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*Emilija Jočić<sup>1</sup>, Miroslav Marjanović<sup>2</sup>*

## **ANALIZA PROGRESIVNOG LOMA KOMPOZITNIH LAMINATA IZLOŽENIH SILI PRITISKA**

### **Rezime:**

U ovom radu sprovedena je analiza progresivnog loma pločastih kompozitnih laminata, opterećenih na pritisak. Hashin-ov kriterijum je korišćen prilikom određivanja inicijacije loma laminata, dok je razvoj oštećenja analiziran primenom modela razmazane pukotine. U cilju poboljšanja efikasnosti proračuna, izabrani model oštećenja je implementiran u numerički model zasnovan na Redijevoj slojevitoj teoriji ploča. Prilikom validacije razvijenog algoritamskog modela, dobijeni rezultati su upoređeni sa numeričkim rezultatima iz literature i dobijena su odlična poklapanja.

*Ključne reči: progresivni lom, razmazana pukotina, slojevita teorija, laminat*

## **PROGRESSIVE FAILURE ANALYSIS OF COMPOSITE LAMINATES LOADED IN COMPRESSION**

### **Summary:**

This paper deals with the progressive failure analysis (PFA) of plate-like composite laminates loaded in compression. The Hashin failure criterion was used to detect failure initiation, while damage evaluation behaviour was calculated using the smeared crack band (SCB) damage model. SCB damage model have been incorporated into the Full-Layerwise Theory (FLWT) framework to increase the computational efficiency of the PFA. To verify the effectiveness of the FLWT-SCB prediction model, the obtained results were compared against the benchmark data from the literature and excellent agreement has been obtained.

*Key words: progressive failure, smeared crack, layerwise theory, laminate*

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<sup>1</sup> Asistent, student doktorskih studija, Građevinski fakultet, Univerzitet u Beogradu, edamjanovic@grf.bg.ac.rs

<sup>2</sup> Dr, Docent, Građevinski fakultet, Univerzitet u Beogradu, mmarjanovic@grf.bg.ac.rs

## 1. INTRODUCTION

Due to their outstanding strength and stiffness, low maintenance costs and corrosion resistance, fiber-reinforced composites have been widely used in the construction of aerospace, marine, mechanical and automotive structures which generally require high reliability levels. These materials have a growing use in civil engineering, as well. The low compressive strength of laminar composites is a limiting factor in fully harnessing the capabilities of such structural elements. In real structural elements, holes and notches are often present, which further decreases the compressive strengths due to presence of stress concentration zones. A crucial factor in determining the response of laminar composites in compression is the presence of manufacturing imperfections, such as fiber misalignment and microstructural voids [1]. Such manufacturing imperfections lead to various failure mechanisms in laminar composites under compression, such as fiber buckling and kinking, matrix yielding and cracking, among others. Failure mechanisms often interact with each other, leading to complex crack pathways when modelling fracture problems in laminar composites.

Progressive failure analysis (PFA) of laminar composites has been studied for a long time and used to predict damage progression and post-failure behaviour in composite materials. For the implementation of failure initiation and propagation algorithms within an existing numerical (i.e. finite element) model, damage models are required [2] such as: discrete damage models (DDM) and continuum damage models (CDM). Generally, discrete techniques lead to models that can accurately predict crack propagation with high fidelity, but at the cost of high computational effort. On the other hand, in CDM approaches, intralaminar cracks are smeared out within the finite element domain and the fracture mechanism is then represented through material stiffness degradation, controlled by damage variables.

The earliest CDM approach is instantaneous softening method (ISM), where the material property associated with the failure mode degrades instantly to zero (or some small value) of the undamaged material properties [3-5]. A comparison of ISM-based models against the experimental results confirmed that the predicted failure occurs at a substantially lower load than the experimentally determined one, underestimating the laminate strength and neglecting the fact that the damage is indeed localized and a failed lamina still has a solid residual load-carrying capability.

Composite materials generally exhibit quasi-brittle post-failure behaviour resulting in a high fracture energy dissipated and therefore a more gradual propagation of fracture [6, 7]. An alternative approach to ISM is gradual softening method (GSM), where the material property associated with the failure mode is degraded gradually (i.e. linearly or exponentially) until it reaches zero [8-10]. The approaches outlined above suffer from a mesh dependency problem related to strain localization during the PFA, associated with the stress concentration zones in plate-like structures (i.e. notches and holes).

The mesh dependency issue can be overcome via the fracture-mechanics augmented smeared crack-band (SCB) model [11], by scaling the fracture energy using a characteristic element length, as described by the crack-band theory [12]. This approach was originally developed for the macroscopic sub-laminate level modeling of laminar composites [13]. Such model leads to a good compromise between computational cost and solution accuracy, and they are thus used in most of the works dealing with composite failure modelling in conjunction with different plate theories. Most SCB models available in the literature generally focus on

PFA of laminar composites under tension [14-16], and the application of these models for the PFA of compressed laminar composites is relatively rare. Some examples of the use of SCB models for the compressive failure modelling of laminar composites include the failure analysis of open-hole compression (OHC) laminar composites [17], and the axial crush simulation of braided composite tubes [18].

The computational costs associated with PFA of laminar composites can be prohibitive, even considering the relative efficiency of smeared crack-band models, especially for larger structures. This is usually due to the requirement of refined, often 3D meshes, to obtain an accurate stress field.

This paper aims to increase the computational efficiency of the progressive failure analysis of laminar composites and preserve the accuracy of the 3D finite element models, by using a layered FLWT-based finite element model [19] for structural analysis and SCB damage model [11] for material modelling. The proposed FLWT-SCB prediction model was previously developed by authors [20] for the progressive failure modelling of open-hole laminar composites under tension, and here the model applicability for compressed members is highlighted. The developed FLWT-SCB prediction model was implemented into an original FLWTFEM framework [21]. To verify the proposed model implementation for compressive failure within the FLWTFEM framework, a series of single element simulations were carried out. For model validation, the obtained results are compared against the experimental load-displacement curves and strain diagrams by Nagaraj et al. [22].

## 2. LAYERWISE THEORY FOR 3D STRUCTURAL ANALYSIS OF LAMINAR COMPOSITES

In the paper, a laminated composite plate made of  $n$  perfectly bonded orthotropic layers is considered (Figure 1, left). The total plate thickness is denoted as  $h$ , while the thickness of the  $k^{th}$  lamina is denoted as  $h_k$ . The plate is supported along the portion  $\Gamma_u$  of the boundary  $\Gamma$  and loaded with loadings  $q_t(x,y)$  and  $q_b(x,y)$  acting to either top or the bottom surface of the plate ( $S_t$  or  $S_b$ ). Piece-wise linear variation of all three displacement components through the thickness is imposed, leading to the layer-wise 3D stress field. The displacement field ( $u, v, w$ ) of an arbitrary point ( $x,y,z$ ) of the laminate is given as:

$$\begin{aligned} u(x, y, z) &= \sum_{I=1}^N U^I(x, y) \Phi^I(z), & v(x, y, z) &= \sum_{I=1}^N V^I(x, y) \Phi^I(z), \\ w(x, y, z) &= \sum_{I=1}^N W^I(x, y) \Phi^I(z) \end{aligned} \quad (1)$$

In Eq. (1),  $N$  is the number of numerical layers,  $U^I(x,y)$ ,  $V^I(x,y)$  and  $W^I(x,y)$  are the displacement components in the  $I^{th}$  numerical layer of the plate in directions  $x$ ,  $y$  and  $z$ , respectively,  $\Phi^I(z)$  are selected to be linear layerwise continuous functions of the  $z$ -coordinate, defined over the considered layer  $I$ .

Based on the assumed displacement field, linear strain field can be easily derived and may be found in [19]. To reduce the 3D model to a plate model, the  $z$ -coordinate is eliminated by the explicit integration of stresses multiplied with the corresponding functions  $\Phi^I(z)$ , introducing the constitutive relations of the laminate which can be found in [19].

The finite element discretization is derived by introducing an assumed interpolation of the displacement field into the weak form of the FLWT. All displacement components are interpolated using the same 2-D Lagrange interpolation polynomials. Element stiffness matrix is obtained in a common way, using 2D Gauss-Legendre quadrature for quadrilateral domains. In the paper, quadratic serendipity (Q8) layered quadrilateral elements have been considered (Figure 1, right). To avoid shear locking, reduced integration is used (2×2 points for Q8). After the derivation of the characteristic element matrices, the assembly procedure is performed in a usual manner.

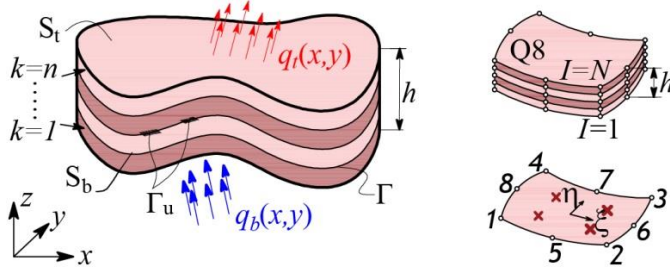


Figure 1 – left: Laminated composite plate with  $n$  material layers and  $N$  numerical interfaces; right: Quadratic serendipity Q8 layered element with linear layerwise interpolation through the thickness and corresponding Gauss quadrature points for the reduced integration.

Once the nodal displacements are obtained the stresses are evaluated from the well-known lamina 3D constitutive equation in the Gauss points at the top (t) and bottom (b) interfaces of the considered lamina and they are given in [19]. The stresses are calculated both in the laminate ( $xyz$ ) and the local lamina ( $123$ ) coordinate systems. This is crucial in the progressive failure analysis (PFA), since the failure criteria require the stresses in the lamina coordinate system. Since the interlaminar stresses calculated from the constitutive equations are discontinuous at layer interfaces, they are re-computed by assuming quadratic distribution through each layer, using the procedure given in detail in [23].

### 3. SMEARED CRACK-BAND COMPRESSIVE DAMAGE MODEL

The current work deals with compressive progressive failure of open-hole laminar composites via the fracture-mechanics augmented SCB damage model [11]. The SCB damage model has been previously described in [20] and is briefly recapitulated here for completeness.

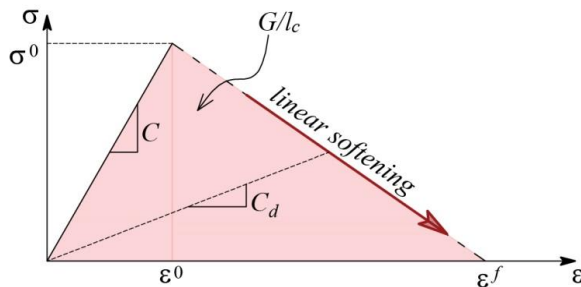


Figure 2 – Stress-strain relationship with linear softening law

In smeared formulations, the fracture energy is distributed (smeared) over the full volume of the element. The response of damaged lamina, in both fiber and matrix direction, was described by distinct bilinear strain-softening curves (see Figure 2), where the peak stress coincides with the fibre and matrix strength, respectively.

This damage law is determined based on the assumption that the total energy needed to fail an element (released strain energy) is equal to the energy needed to create a crack that passes through it. The released strain energy of a failed element is determined as a product of the area under the stress-strain curve, defined in Figure 2, and characteristic element length  $l_c$ . In the present work,  $l_c$  is defined as the square root of a layered finite element area. This energy is set to be equal to the dissipated fracture energy of the composite material (fracture toughness):

$$\varepsilon^f = \frac{2G}{\sigma_0 l_c} \quad (2)$$

In Eq. (2),  $\varepsilon^f$  is maximum strain, while  $\sigma_0$  is the material strength. The fracture toughness  $G$  is a material property that must be specified for each failure mode. By including the characteristic element length into the material damage law, the constant dissipated fracture energy is achieved, regardless of the element dimensions.

### 3.1. DAMAGE INITIATION AND EVOLUTION

Initiation of damage occurs when stresses in the weakest lamina exceed the allowable fiber or matrix strength. In this paper, the Hashin failure criterion [24] is used, a quadratic criterion in a piecewise form based on the material strengths, where each smooth branch represents a failure mode. These criteria consider four different damage initiation modes: fiber tension, fiber compression, matrix tension, and matrix compression. The quadratic Hashin failure criteria are given in [20].

Once the failure has been initiated, the damaged material must be unloaded in a proper manner, so the stresses can be redistributed to the remaining undamaged material. As damage progresses, PFA is performed through the material stiffness degradation, controlled by damage variables. For each failure mode  $J$  ( $J = \text{fiber tension, fiber compression, matrix tension, matrix compression}$ ) these variables are defined such that they have values between zero (undamaged status) and one (complete damaged status). The evolution of each damage variable is governed by equivalent strain  $\varepsilon_{J,eq}$ . In this way, each damage mode is represented as a 1D stress-strain problem (Figure 2) instead of the actual 3D stress-strain relation. The equivalent strain and corresponding equivalent stress for each failure mode were defined in detail in [20].

Using the equivalent strain and stress, a damage variable for each failure mode  $J$  can be calculated using the following relation:

$$d_J = \frac{\varepsilon_{J,eq}^f (\varepsilon_{J,eq} - \varepsilon_{J,eq}^0)}{\varepsilon_{J,eq}^f (\varepsilon_{J,eq}^f - \varepsilon_{J,eq}^0)}, \quad \varepsilon_{J,eq}^0 \leq \varepsilon_{J,eq} \leq \varepsilon_{J,eq}^f \quad (3)$$

In Eq. (3),  $\varepsilon_{J,eq}^0$  is the equivalent strain at the initial failure state ( $d_J = 0$ ) and  $\varepsilon_{J,eq}^f$  is the equivalent strain at the final failure state ( $d_J = 1$ ). Since damage evolution is an irreversible

process, damage variable is equal to the maximum of its current value and the value obtained from Eq. (3). The equivalent strain at the final failure state  $\varepsilon_{J,eq}^f$  is computed from Eq. (2).

#### 4. MODEL VALIDATION

This section presents a series of single-element analyses to verify the applicability of the FLWT-SCB prediction model for the progressive failure analysis of laminar composites loaded in compression. Such simulations provide a convenient method to verify the failure initiation and progression of each failure mode independently. In the current verification tests, the structure is a IM7/8552 unidirectional lamina (see Table 1) of thickness  $h_i = 0.125\text{mm}$ , modelled as a single element of size  $1\text{ mm} \times 1\text{ mm}$  and subjected to uniaxial compression. All the material properties are adopted according to reference [22]. The quasi-3D stress analysis was performed using a single Q8 layered quadrilateral element with reduced integration in all numerical simulations. Each lamina is modelled as a single numerical layer, adopting the linear distribution of displacements along the lamina thickness. The use of a single element test is typical for this class of problems, and it is necessary to verify the model reliability. In all simulations, the specimen was clamped on one side and loaded using a 2mm displacement on the opposite side. At the clamped end, all DOFs were constrained.

Table 1 – Material properties of IM7/8552 carbon fibre reinforced polymer

Properties	Values	Properties	Values	Properties	Values
$E_1$	150 GPa	$\nu_{23}$	0.48	$S = T$	90 MPa
$E_2 = E_3$	11 GPa	$X_T$	2560 MPa	$G_{ft}$	120 KJ/m <sup>2</sup>
$G_{12} = G_{13}$	5.8 GPa	$X_C$	1690 MPa	$G_{fc}$	80 KJ/m <sup>2</sup>
$G_{23}$	2.9 GPa	$Y_T = Z_T$	73 MPa	$G_{mt}$	2.6 KJ/m <sup>2</sup>
$\nu_{12} = \nu_{13}$	0.34	$Y_C = Z_C$	250 MPa	$G_{mc}$	4.2 KJ/m <sup>2</sup>

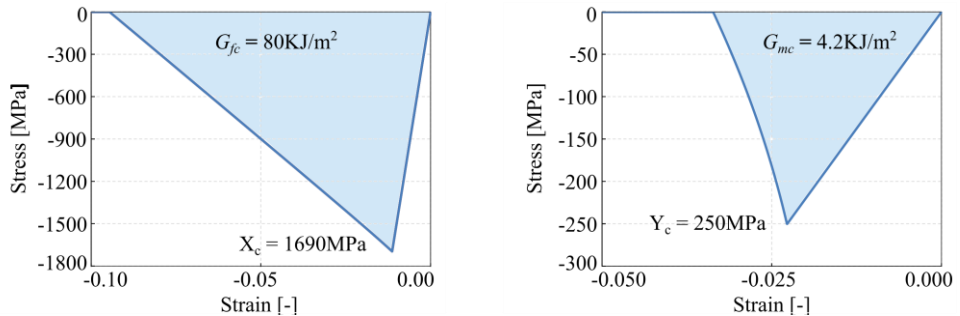


Figure 3 – left: Compressive stress - strain curve of a  $1\text{mm} \times 1\text{mm}$  single element lamina loaded in longitudinal (fiber) direction; right: Compressive stress - strain curve of a  $1\text{mm} \times 1\text{mm}$  single element lamina loaded in transverse (matrix dominated) direction.

The first simulation involves a specimen made from a single unidirectional layer ( $0^\circ$ ) under longitudinal compression (in the fiber direction,  $0^\circ$ ), which consequently results in a fibre failure mode. The stress-strain curve for this case is plotted in Figure 3a. Next, the same specimen made from a single unidirectional layer ( $0^\circ$ ) is subjected to transverse compression



(perpendicular to the fiber), resulting in a matrix failure mode. The stress-strain curve for this case has been shown in Figure 3b. The final assessment is the compressive loading of a single element consisting of a quasi-isotropic laminate consists of 16 layers assembled in  $[90^\circ/45^\circ/0^\circ/-45^\circ]_{2s}$  stacking sequence. Since the stacking sequence is symmetric, only half of the laminate through the thickness was modelled to reduce the number of DOFs.

In the symmetry plane, displacements in the thickness direction were set to zero. Figure 4 illustrates the stress-strain response predicted by the developed FLWT-SCB framework, along with reference numerical results [22].

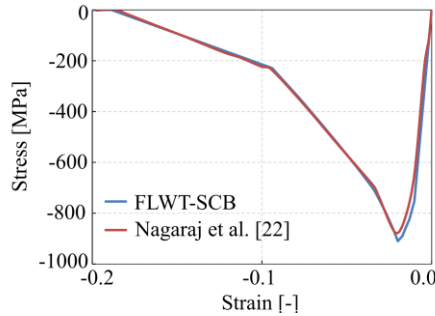


Figure 4 – Compressive stress - strain curve of a  $1\text{mm} \times 1\text{mm}$  single element laminate with  $[90^\circ/45^\circ/0^\circ/-45^\circ]_{2s}$  stacking sequence, considering: present FLWT-SCB model (blue lines) and the damage model of Nagaraj et al. [22] (red lines).

The stress-strain response of a specimen made from a single unidirectional layer ( $0^\circ$ ), loaded in compression parallel and perpendicular to the fiber, follows the bilinear path described by the FLWT-SCB damage model with linear softening, as shown in Figure 3. The peak stress in both cases is equal to the fibre (1690 MPa) and matrix (250 MPa) material strengths. Furthermore the area under the stress-strain curve is equal to the fracture energy in the longitudinal ( $80 \text{ KJ/m}^2$ ) and transverse ( $4.2 \text{ KJ/m}^2$ ) directions. Finally, the stress-strain response of the single element  $[90^\circ/45^\circ/0^\circ/-45^\circ]_{2s}$  laminate obtained by FLWT-SCB prediction model is in good agreement with reference numerical results [22], as shown in Figure 4.

## 5. CONCLUSIONS

The present paper aims to increase the computational efficiency of the progressive failure analysis of laminar composites, preserve the accuracy of the 3D finite element models, by using a layered FLWT-based finite element model [19] for structural analysis and SCB damage model [11] for material modelling. The proposed FLWT-SCB prediction model was previously developed by authors [20] for the progressive failure modelling of open-hole laminar composites under tension. The developed FLWT-SCB prediction model was implemented into an original FLWTFEM framework. In SCB approaches, the material stiffness matrix degradation was controlled by damage variables, which evolution are governed by an equivalent strains appropriately defined for each failure mode. The response of damaged lamina, in both fiber and matrix direction, was described by distinct bilinear strain-softening curves, where the peak stress coincides with the fibre and matrix strength, respectively. The mesh dependency problem was reduced by scaling the fracture energy using a characteristic element length, as described by the crack-band theory [12]. The failure initiation and modes of

failure are determined using the Hashin failure criterion. The use of layered quadrilateral elements in the damage modelling of laminar composites is relatively unexplored in the literature, where the standard approach is to use linear solid elements.

To verify the proposed model implementation for compressive failure within the FLWTFEM framework, a series of single elements tests was carried out. The quasi-3D stress analysis was performed using a single Q8 layered quadrilateral element with reduced integration in all tests. For model validation, the obtained results are compared against the experimental load-displacement curves and strain diagrams by Nagaraj et al. [22].

FLWT-SCB demonstrated a bilinear stress-strain response of single element laminas, loaded in compression parallel and perpendicular to the fiber, where the peak stress coincides with the fibre and matrix strength, respectively. Furthermore the area under the stress-strain curve is equal to the fracture energy in the longitudinal and transverse directions. Finally, the stress-strain response of the single element  $[90^\circ/45^\circ/0^\circ/-45^\circ]_{2s}$  laminate obtained by FLWT-SCB prediction model is in good agreement with reference numerical results [22].

The advantages of layered FLWT-based finite element model were demonstrated, in particular the savings in computational costs and the capability to provide the accurate 3D stress field, which is curious for accurate prediction of damage initiation and evolution in problems with highly localized stress peaks. Moreover, the use of layered quadrilateral elements leads to relaxation of element aspect-ratio and size constraints, from which 3D models suffer. These aspects could have significant advantages in improving the computational efficiency during the progressive failure analysis of large-scale composite structures.

Future works includes the extension of the FLWT-SCB framework's capabilities towards PFA of an open-hole fiber-reinforced laminates loaded in compression, the inclusion of cohesive zone modelling to account for delamination, as well as the application of the framework to the prediction of behaviour of cross-laminated timber (CLT) panels.

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