1.71	0.83	200	32.5
S2b	40.0	12.9	16.0	3.39	0.00	3.50	1.40	22.82	1.71	0.83	200	21.9
S3a	40.0	12.9	16.0	4.52	0.00	3.50	1.40	22.82	1.71	0.83	200	29.8
S3b	40.0	12.9	16.0	4.52	0.00	3.50	1.40	22.82	1.71	0.83	200	22.9

TABLE 9 Calculated data according to the proposed method for the experiment—Gilbert and Nejadi[21]

Specimen (1)	σ_s [MPa] (14)	C (15)	ρ (16)	δ (17)	ρ_C (18)	k_s (19)	$l/u_t = N$ (7)/(13) (20)	d_{req} [cm] (21)	d_{prov} [cm] (3)	Δ [%] (22)	Average Δ [%] (23)
B1a	226	0.052	0.047	0.14	0.013	0.0	290	28.80	30	4%	−3.6%
B1b	155	0.076	0.047	0.14	0.019	0.0	470	33.20	30	−11%	
B2a	225	0.052	0.047	0.10	0.012	0.0	282	28.54	30	5%	
B2b	153	0.077	0.047	0.10	0.017	0.0	445	32.08	30	−7%	
B3a	213	0.055	0.070	0.10	0.012	0.0	263	30.47	30	−2%	
B3b	128	0.092	0.070	0.10	0.021	0.0	443	34.43	30	−15%	
S1a	254	0.046	0.038	0.19	0.013	0.0	139	13.29	13	−3%	
S1b	197	0.060	0.038	0.10	0.017	0.0	177	13.12	13	−2%	
S2a	250	0.047	0.057	0.10	0.013	0.0	108	12.39	13	4%	
S2b	172	0.068	0.057	0.10	0.019	0.0	160	13.35	13	−3%	
S3a	218	0.054	0.076	0.10	0.015	0.0	117	13.68	13	−6%	
S3b	160	0.073	0.076	0.10	0.021	0.0	153	13.88	13	−8%	

shrinkage strain, shown in Table 8, were measured on the companion samples over 400 days. The measured splitting tensile strength at 14 days was 2.0 MPa. All required data are summarized in Table 8. The measured deflections are shown in column 13.

Required effective depths d_{req} are shown in Table 9. The relative change (column 22) between the required effective depth d_{req} by Equation (18) and the actual one shows a good match. The mean value of Δ (column 23) is only −3.6% and most of the values are on the safe side.

The comparisons presented in Chapter 3, in relation to the reference model of the calculation of deflection, as well as in Chapter 4, in relation to the selected experiments, confirm the efficiency of the proposed procedure.

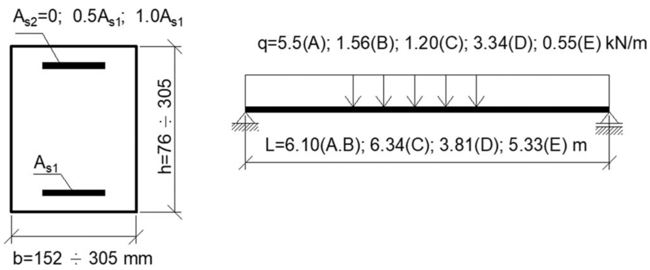

FIGURE 5 Layout of test specimens[19]

5 | CONCLUSIONS

An allowable span-to-depth ratio for deflection control is proposed. Deflections calculated by numerical integration of curvatures according to Eurocode 2 and MC 2010 were used as reference values for the formulation of the expressions. Expressions are given for cracked elements and for elements without cracks. The procedure was developed for one-way slabs or rectangular cross-section beams. The limitation applies to the total deflection under constant long-term (quasi-permanent) load.

The following parameters explicitly participate in the proposed expressions:

- Cross-sectional dimensions, area and position of tensile and compressive reinforcement;
- Material properties (moduli of elasticity, tensile strength of concrete);
- Long-term properties of concrete (effective creep coefficient, final shrinkage strain);
- Deflection limit (given as a fraction of the span length);
- Stress in tensile reinforcement in the representative cross-section due to quasi-permanent load.

Tensile and compressive reinforcement can be selected independently of ULS design requirements (areas can be

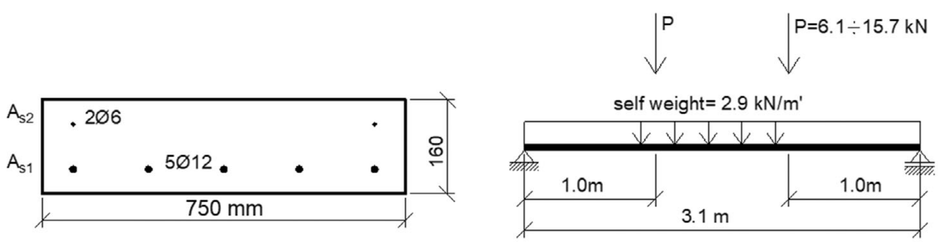

FIGURE 6 Layout of test specimens[20]

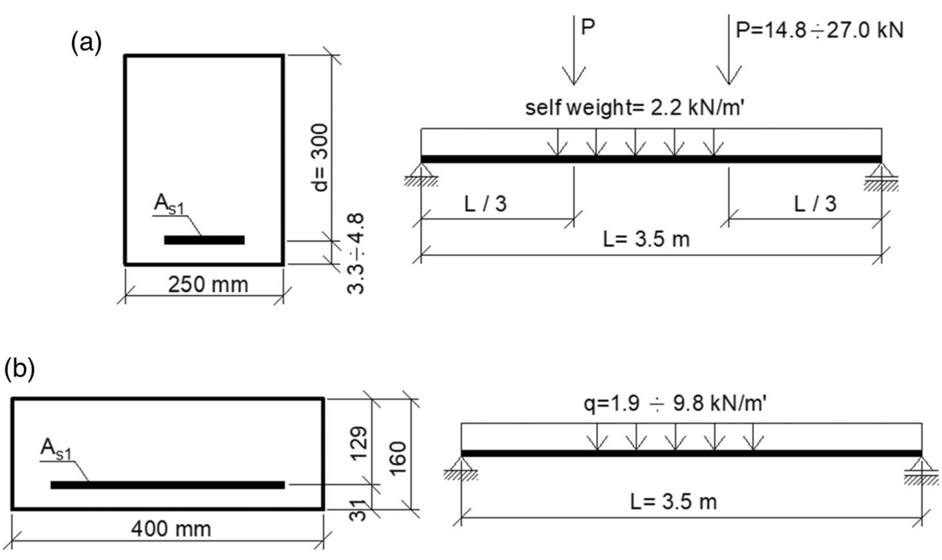

FIGURE 7 Layout of test specimens[21]—(a) beams and (b) slabs

enlarged to reduce deformation due to creep of concrete). The tensile strength of concrete is a direct parameter and the appropriate choice of its value takes into account the concrete class, the age of the concrete at the time of loading and the ratio of maximum service load and quasi-permanent load. The effective creep coefficient covers different load histories and environmental conditions. Quasi-permanent load is described by the effective reinforcement ratio and the stress in tensile reinforcement in a representative cross-section (cross-section with the maximum bending moment in the span). This approach allows that the ratio of the quasi-permanent and limit load is not predefined.

The expressions are simple and easy to apply, which is especially suitable for conceptual design. The formulation allows simplification if some parameters are neglected (e.g., compressive reinforcement) or predefined (e.g., effective creep coefficient). An analysis of the effective creep coefficient and total shrinkage strain according to Annex B of EN 1992-1-1, for a number of time histories and load compositions, was performed and representative values for "dry" and "humid" conditions were established.

The basic procedure was developed for a simple beam. A procedure based on the theory of elasticity is proposed for different support conditions. The ratio of deflections is expressed by the bending moment in representative cross-sections and the corresponding deflection coefficients.

The expressions are also valid in the case of redistribution of internal forces in the ULS design because the quasi-permanent load is completely described by the effective reinforcement ratio and the stress in tensile reinforcement.

A comparative study between the proposed span-to-depth ratio limits and those obtained from the calculated long-term deflections by numerical integration of curvatures (EN 1992-1-1, MC 2010, almost 9000 samples) showed very good agreement.

The results obtained with the proposed expressions have also been compared with measured long-term deflections from three published experimental studies (33 specimens).

Good compliance and a shift to the side of safety were shown.

The procedure is suitable for deflection verification according to Eurocode 2. It can also be used for conceptual design. The application is illustrated with numerical examples in Appendix A.

NOTATION

b	width of the rectangular cross section
d	effective depth of the section
f_{ct}	concrete tensile strength
$f_{ct,red}$	reduced value of the concrete tensile strength, Equation (5)
f_{ctm}	concrete mean tensile strength
h	height of the cross section
k_s	ratio of the compressive to tensile reinforcement area, Equation (20)
l	span of the element
sys	structural system
A_{s1}	tensile reinforcement area
A_{s2}	compressive reinforcement area
C	nondimensional parameter, Equation (10)
E_c	modulus of elasticity of concrete associated with creep coefficient (tangent modulus)
E_s	modulus of elasticity of reinforcement steel
F_N	multiplier that accounts for the selected deflection limit, Equation (25)
F_{sys}	multiplier that accounts for the structural system, Equation (24)
M_{cr}	cracking moment
M_D	maximum bending moment in the span
M_{qp}	bending moment due to the quasi-permanent load
$M_{char.}$	bending moment due to the characteristic load
N	number denoting the fraction of the span length used as the deflection limit (l/N)
α	modulus ratio, $\alpha = E_s/E_c$
δ	nondimensional cover to the center of the tension reinforcement, Equation (19)
ε_{sh}	final value of the shrinkage strain
$\varphi(\infty, t_i)$	creep coefficient for loading at age t_i
φ_{eff}	effective creep coefficient, Equation (12)
ρ	effective reinforcement ratio of the tensile reinforcement, Equation (11)
ρ_C	effective reinforcement ratio that corresponds to the load that produces the maximum moment in the span, which is equal to the cracking moment, Equation (21)
σ_s	stress in the tensile reinforcement due to quasi-permanent load at the section subjected to the maximum bending moment in span M_D, which is calculated based on the cracked section

DATA AVAILABILITY STATEMENT

The data that supports the findings of this study are available in the supplementary material of this article.

ORCID

Snežana Mašović https://orcid.org/0000-0002-4128-7410

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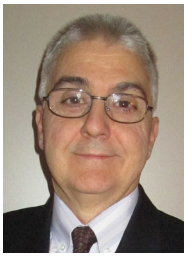

Nenad Pecić, Faculty of Civil Engineering, University of Belgrade, Belgrade, Serbia. Email: peca@imk.grf.bg.ac.rs

Snežana Mašović, Faculty of Civil Engineering, University of Belgrade, Belgrade, Serbia. Email: smasovic@grf.bg.ac.rs

Saša Stošić, Faculty of Civil Engineering, University of Belgrade, Belgrade, Serbia. Email: sasa@grf.bg.ac.rs

How to cite this article: Pecić N, Mašović S, Stošić S. Span-to-depth ratio limits for deflection control of reinforced concrete elements. Structural Concrete. 2022. 1–16. https://doi.org/10.1002/suco.202200132

APPENDIX A

Example 1a. (conceptual design)

Determine the appropriate thickness of a simply supported slab with a span of 6.0 m which is loaded with an additional permanent load of 3.5 kN/m^2 and a variable load of 3.0 kN/m^2 ($\psi_2 = 0.3$). The deflection limit for a quasi-permanent load is span/250. Other required data: C30/37, $E_{cm}=33\,\text{GPa}$, $f_{ct}=f_{ctm}=2.9\,\text{MPa}$, $\varphi_{eff}=2.5$, $\varepsilon_{sh}=0.5‰$ ("dry conditions"), B500 steel, $E_s=200\,\text{GPa}$.

A suitable assumption for the conceptual design is $\rho=0.02$ (which roughly corresponds to the reinforcement ratio of 0.33%–0.37%, depending on the concrete class). Also, if the reinforcement is adopted according to ULS design, σ_s is about 250 MPa for quasi-permanent load, and a suitable assumption is $C=0.045$:

(e.g., for C30/37: $f_{ct,red}=0.7\cdot f_{ctm}=0.7\cdot 2.9=2.0\,\text{MPa}$, $\alpha=\frac{E_s}{E_c}=\frac{E_s}{1.05E_{cm}}=\frac{200}{1.05\cdot 33}=5.8$ $C=\frac{\alpha f_{ct,red}}{\sigma_s}=\frac{5.8\cdot 2.0}{250}=0.046$. Similarly, for C25/30: $C=0.045$).

Other required data for Equation (18):

$\delta=0.10$ (assumed, required cover 15–20 mm); $F_{sys}=1$; $F_N=1$;

$\rho_C=0.045\cdot(0.22\cdot 0.045+0.10^2+0.3\cdot 0.10+0.17)=0.0099$; $B(k_s)=1$ (no compressive reinforcement).

$$\left(\frac{l}{d}\right)_{\lim}=\frac{500\cdot(3+0.1)}{250\cdot(1+0.2\cdot 2.5)+130\cdot 0.5}\times\left(1+\frac{0.5}{\sqrt{0.0200-0.0099}}\right)\times 1\times 1\times 1=21.0$$

where $d=600/21.0=28.6\,\text{cm}$ and $h=28.6+2.0+\varphi/2\approx 31\,\text{cm}$. (Note: with total height $h=31$ cm, the required ULS reinforcement is about 7.36 cm^2/m, $\rho=0.015$ and $\sigma_s=275\,\text{MPa}\cdot(l/d)_{\lim}$ by Equation (18) is 23.8, while $600/28.5=21.0<23.8$ – OK).

Example 1b. (deflection control)

Chapter 2 notes that the required effective depth (height) can be reduced by adding tensile reinforcement compared to that required by the ULS design. It is adopted for design: $h=0.9\cdot 31\approx 28$ cm.

For the height $h=28$ cm and the assumed $d=25.5$ cm, it follows: $M_{Ed}=84.04\,\text{kNm/m}$, $A_{s,req}=7.84\,\text{cm}^2/\text{m}$ (ULS), $M_{qp}=51.3\,\text{kNm/m}$ and $\sigma_s=272\,\text{MPa}$.

The required input for Equation (18):

$$\alpha=\frac{E_s}{E_c}=\frac{E_s}{1.05E_{cm}}=\frac{200}{1.05\cdot 33}=5.8;$$

$$C=\frac{\alpha f_{ct,red}}{\sigma_s}=\frac{5.8\cdot 2.0}{272}=0.043;$$

$$\delta=\frac{2.5}{28.0}=0.089;$$

$$F_{sys}=1;\ F_N=1;\ B(k_s)=1;$$

$$\rho=\frac{5.8\times 7.84}{25.5\times 100}=0.0178;$$

$$\rho_C=0.043\cdot\left(0.22\cdot 0.043+0.089^2+0.3\cdot 0.089+0.17\right)=0.0092.$$

$$\left(\frac{l}{d}\right)_{\lim}=\frac{500\cdot(3+0.089)}{272\cdot(1+0.2\cdot 2.5)+130\cdot 0.5}\times\left(1+\frac{0.5}{\sqrt{0.0178-0.0092}}\right)\times 1\times 1\times 1=20.9$$

where $d=600/20.9=28.7\,\text{cm}$ and $h=28.7+2.0+\varphi/2\approx 31.2\,\text{cm}>28$ cm.

The required effective depth is 13% greater than the actual one.

Additional tensile reinforcement in the amount of 35% ($A_s=1.35\cdot 7.84=10.58\,\text{cm}^2/\text{m}$) results in $\sigma_s=203\,\text{MPa}$ and:

$$C=\frac{\alpha f_{ct,red}}{\sigma_s}=\frac{5.8\cdot 2.0}{203}=0.057;$$

$$\rho=\frac{5.8\times 10.58}{25.5\times 100}=0.0241;$$

$$\rho_C=0.057\cdot\left(0.22\cdot 0.057+0.089^2+0.3\cdot 0.089+0.17\right)=0.0125;$$

$$\left(\frac{l}{d}\right)_{\lim}=\frac{500\cdot(3+0.089)}{203\cdot(1+0.2\cdot 2.5)+130\cdot 0.5}\times\left(1+\frac{0.5}{\sqrt{0.0241-0.0125}}\right)\times 1\times 1\times 1=23.6$$

where $d=600/23.6=25.4\,\text{cm}$ and $h=25.4+2.0+\varphi/2\approx 28\,\text{cm}$.

The deflection of the slab obtained by numerical integration of the curvatures is 2.4 cm, which corresponds to $N=600/2.4=250$, as required.

Example 2a. (conceptual design)

Determine the appropriate height of a continuous slab with two equal spans of 6.0 m which is loaded with an additional permanent load of 3.5 kN/m^2 and a variable load of

3.0 kN/m^2 ($\psi_2 = 0.3$). The deflection limit for a quasi-permanent load is span/300. Other required data C30/37, $E_{cm} = 33\,\text{GPa}$, $f_{ct} = f_{ctm} = 2.9\,\text{MPa}$, $\varphi_{eff} = 1.8$, $\varepsilon_{sh} = 0.3‰$ ("humid conditions"), B500 steel, $E_s = 200\,\text{GPa}$.

As in Example 1, $\sigma_s = 250\,\text{MPa}, \rho = 0.02$ and $C = 0.045$ are assumed.

Other required inputs:

$\delta = 0.15$ (assumed, required cover is about 25 mm); $F_{sys} = 1.35$ (one end restrained span);

$F_N = 250/300 = 0.833$, $B(k_s) = 1$ (no compressive reinforcement); $\rho_C = 0.045 \cdot (0.22 \cdot 0.045 + 0.15^2 + 0.3 \cdot 0.15 + 0.17) = 0.0111$.

$$\left(\frac{l}{d}\right)_{\text{lim}} = \frac{500 \cdot (3+0.15)}{250 \cdot (1+0.2 \cdot 1.8) + 130 \cdot 0.3} \times \left(1 + \frac{0.5}{\sqrt{0.0200 - 0.0111}}\right) \times 1 \times 1.35 \times 0.833 = 29.4$$

Hence $d = 600/29.4 = 20.4\,\text{cm}$ and $h = 20.4 + 3.0 = 23.4\,\text{cm}$ (Note: with a total height $h = 24$ cm, the required ULS reinforcement is about 4.93 cm^2/m, $\rho = 0.0136$ and $\sigma_s = 268\,\text{MPa}$. The ratio $(l/d)_{\text{lim}}$ by Equation (18) is 41.0, while $600/21.0 = 28.6 < 41.0$ – OK).

Example 2b. (deflection control)

As in Example 1b, a slightly lower thickness can be used, relying on the effect of additional tensile reinforcement. Assumed $h = 20$ cm.

For the adopted height $h = 20$ cm and the assumed $d = 17.0$ cm, it follows: $M_{Ed} = 40.44\,\text{kNm/m}$ (span moment) and $A_{s,req} = 5.68\,\text{cm}^2/\text{m}$ (ULS). M_{qp} equals 23.79 kNm/m and σ_s is 262 MPa.

The required input for Equation (18):

$$\alpha = \frac{E_s}{E_c} = \frac{E_s}{1.05 E_{cm}} = \frac{200}{1.05 \cdot 33} = 5.8$$

$$C = \frac{\alpha f_{ct,red}}{\sigma_s} = \frac{5.8 \cdot 2.0}{262} = 0.0443$$

$$\delta = \frac{3.0}{20.0} = 0.15$$

$$\rho_C = 0.0443 \cdot \left(0.22 \cdot 0.0443 + 0.15^2 + 0.3 \cdot 0.15 + 0.17\right) = 0.0110$$

$$\rho = \frac{5.8 \times 5.68}{17 \times 100} = 0.0194$$

$$\left(\frac{l}{d}\right)_{\text{lim}} = \frac{500 \cdot (3+0.15)}{262 \cdot (1+0.2 \cdot 1.8) + 130 \cdot 0.3} \times \left(1 + \frac{0.5}{\sqrt{0.0194 - 0.0110}}\right) \times 1 \times 1.35 \times 0.833 = 28.9$$

where $d = 600/28.9 = 20.8\,\text{cm}$ and $h = 20.8 + 3.0 = 23.8\,\text{cm} > 20\,\text{cm}$.

The required effective depth is 19% greater than the actual one.

The check is repeated with Ø12/125 mm (9.04 cm^2/m). The corresponding σ_s is 167 MPa:

$$C = \frac{\alpha f_{ct,red}}{\sigma_s} = \frac{5.8 \cdot 2.0}{167} = 0.069$$

$$\rho_C = 0.069 \cdot \left(0.22 \cdot 0.069 + 0.15^2 + 0.3 \cdot 0.15 + 0.17\right) = 0.0175$$

$$\rho = \frac{5.8 \times 9.04}{17.0 \times 100} = 0.0308$$

$$\left(\frac{l}{d}\right)_{\text{lim}} = \frac{500 \cdot (3+0.15)}{167 \cdot (1+0.2 \cdot 1.8) + 130 \cdot 0.3} \times \left(1 + \frac{0.5}{\sqrt{0.0308 - 0.0175}}\right) \times 1 \times 1.35 \times 0.833 = 35.5$$

where $d = 600/35.5 = 16.9\,\text{cm}$ and $h = 16.9 + 3.0 = 19.9\,\text{cm} \approx 20\,\text{cm}$.

(Note: The deflection of the slab obtained by numerical integration of the curvatures is 1.9 cm, which corresponds to $N = 600/1.9 = 315 > 300$, as required. The redistribution of the bending moment has been performed so that the long-term curvatures meet the support conditions.

The reinforcement above the support is the one required by ULS—not changed—10.4 cm^2/m).