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WAVE PROPAGATION DUE TO A MOVING LOAD

M. Petronijević[1], M. Radišić[1], M. Nefovska Danilović[1]

[1] Faculty of Civil Engineering
e-mail: pmira@grf.bg.ac.rs
University of Belgrade, Bulevar kralja Aleksandra 73, 11000 Belgrade
e-mail: mradisic@grf.bg.ac.rs
e-mail: marija@grf.bg.ac.rs

Abstract. The wave propagation on the surface of a half-space due to a moving load is analyzed using the Integral Transform Method. By using of the Helmholtz's decomposition and threefold Fourier transformation the body wave equation is transformed in the wave number-frequency domain and solved numerically. The obtained displacement field is transformed in time domain by the Inverse Fourier Transform. The analysis is carried out using computer program written in MATLAB program language. The load is vertical sinusoidal force P=1 MN moving along the line defined by x=0 with constant speed. The influence of source velocity on the displacements of the half-space and on the frequency content of the displacements at three locations at different distances from the load line is presented.

1. Introduction

Needs for easily accessible, available and mobile public transport have caused higher level of traffic induced vibrations in the urban zones. The rail/road - vehicle system imperfections, such as road and wheel roughness, are the main causes of ground vibrations. These vibrations induce waves that propagate through the soil and affect surrounding buildings. Different methods of analysis may be applied for evaluation of the ground response to moving sources. For simple geometry of the soil region analytical or semi-analytical solutions in the wavenumber–frequency domain are the most applicable, like Integral Transform Method (ITM), [1]. As an alternative to the semi-analytical solution, the thin-layer method can be employed, [2]. For the analysis of subsoil with complex geometry a numerical method, like Finite Element Method (FEM) with different type of transmitting boundaries, or Boundary Element Method (BEM) [3], are available in the time or frequency domain. The overview of the numerical methods for the analysis of ground vibrations due to the moving load is given by [4].

This paper presents the analysis of ground vibrations caused by a moving force with a constant speed in the *x*-direction along the surface of visco-elastic half-space. The force is half-cosine load, which represents the distribution of the moving wheel force. For this case the response is expressed in terms of a double integral with respect to k_y and ω. The solution is presented in the moving frame of reference. In that case it is the same as the solution for a stationary force. The dynamic responses at different load speed (subsonic, transonic and supersonic) are calculated using the ITM and presented. The results are compared with the results obtained by Chouw using the BEM [3].

2. Half-space solution according to ITM

The ITM was applied in the last years to several problems of halfspace dynamics [5] and soil structure interaction [6], in particular, to problems of road/track/soil interaction, [7]. Method is based on the Helmholz's decomposition of the Lamé's equations and their threefold Fourier transform from the time-space to the frequency-wavenumber domain. Therefore, it is restricted to linear systems and to the frequency domain analysis. The short description of this method will be presented in the following.
The Lamé's equations of motion of the continuum

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}=\rho\ddot{\mathbf{u}} \tag{1}$$

can be brought into the form of wave equations

$$\nabla^2\varphi=\frac{1}{c_p^2}\ddot{\varphi},\qquad \nabla^2\boldsymbol{\psi}=\frac{1}{c_s^2}\ddot{\boldsymbol{\psi}} \tag{2}$$

if the displacement vector is expressed by the scalar field φ and the vector field ψ, according to Helmholtz's principle, as

$$\nabla u=\nabla\varphi+\nabla\times\boldsymbol{\psi}\ . \tag{3}$$

In Eqs. (2) c_p and c_s are the velocities of the dilatational and shear waves, respectively

$$c_p^2=\frac{\lambda+2\mu}{\rho},\qquad c_s^2=\frac{\mu}{\rho} \tag{4}$$

where ρ is the mass density of the material and λ and μ are the Lamé's constants.
If we assume that $\psi_z=0$, the displacement components can be obtained form Eq. (3) in the following form

$$\begin{aligned} u_x &= \varphi_{,x}-\psi_{y,z}\\ u_x &= \varphi_{,y}-\psi_{x,z}\\ u_z &= \varphi_{,z}-\psi_{x,y}+\psi_{y,x} \end{aligned}\ . \tag{5}$$

By a threefold Fourier transform

$$\hat{f}(k_x,k_y,\omega)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y,t)e^{-i(k_x x+k_y y+\omega t)}dxdydt\ , \tag{6}$$

Eqs. (2) can be transformed into a system of 2 decoupled ordinary differential equations in the frequency-wavenumber domain

$$\begin{aligned} &-\lambda_1^2\hat{\varphi}+\frac{\partial^2\hat{\varphi}}{\partial z^2}=0\\ &-\lambda_2^2\hat{\psi}_i+\frac{\partial^2\hat{\psi}_i}{\partial z^2}=0,\quad i=x,y,z \end{aligned} \tag{7}$$

where

$$\lambda_1^2 = k_x^2 + k_y^2 - k_p^2, \quad k_p = \frac{\omega}{c_p}$$
$$\lambda_2^2 = k_x^2 + k_y^2 - k_s^2, \quad k_s = \frac{\omega}{c_s} \tag{8}$$

The solution of differential equations (7) in the transformed domain should satisfy the Sommerfield's radiation condition, which means that there is no propagation of waves from infinity toward the source. Therefore $A_1=B_{x1}=B_{y1}=0$, giving the solutions in the form

$$\hat{\varphi} = A_2 e^{-\lambda_1 z}, \qquad \hat{\psi}_x = B_{x2} e^{-\lambda_2 z}, \qquad \hat{\psi}_y = B_{y2} e^{-\lambda_2 z} \quad . \tag{9}$$

Substituting Eq. (9) into the Eqs. (5) results in the following relation between the displacement vector $\hat{\mathbf{u}}$ and vector of unknown coefficients $\mathbf{C}$

$$\hat{\mathbf{u}} = \mathbf{A}^u \cdot \mathbf{C} \tag{10}$$

where

$$\hat{\mathbf{u}} = \begin{Bmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{u}_z \end{Bmatrix}, \quad \mathbf{A}^u = \begin{bmatrix} ik_x & 0 & \lambda_2 \\ ik_y & -\lambda_2 & 0 \\ -\lambda_1 & -ik_y & ik_x \end{bmatrix}, \quad \mathbf{C} = \begin{Bmatrix} A_2 \\ B_{2x} \\ B_{2y} \end{Bmatrix}. \tag{11}$$

The unknown coefficients A_2, B_{x2} and B_{y2} can be obtained from the boundary conditions at the surface of the half space, defined as

$$\begin{Bmatrix} \hat{\sigma}_{zx}(k_x,k_y,z=0,\omega) \\ \hat{\sigma}_{zy}(k_x,k_y,z=0,\omega) \\ \hat{\sigma}_z(k_x,k_y,z=0,\omega) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \hat{p}_z(k_x,k_y,\omega) \end{Bmatrix} = \hat{\mathbf{p}}(k_x,k_y,\omega) \tag{12}$$

where $\hat{p}_z(k_x,k_y,\omega)$ is the Fourier's transform of the applied moving load $p_z(x,y,t)$

$$\hat{p}_z(k_x,k_y,\omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_z(x,y,t) e^{-i(k_x x + k_y y + \omega t)} dxdydt \,. \tag{13}$$

Using well known relations between stress and displacements, the stress in wavenumber domain can be written as

$$\boldsymbol{\sigma} = \mathbf{A}^\sigma \mathbf{C} \tag{14}$$

where

$$\mathbf{A}^\sigma = \mu \begin{bmatrix} -2ik_x\lambda_1 & k_x k_y & -\left(\lambda_2^2 + k_x^2\right) \\ -2ik_y\lambda_1 & \left(\lambda_2^2 + k_y^2\right) & -k_x k_y \\ 2k_r^2 - k_s^2 & 2ik_y\lambda_2 & -2ik_x\lambda_2 \end{bmatrix} \tag{15}$$

and $k_r^2 = k_x^2 + k_y^2$.

Substituting solution for $\mathbf{C}$ obtained from the Eq. (14) into the Eq. (10), regarding the Eq. (12), gives the displacement vector in the frequency-wavenumber domain in the form

$$\hat{\mathbf{u}}(k_x,k_y,\omega) = \hat{\mathbf{H}}(k_x,k_y,\omega)\hat{\mathbf{p}}(k_x,k_y,\omega) \; , \tag{16}$$

where $\hat{\mathbf{H}} = \left[\mathbf{A}^u(\mathbf{A}^\sigma)^{-1}\right]$ is the transfer function matrix (compliance) of the half-space.

The response in the frequency-wavenumber domain requires the transformation in the space-time domain by usage of the inverse Fourier transform

$$f(x,y,t) = \frac{1}{8\pi^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\hat{f}(k_x,k_y,\omega)e^{i(k_x x+k_y y+\omega t)}dk_x dk_y d\omega \; . \tag{17}$$

In these evaluations, damping is taken into account by using complex values for the Lamé's constants according to the principle of correspondence

$$\hat{E} = E\left(1+2i\xi\right), \qquad \hat{G} = G\left(1+2i\xi\right) \tag{18}$$

where ξ is the damping coefficient.

2.1. Moving load

Consider the vertical load that moves in x-direction along the surface of a half-space with a constant speed v, starting from the point x_k=0

$$p_z(x,y,t) = p_o \cdot p_1(x-vt)p_2(y) \; . \tag{19}$$

By substituting Eq. (19) into Eq. (13) and applying the shifting theorem obtained is the moving force in the frequency-wavenumber domain as

$$\hat{p}_z(k_x,k_y,\omega) = 2\pi p_o \delta(\omega + k_x v)\tilde{p}_1(x)\tilde{p}_2(y) \tag{20}$$

where $\tilde{p}_1(x)$ and $\tilde{p}_2(y)$ are wavenumber transform of $p_1(x)$ and $p_2(x)$.

The solution in the space domain is found by substituting Eq. (20) into Eq. (16) and applying inverse Fourier transform. Taking into account shifting theorem obtained is final result in the form

$$\mathbf{u}(x,y,t) = \frac{1}{\left(2\pi\right)^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\hat{\mathbf{H}}(k_x,k_y,\tilde{\omega}-k_x v)\hat{\mathbf{p}}(k_x,k_y,\tilde{\omega})e^{ik_x\bar{x}}e^{ik_y y}e^{i\tilde{\omega}t}dk_x dk_y dt \; , \tag{21}$$

where

$$\bar{x} = x - vt, \qquad \tilde{\omega} = \omega + k_x v \quad , \tag{22}$$

represent the moving coordinate system $\bar{x}$ and frequency $\tilde{\omega}$ at the source, respectively.

From Eq. (20) follows that $\omega = -k_x v$ i.e $\tilde{\omega} = 0$, which means that integral (21) is constant in time and the response of the half-space due to the moving load can be expressed in the moving frame of reference as

$$\mathbf{u}(\bar{x},y) = \frac{1}{\left(2\pi\right)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\hat{\mathbf{H}}(k_x,k_y,-k_x v)\hat{\mathbf{p}}(k_x,k_y)e^{ik_x\bar{x}}e^{ik_y y}dk_x dk_y \; . \tag{23}$$

The obtained integral is the same as in the case of stationary force. The only difference is that in the case of moving load the compliance of the soil has to be calculated with the shifted frequency $\omega = -k_x v$ [7].

3. Evaluation of numerical model

The response of the half-space is calculated using computer program using Matlab [8]. The characteristics of the half-space and the moving force distribution are taken as in [3]. The characteristics of the half-space are: c_s=120 m/s, c_p=240 m/s, ρ=2000 kg/m^3, ξ=5%.

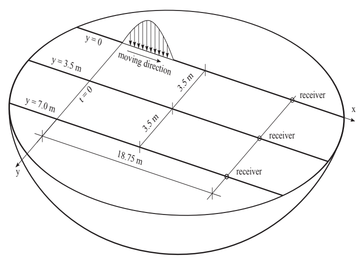

Figure 1.

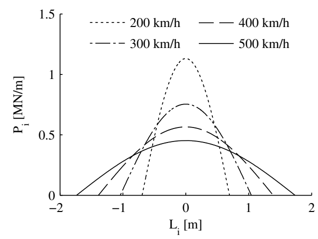

Figure 2.

The force is half-cosine load that moves in *x*-direction with a constant speed *v*, starting from the point x_k=0, Fig. 1. The force is half-cosine load

$$P_{z_i}(x,y,t) = P_i \cos\frac{\pi \overline{x}}{l_i}\delta(x - v_i t)\delta(y) \tag{24}$$

where $P_i = P_t \pi / l_i$ is the maximum, $P_t = 1$ *MN* is the total load, l_i is the length of half-cosine, Fig. 2. Load duration is *0.025 s*. The responses are obtained for the following velocities v_i of the force: 200, 300, 400, and 500 *km/h*, respectively.
The displacements at the receiver *18.75 m* from the starting point, at a distance *y=0, 3.5* and *7 m*, respectively are displayed in Fig. 3. The higher the source speed the quicker response occurs. The displacement $u_y=0$, for $y=0$. The displacements decrease with increasing distances from the load path. The dependency of the displacements on the ratio between the source speed and the wave speed in the soil is obvious. The highest displacements u_x, u_y, u_z occur when the force velocity is 400, 500 and 200 *km/h*, respectively.

4. Conclusion

In this paper presented is the application of the ITM to dynamic analysis of the half-space due to a moving load. The advantage of the ITM is based on the fact that in the wavenumber domain the displacement due to the moving force in the moving frame of reference is equal to the displacement due to the stationary force calculated with the shifting frequency.
The displacements due to the half-cosine force obtained by present approach are in a good agreement with those obtained by the Boundary element method [3] .

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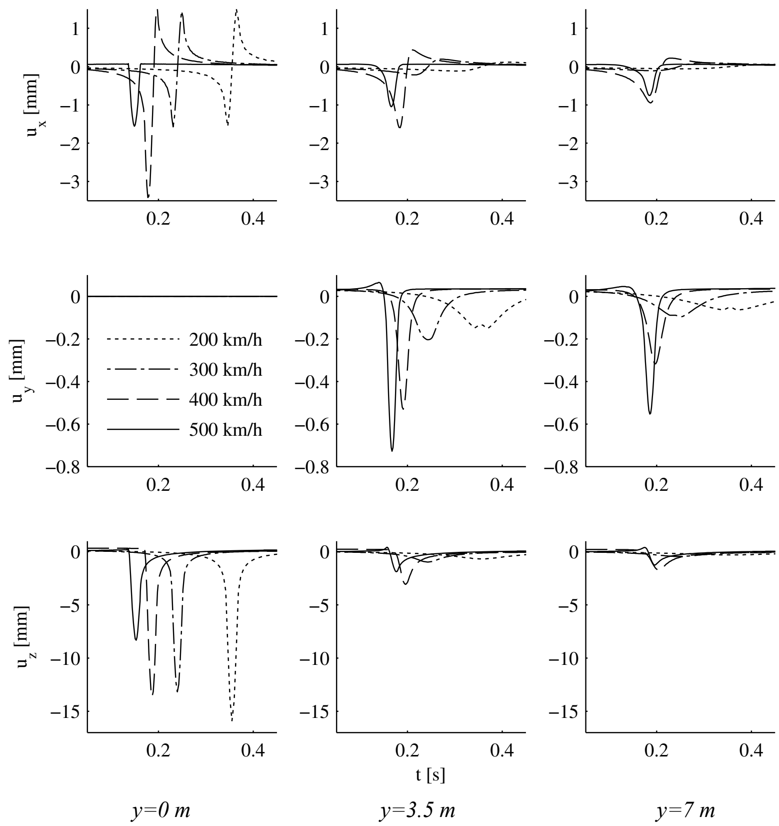

Figure 3. Influence of the moving force velocity on the displacements of the half-space surface at *x=18,5 m*

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