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## **PROCENA ODGOVORA KONSTRUKCIJE USLED VIBRACIJA OD SAOBRAĆAJA PRIMENOM TRANSFER FUNKCIJA**

### ***Rezime:***

Kao rezultat ubrzanog razvoja gradova i izgradnje brzih železnica i puteva sve češće se u objektima javljaju vibracije izazvane saobraćajem. Ovaj problem postaje važan jer vibracije mogu negativno uticati na kvalitet života i zdravlje ljudi. U okviru ovog rada izvršena je analiza uticaja krutosti tla na vibracije od saobraćaja koje se javljaju u stambenoj osmospratnoj zgradi, uzimajući u obzir dinamičku interakciju tla i objekta primenom metode podstruktura i metode transfer funkcija. Rezultati su prikazani u vidu transfer funkcija za dve karakteristične tačke unutar zgrade. Pokazano je da se rezonantni odgovor konstrukcije javlja pri nižim frekvencijama za mekše tlo odnosno pri višim za tlo veće krutosti, što ukazuje da interakcija tla i objekta može značajno uticati na nivo vibracija u zgradama.

*Кljučне речи: Interakcija tla i objekta, Dinamička krutost, Vibracije usled saobraćaja*

## **ASSESSMENT OF TRAFFIC-INDUCED BUILDING VIBRATIONS USING THE TRANSFER FUNCTIONS METHOD**

### ***Summary:***

As a result of rapid development of cities and construction of high-speed rail/road infrastructure, vibration serviceability of buildings due to traffic-induced vibrations has become a growing issue that affects the comfort of living and health of building occupants. In this paper a numerical study in which the influence of soil stiffness on the traffic-induced vibrations of an eight-story residential building is investigated considering soil-structure interaction and applying the substructuring approach and transfer function method. The results are presented in terms of transfer functions for selected points of interests inside the building. The results show that the resonant response of the structure occurs at lower frequencies for soft soil and at higher frequencies for stiff soil, indicating that soil-structure interaction can significantly affect the vibration response of the building.

*Keywords: Soil-structure interaction, Dynamic stiffness, Traffic-induced vibrations*

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## 1. INTRODUCTION

Rapid development of cities led to the need for high-speed transport and high-rise buildings. The existence of such systems is stricken by the traffic induced ground borne vibrations that affect the comfort of living and the health of the occupants. In order to mitigate the influence of the traffic induced vibrations on buildings, one must predict the response of the building by using a reliable and efficient numerical model that takes into account the soil-structure interaction (SSI).

The SSI is usually performed using the substructuring approach. The system is decomposed into two substructures: the structure and the soil-foundation system. The soil-foundation interaction (SFI) is solved using transform techniques such as Thin Layer Method [1], Boundary Element Method [2] or Integral Transform Method [3] that takes into account the unbounded domain of the soil medium. The aim of the SFI analysis is to provide dynamic stiffnesses of the foundations - impedances, that are assigned to the structure model at the soil-structure interaction joints.

The usage of SSI is still not common in everyday engineering practice. Codes and standards contain limited information and the literature is usually very complex to understand without extensive knowledge. This triggered many researchers to try to simplify the SSI analysis. Analytical solutions of impedances for foundations of regular geometries over homogeneous soil sediments are collected and presented in the literature [4], [5]. The geometry of every structure is unique as well as the dynamic response of the structure. The tendency of researchers is to generalize the response as much as possible and to describe it using geometrical parameters such as bay spans, story heights, number of bays and stories and the characteristics of the structural elements of the system such as soil, column and floor resonances [6], [7]. This is usually done by using transfer functions (TFs) - a frequency dependent functions which output the response of the system for different inputs. In general, input and output could be any parameter. In SSI analysis they are usually displacements, velocities or accelerations.

The aim of this paper is to show the importance of the SSI in the dynamic analysis of structures and to present the methodology of performing a simplified SSI analysis on a dwelling building founded on a homogeneous soil sediment. The analysis is performed by upgrading the FEM model of the building modeled in CSI SAP2000 with impedances obtained using analytical expressions from the literature. The results are presented in terms of transfer functions for selected points of interests inside the building. Since the impact of traffic induced ground borne vibrations on humans is investigated, only vertical vibrations are considered.

## 2. METHODOLOGY

This paper presents the methodology for problems where the input ground motion at the foundation level of the structure is known. Therefore, only inertial soil-foundation interaction is considered. The methodology for calculation building response including SSI considers:

- creating FEM model of the structure,
- obtaining dynamic stiffnesses of the foundations – impedances,
- implementing impedances in the FEM model of the structure,
- calculation of transfer functions for considered observation points of the structure,

- obtaining the response of the structure by combining transfer functions with the free field vibrations due to an arbitrary excitation.

Dynamic stiffnesses of the foundations - impedances, are obtained using analytical solutions from the literature [5]. Only vertical impedances are calculated, since the impact of vibrations on humans is investigated. In general, the impedance of the foundation is a frequency dependent complex function of the following form:

$$\bar{k} = k + i\omega c \quad (1)$$

where  $k$  and  $c$  are frequency dependent stiffness and dashpot coefficients and  $\omega$  is circular frequency of the system.

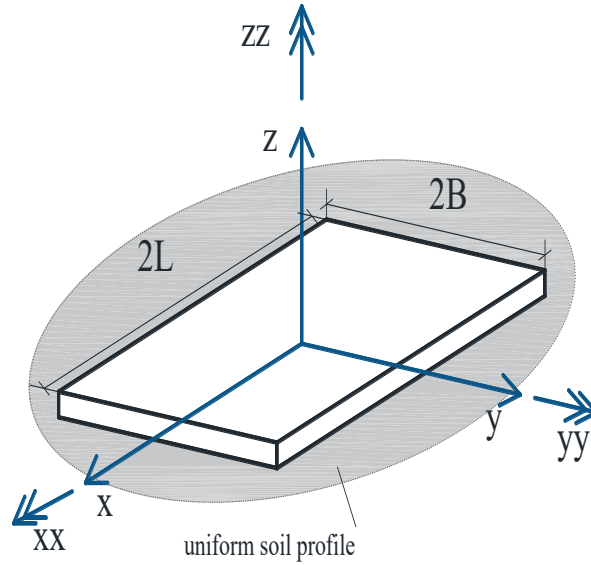


Figure 1 - Disposition of the surface rectangular foundation over uniform soil profile

However, solutions for surface massless rigid rectangular foundations over uniform soil profile, Figure 1, can be given in terms of foundation static stiffness  $K$  and dynamic stiffness modifier  $\alpha$  [5] as follows:

$$k_z = K_{z,sur} \alpha_z \quad (2)$$

$$K_{z,sur} = \frac{GB}{1-\nu} \left[ 3.1 \left( \frac{L}{B} \right)^{0.75} + 1.6 \right] \quad (3)$$

$$\alpha_z = 1 - \frac{\left( 0.4 + \frac{0.2}{L/B} \right) \alpha_0^2}{\left( \frac{10}{1 + 3(L/B - 1) + \alpha_0^2} \right)} \quad (4)$$

$$\alpha_0 = \frac{\omega_1 B}{c_s} , \quad (5)$$

where  $G$  is the shear modulus,  $L$  and  $B$  are foundation dimensions,  $\nu$  is Poisson's ratio and  $\alpha_0$  is dimensionless frequency. The dashpot coefficient of the foundation  $c_z$  accounts for two types of damping: damping ratio of the soil  $\beta_s$  ( $\beta_s = 2\%$  in all calculations), and radiation damping ratio  $\beta_z$ :

$$c_z = 2k_z \frac{\beta_z + \beta_s}{\omega}, \quad (6)$$

$$\beta_z = \frac{4\psi \frac{L}{B} \alpha_0}{\frac{K_{z,sur}}{GB} 2\alpha_z}, \quad (7)$$

$$\psi = \sqrt{2(1-\nu)(1-2\nu)} \quad \text{limited to } \psi \leq 2.5$$

where  $c_s$  is the shear wave velocity of the soil. The foundation impedances are implemented into the model of the structure as frequency dependent springs, Figure 2.

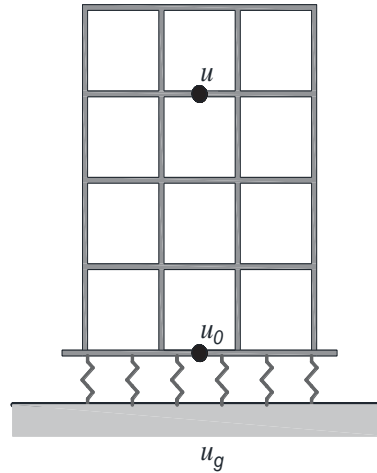


Figure 2 - SSI substructuring approach

In order to easily obtain the response of the structure due to arbitrary loads, frequency dependent TFs are calculated. They represent the structure response spectrum  $u$  due to a displacement impulse spectrum applied at the building foundation,  $u_0$ , such that:

$$TF = \frac{u}{u_0} \quad (8)$$

The response spectrum of the building at observation point  $j$  due to the load spectrum  $u_g$ , is obtained as

$$u_j(g) = TF_j u_g \quad (9)$$

The transfer function at observation point  $j$  is a set of results of multiple steady state analyses performed for discrete frequencies in a desired frequency range.

### 3. MODEL

The structure analyzed in this paper is an eight-story dwelling reinforced concrete (RC) building supported by columns and shear walls. The global 3D FEM model of the building is modeled in CSI SAP2000. The structure is founded on rectangular massless rigid surface foundations. The foundations under the building are square footings ( $L=B=3$  m) under the columns, and 3 m wide strip footings under the walls ( $L=7$  m for walls in the x direction and  $L=8$  m for walls in the y direction).

Horizontal elements are slabs 24 cm in thickness, and perimeter beams B1 30x60 cm. Vertical elements are shear walls W1 500x30 cm and W2 600x30 cm. There are three types of column dimensions, the corner ones with dimension C3 30x30 cm, edge ones C2 30x60 cm, and the internal C1 60x60 cm. All beams and columns are modeled as frame elements, walls and slabs as shell elements with material properties of concrete class C30/37. Layout of the building is presented in Figure 3.

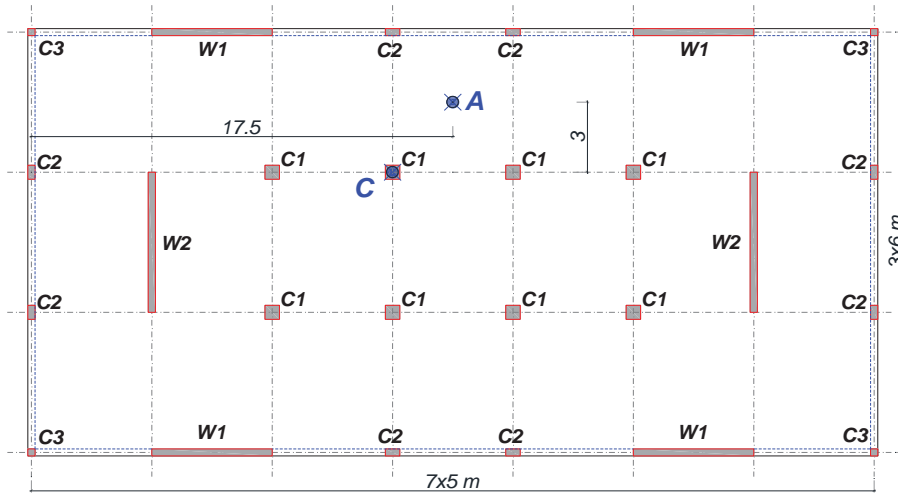


Figure 3 - Plan geometry with observation points A and C

Three different types of soil are considered: soft (S1), medium stiff (S2) and stiff (S3) soil. Soil parameters are given in Table 1. For each soil type, it is adopted Poisson's ratio  $\nu=0.3$  and soil mass density  $\rho_0=2000$  kg/m<sup>3</sup>.

Table 1 - Soil types and parameters

| Soil type   | S1-soft | S2-medium | S3-stiff |
|-------------|---------|-----------|----------|
| $c_s$ [m/s] | 100     | 200       | 300      |

The dynamic stiffnesses are obtained using foundation dimensions, soil properties from Table 1 and Equations (2)-(7). The values of dynamic stiffnesses shown in Table 2 are given in terms of  $k_z$  and  $c_z$  for three soil types at the characteristics frequencies, separately for columns and walls in both directions. Figure 4 shows the real and imaginary part of the vertical impedance of the square foundation below the column C1. The impedances are implemented into the FEM model of the structure using the frequency dependent spring elements.

Values of frequency dependent springs under the columns are taken from Table 2. Under the walls, the spring stiffness given in Table 2 are modeled as series of equivalent frequency

dependent springs attached to the nodes of the wall's finite elements. The stiffness of equivalent frequency dependent springs was calculated as dynamic stiffness from Table 2 divided by the length the of the wall and multiplied by the dimension of the finite element (0.5 m).

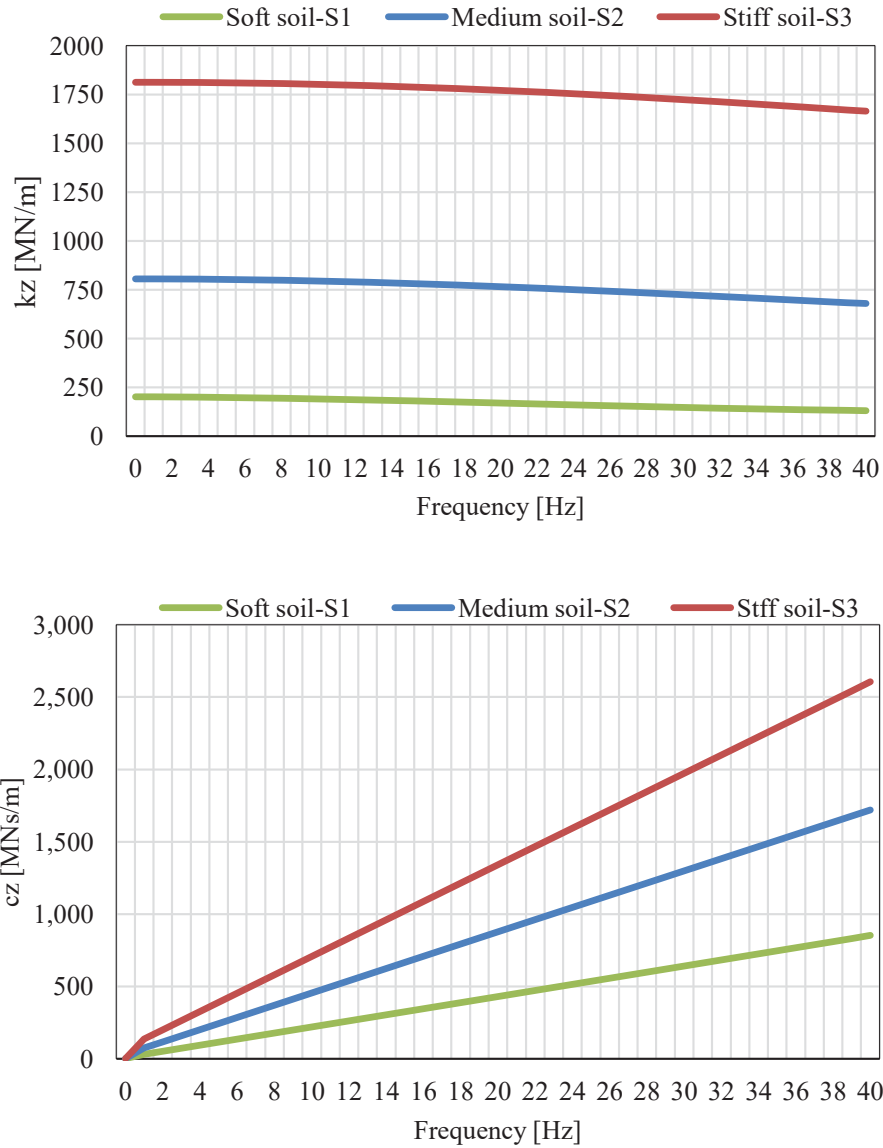


Figure 4 - Real ( $k_z$ ) and imaginary ( $c_z$ ) part of the vertical impedance below column C1

Table 2 - Values of  $k_z$  and  $c_z$  for three soil types for considered frequency range

|              | Columns |     |      | Walls in X direction |      |      | Walls in Y direction |      |      | Freq. [Hz] |
|--------------|---------|-----|------|----------------------|------|------|----------------------|------|------|------------|
|              | S1      | S2  | S3   | S1                   | S 2  | S 3  | S 1                  | S2   | S3   |            |
| $k_z$ [MN/m] | 201     | 806 | 1813 | 319                  | 1278 | 2875 | 346                  | 1383 | 3112 | 0          |
|              | 170     | 766 | 1772 | 220                  | 1087 | 2644 | 234                  | 1155 | 2829 | 20         |
|              | 130     | 679 | 1665 | 183                  | 881  | 2259 | 199                  | 936  | 2393 | 40         |
| $c_z$ [MN/m] | 0       | 0   | 0    | 0                    | 0    | 0    | 0                    | 0    | 0    | 0          |
|              | 430     | 877 | 1340 | 996                  | 2018 | 3068 | 1138                 | 2303 | 3499 | 20         |
|              | 852     | 1.2 | 2606 | 1982                 | 3985 | 6015 | 2265                 | 4551 | 6866 | 40         |

A modal analysis of the structure is calculated, in order to observe the modes in which the vertical vibrations of the object occur and to choose the observation points. Vertical mode shapes for three soil types are shown in Figure 5.

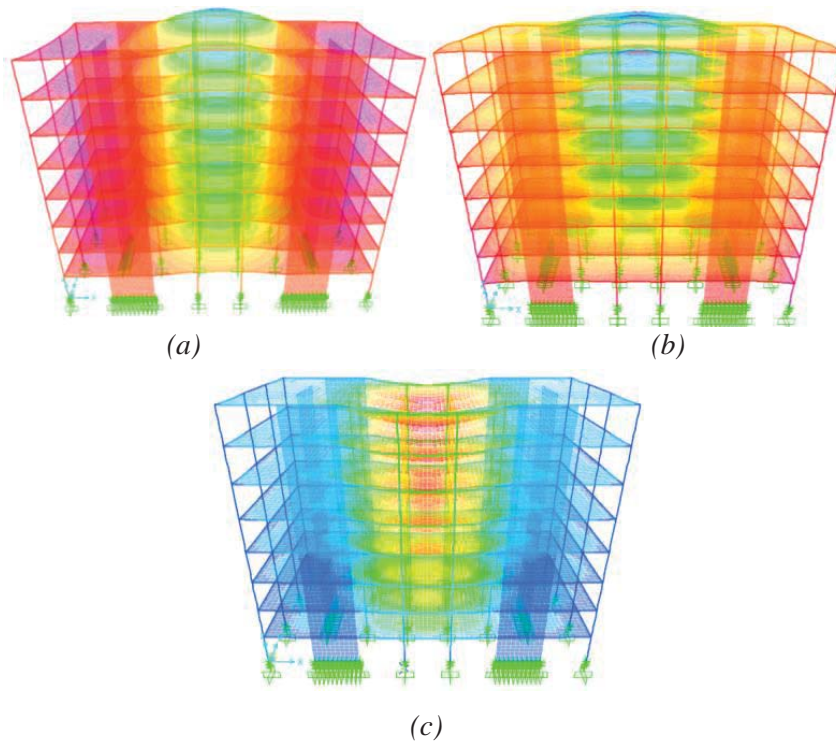


Figure 5 - First vertical mode shape: (a) soil S1 ( $f_{1v} = 5.3$  Hz), (b) Soil S2 ( $f_{1v} = 7.1$  Hz) and (c) soil S3 ( $f_{1v} = 8.1$  Hz)

The position of two observation points A and C are shown in Figure 3. These points are chosen to describe the behavior of structure as the points with greatest vertical deflections. The TFs at observation points are calculated using the steady state analysis and a unit displacement harmonic excitation  $u_0 = 1$  applied to the spring supports. The magnitudes of TFs of observation points are determined on each floor in the frequency range 0-40 Hz, and presented in Figure 6.

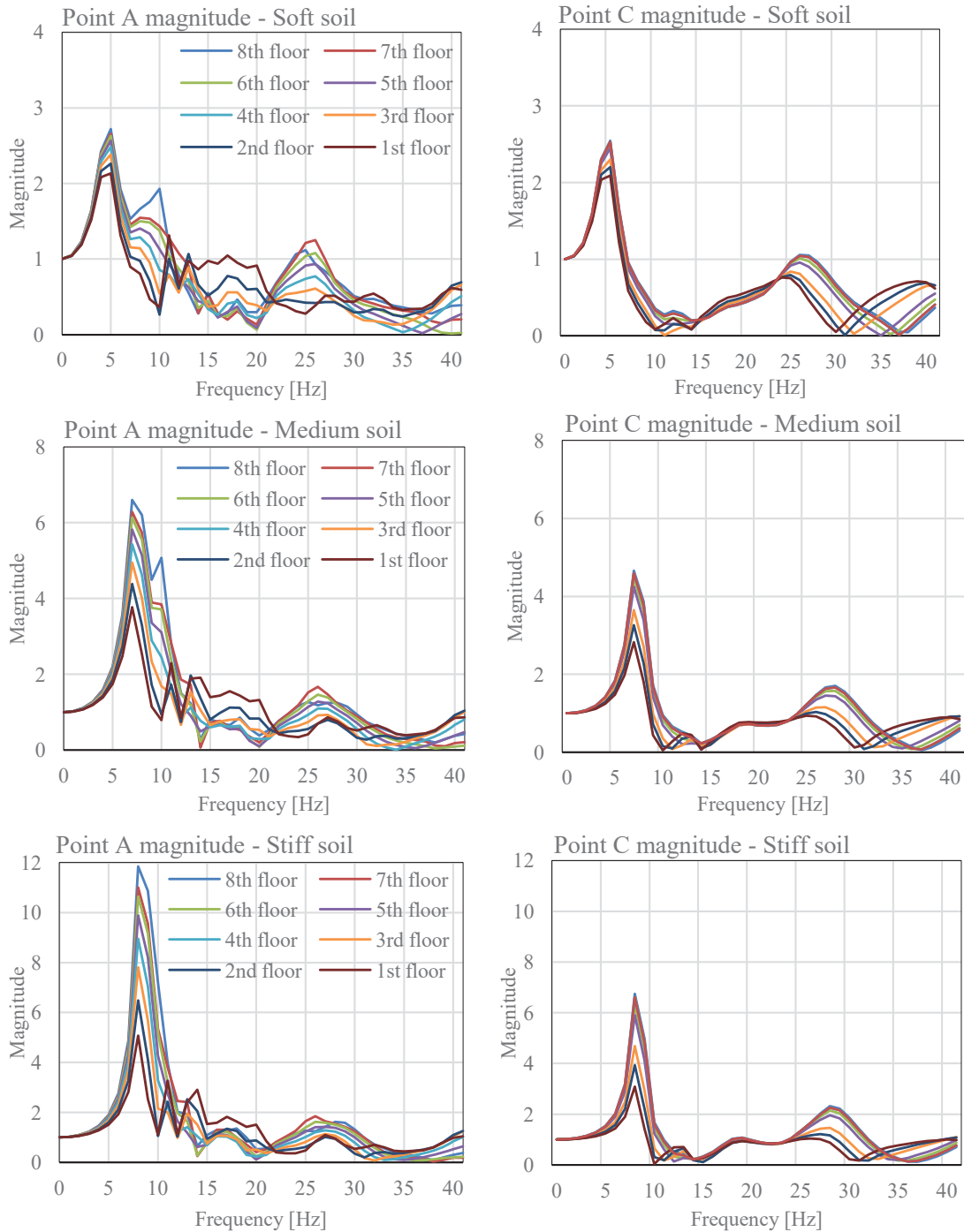


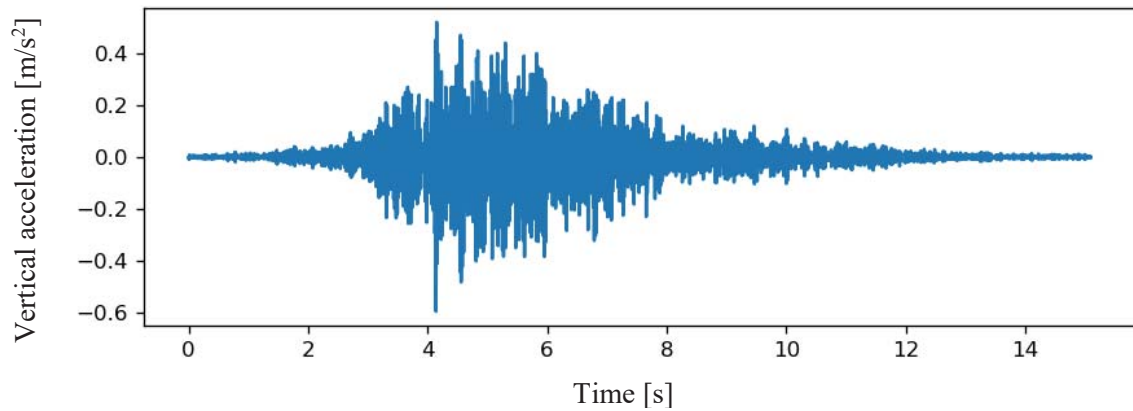
Figure 6 - TFs for point A and C for soil types S1, S2 and S3



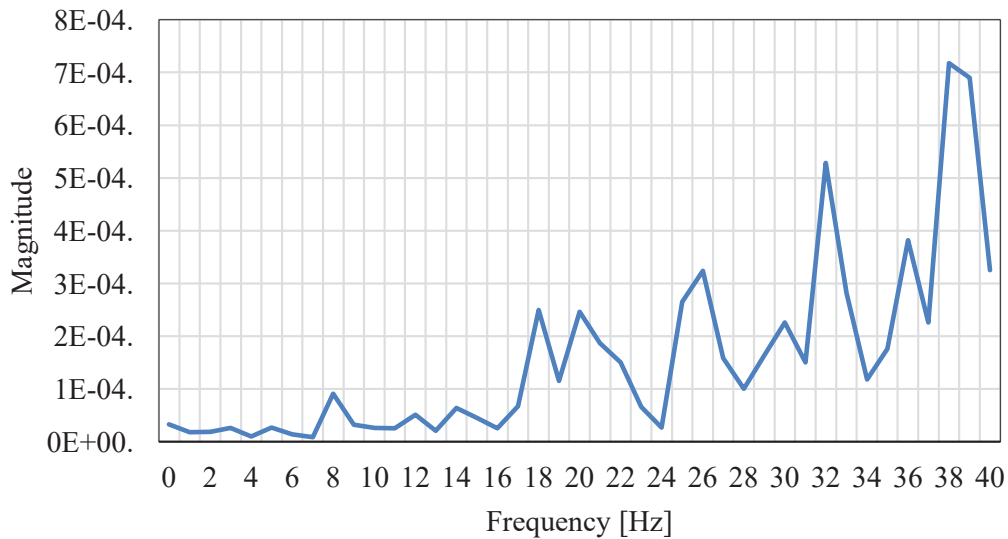
The free field motion  $u_g$  is obtained using in-situ tests performed in Belgrade [11]. Figure 7 shows vertical acceleration time history induced by train and the corresponding frequency spectrum. According to Equation (9), the dynamic response at observation point A due to the measured free-field motion presented in Figure 8 is calculated as the product of the TF at observation point A,  $TF_A$  (Figure 6), and the  $u_g$  spectrum (Figure 7b):

$$u_A(g) = TF_A u_g \quad (10)$$

Dynamic response for observation point C is calculated analogously.



(a)



(b)

Figure 7 - (a) vertical acceleration time history induced by train and (b) frequency spectrum for train passage

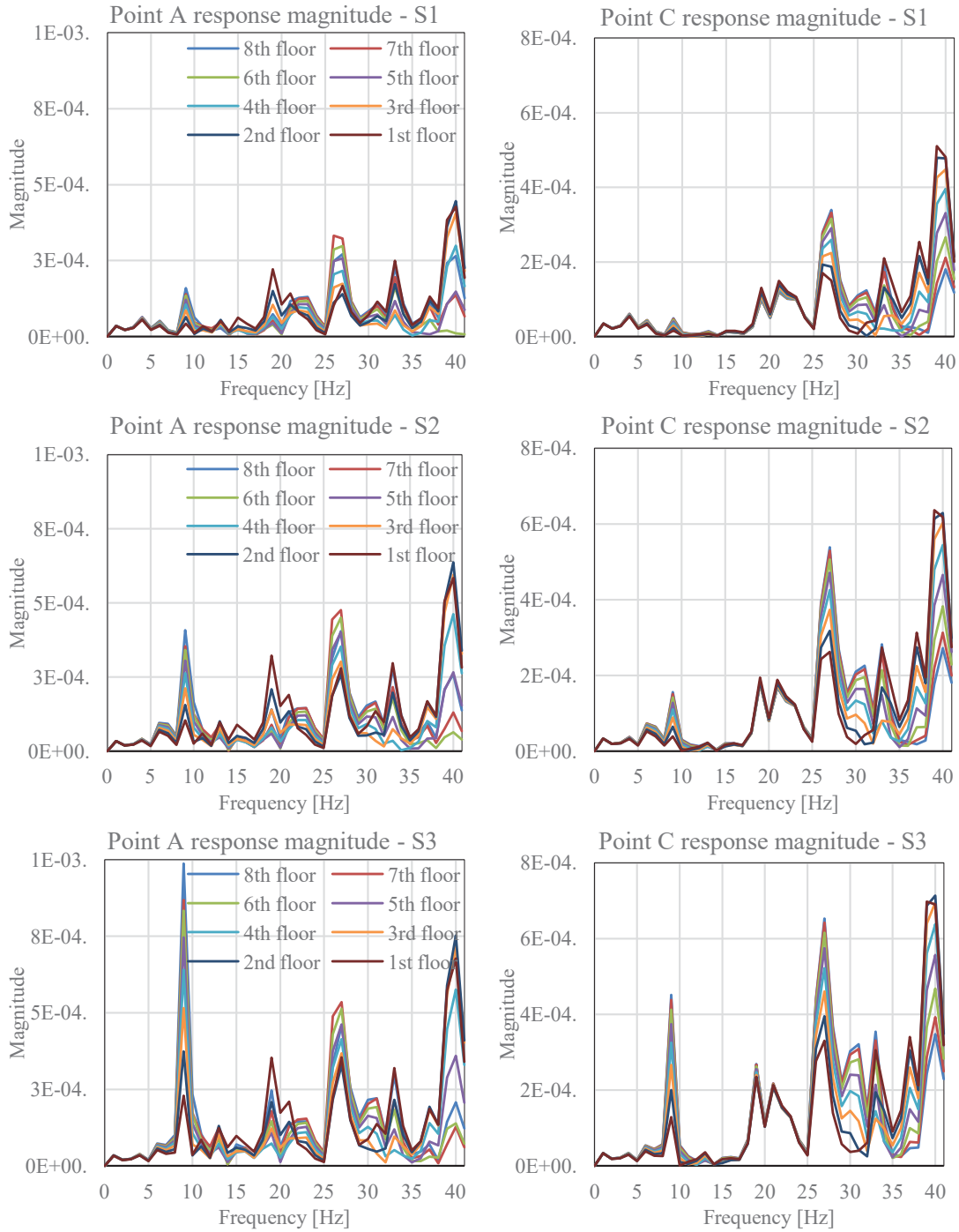


Figure 8 - Dynamic response at points A and C due to measured free field motion

#### 4. DISCUSSION AND CONCLUSIONS

Influence of SSI on the response of an eight story residential building subjected to train-induced free field motion is investigated in this paper. The building is supported by shallow square foundations under the columns, and strip foundations under the walls. The dynamic response of the building due to the measured free-field motion is obtained for three different types of soil: soft (S1), medium (S2), and stiff soil (S3). The response of the building is obtained at two observation points with highest deflections at each story. Based on the performed numerical study, the following conclusions are drawn:

- As expected, for all TFs amplification factor (magnitude in Figure 6) is equal one for zero frequency, while for frequencies larger than 40 Hz it is less than one. This means that free field motion is affected by the building at higher frequencies (i.e.  $f > 40\text{Hz}$ ) [6].

- In the case of soft soil, the building resonance occurs at 5 Hz with highest amplification factors between 2.1 and 2.55 (corresponding to 1st and 8th floor) at observation point C and between 2.1 and 2.7 at observation point A. The building resonance frequency is fully affected by the soil-building resonance frequency [6] determined by the soil stiffness and total mass of the building and is equal 5.7 Hz. In addition, amplification factors between 1 and 1.5 are detected for observation point A for frequencies between 8 and 9 Hz due to the coupling of floor resonance frequency at 10Hz and the soil-building resonance frequency.

- For medium and stiff soil, the building resonance occurs at 7 Hz and 8 Hz, respectively, which is lower than the corresponding soil-building resonance frequencies of 11 Hz and 17 Hz respectively. This means that there is a certain coupling of the soil-building resonance frequencies and the so-called column frequency determined by the wave velocity of the column/wall – floor model and the building height  $H$  (for details see ref [6]). The column frequency of the investigated building is 13 Hz, thus building resonance is shifted from 11 Hz to 7 Hz and from 17 Hz to 8 Hz for medium and stiff soil, respectively.

- At the building resonance frequency, drastically higher amplification factors are detected for medium and stiff soil in comparison to the soft soil (up to 6.6 for medium soil and 11.8 for stiff soil). Consequently, soft soil can significantly reduce the response of the structure

- Generally, higher amplification factors are detected at the observation point A than at the observation point C. This is more evident for medium and stiff soil.

- Amplification for both observation points A and C occurs in the range between 25 Hz and 26Hz. This amplification ranges from 1.2 to 2.0 from soft to stiff soil scenario, respectively.

#### ACKNOWLEDGEMENTS

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