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STABILITY ANALYSIS OF MULTI-STORY STEEL FRAMES SUBJECTED TO DIFFERENT AXIAL LOAD

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Abstract:

This paper presents a stability analysis of multi-story steel frames in the elastic and elasto-plastic domain. The concept of the tangent modulus theory is applied. Numerical analysis is carried out using FEM where corresponding stiffness matrices are based upon the trigonometric and hyperbolic interpolation functions of normal forces. Also, the calculation algorithm is based on the global stability analysis of the considered frames. The numerical analysis is performed using the self-developed computer program ALIN. A six-story three-bay steel frame was chosen as a benchmark numerical example. Sway and non-sway frames that are clamped at the base are analyzed separately. Two load cases are considered: when the axial forces are applied at the top of the frame and when these forces are applied at each story of the frame. From the obtained results it is obvious the weakness of the traditional elastic stability analysis. Therefore, stability analysis in the inelastic domain is recommended, especially in the case of rigid structures.

Key words: steel frames, inelastic buckling, tangent modulus, finite element method.

1. Introduction

Calculation of compressed frame columns requires investigation of stability phenomena. The dominant compressive stresses can cause structural instability, loss of load-bearing capacity and at the end collapse of the construction, even in cases where the allowable stresses are not exceeded. So, the calculation of such structures, especially taking into account their stability, requires the application of modern and complex numerical methods.

The first investigations in this field were based on Euler's theory of buckling of isolated columns [1]. They were mainly based on solving the differential equation of buckling according to the second-order theory. However, in the case of the complex structures, it was necessary to introduce some approximations. So, all the compressed elements were considered "isolated" from the structure as a whole. These isolated columns are supported only by the adjacent columns and beams. Basically, the presence of the other structural elements connected to the considered one is introduced by the corresponding boundary conditions. Thus, stability analysis of columns is simplified and the results may be obtained through the corresponding diagrams and approximate formulas. The results of some elementary cases are given in [2]. It is important to point out that these are approximate solutions are bases for many design codes, especially for steel structures, for example [3]-[5]. Regardless of its frequent use in engineering practice, this approach has major limitations, as it is explained in [6]. That is the reason why lately a great effort has been devoted to improving these approximate calculation procedures. Thus, besides other, investigations [7]-[9] provided significant contributions in this field.

Certainly, it has to be pointed out usage of finite element method (FEM) as one of the most efficient methods for stability analysis of the frame structures [10], [11]. It is used in modern commercial programs which deal with stability analysis of the frames. In the usual approach, the finite element method analysis is based upon the integral structural model and the geometric stiffness matrix as a part of the tangent stiffness matrix. This paper presents the procedure where the stiffness matrix is derived using the interpolation functions related to the exact solution of the differential equation of bending of a beam according to the second-order theory. On the basis of the obtained solutions, the method for determining the critical load in all columns is based on the global stability analysis.

Besides the geometrical nonlinearity, the physically (or materially) nonlinearity is also considered in this analysis. The tangent modulus theory [12] is used, as one of the most efficient methods for solving such kind of problems. It means that stiffness matrices are derived using the tangent modulus (E_t) that is stress dependent and follows changes of the member stiffness in the inelastic domain.

So, the most important aspect of this research will be devoted to the investigation of the suitable numerical methods to obtain the solution of the corresponding transcendental stability equation. After finding the appropriate algorithm, the problem will be extrapolated to the non-elastic material behavior, i.e. to the stability problems in the plastic, or rather, elastic-plastic range. The numerical investigation of multi-story steel columns with different boundary conditions and different loads will be carried out in self-developed code ALIN.

2. Calculation of the critical load applying the tangent modulus theory

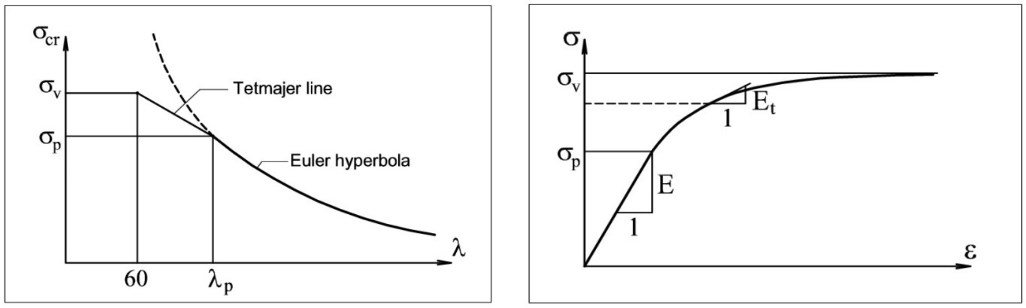
In this investigation, it is used finite element method, as the most convenient for numerical analysis of frames related to their stability. Applying FEM, the critical load can be obtained as the non-trivial solution from the homogeneous matrix equation:

$$\boldsymbol{K} \cdot \boldsymbol{q} = 0 \tag{1}$$

In Eq. (1) **K** represents global stiffness matrix for the entire frame including the corresponding boundary conditions, while **q** is the vector of generalized coordinates. In the elastic stability analysis relationship between stresses and strains is a linear, so the Young's modulus (E) has a constant value. Taking into consideration well known expression for the Euler’s critical force, critical stress in a member may be expressed as a function of the modulus of elasticity and the slenderness ratio:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{l_i^2 A} = \pi^2 \frac{E}{\lambda_i^2} \tag{2}$$

where $\lambda_i = l_0 / i$ is slenderness ratio of the member; l_0 is effective length of the member; $i = \sqrt{I / A}$ is the radius of gyration; I is moment of inertia and A is cross-sectional area. Eq. (2) is represented by a hyperbolic curve and this function is valid until the critical stress is less than the proportionality limit (σ_p), as shown in Fig.1a. This stability formulation for elastic buckling is given in [2] and it is also bases of the regulations for design of steel structures, for example [4].



(a) (b)

Figure 1. (a) Buckling stress diagram - Euler hyperbola and Tetmajer line
(b) σ-ε diagram of structural steel

However, this analysis becomes more complicated when some compressed columns enter into the phase of nonlinear material behavior although the critical load has not been reached. It means that stresses in such members exceed the proportional limit value and that buckling occurs in the plastic domain. One of the first solutions to this problem, which is still used in engineering practice, was proposed by Tetmajer, Fig.1a.

Besides many other investigations related to this problem, it is important to emphasize Engesser's solution [13] because has established a tangent modulus theory. Namely, he replaced Young's modulus E with the tangent modulus E_t, that represents the slope of the tangent on the stress-strain diagram at any point. More about this theory can be found in [12], [14]. Another well-established method for obtaining the tangent modulus is the Ramberg–Osgood [15] equation that relates Young's modulus to the tangent modulus.

In this paper are considered multi-story steel frames that are made of steel. In order to implement numerical stability analysis in the inelastic domain, we have to know the physical and mechanical properties of such material. The diagram graph that represents the relationship between the strain and stress of the axially loaded steel member is presented in Fig.1b. The proportional limit is marked as σ_p, and yield stress with σ_v.

In this analysis it is used an empirical relationship between two moduli that is suggested in many relevant investigations [14], [16], where it is assumed that this expression is valid for ratio $\sigma/\sigma_p > 0.5$:

$$E_t = 4E \cdot \left[\frac{\sigma}{\sigma_y} \left(1 - \frac{\sigma}{\sigma_y} \right) \right] \tag{3}$$

This empirical formula was derived from inelastic column curves and represents the behavior of structural steel columns in the inelastic domain. It was applied in this investigation in order to develop corresponding computer program ALIN for the nonlinear elastic-plastic analysis of frame structures.

The main goal of this analysis was, conditionally speaking, to formulate the exact matrix stability analysis of multi-story frames. So, it was necessary to use trigonometric shape functions related to the exact solution of the differential equation of bending of a beam according to the second-order theory. The advantages of such an approach can be found in numerous literatures, for example [17].

The most important aspect of this research was devoted to the formulation of a suitable algorithm and corresponding computer program that can solve solving such kind of problems. Namely, in this case instead of the generalized eigenvalue problem, for which there are several well-established methods, herein the buckling problem is reduced to the solution of the transcendental equation that depends, in a very complicated way, upon the axial forces in the columns and beams. Therefore, it was necessary to formulate corresponding stiffness matrices for nonlinear material behavior. In this paper it is only given stiffness matrix for the member that is clamped at both ends and subjected to the compressive force:

$$K = \frac{E_t I}{l^3 \Delta_t} \begin{bmatrix} \omega_t^3 \sin \omega_t & \omega_t^2 l (1-\cos \omega_t) & -\omega_t^3 \sin \omega_t & \omega_t^2 l (1-\cos \omega_t) \\ & \omega_t l^2 (\sin \omega_t - \omega_t \cos \omega_t) & -\omega_t^2 l (1-\cos \omega_t) & \omega_t l^2 (\omega_t - \sin \omega_t) \\ & & \omega_t^3 \sin \omega_t & -\omega_t^2 l (1-\cos \omega_t) \\ & symm. & & \omega_t l^2 (\sin \omega_t - \omega_t \cos \omega_t) \end{bmatrix} \tag{4}$$

where is: $E_t = 4E \cdot \left[\frac{P_{cr,i}}{A \cdot \sigma_v} \left(1 - \frac{P_{cr,i}}{A \cdot \sigma_v} \right) \right]$ (5)

$$\omega_t = \sqrt{\frac{P_{cr,i}}{E_t \cdot I}} \cdot l = \frac{1}{2} A \sigma_v l \cdot \sqrt{\frac{1}{EI \left(A\sigma_v - P_{cr,i} \right)}} \tag{6}$$

$$\Delta_t = 2 \cdot (1 - \cos \omega_t) - \omega_t \cdot \sin \omega_t \tag{7}$$

The procedure for deriving this matrix is given in [17]-[19]. It is clear that it has a same form as for the linear behavior of the material, but it is essentially totally different. Namely, instead of constant modulus E, there is stress dependent function E_t. Also, values of ω and Δ are replaced by ω_t and Δ_t, respectively.

3. Program for the stability analysis in elasto-plastic domain

In order to successfully use this theoretical approach, it was necessary to formulate an appropriate computer program. This code was named ALIN and it was developed using C++ programming language. It should be emphasized that one of the main contributions of this investigation consists of the development of the part of this program which provides efficient solutions for stability problems in the elasto-plastic domain. Those solutions are obtained using the "exact" expressions of the stiffness matrix which are derived previously. So, in this case, instead of the generalized eigenvalue problem (that is applied in „classical" stability analysis), for which there are several well-established methods (e.g. the subspace iteration, Lanczos method, etc), herein the buckling problem is reduced to the solution of the transcendental equation that depends, in a very complicated way, upon the axial forces in the columns and beams.

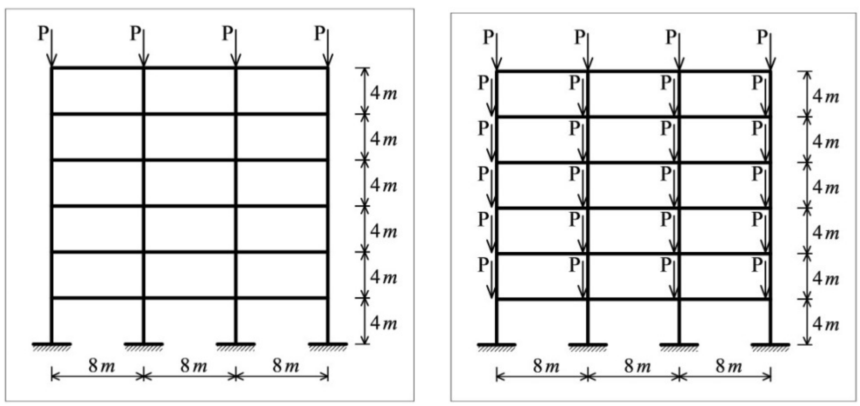
In the framework of this paper, the algorithm for the calculation of the critical load in the elastic and inelastic domain will be briefly presented. Firstly, this program has the ability to determine the critical load in the elastic range. It means that in first iteration axial forces are calculated according to the first order theory. These forces are in the first iteration to determine the stiffness matrix according to the second order theory. This calculation is performed iteratively until the displacement difference in two consecutive iterations becomes smaller than some pre-set small value. At the end of this procedure stiffness matrix of the whole system should be reduced, and only the active degrees of freedom should be considered. Obtained reduced stiffness matrix must satisfy the condition for the existence of the nontrivial solution, i.e. that the determinant of this matrix is equal to zero. In the end, this procedure gives the value of the critical force in the elastic domain, i.e. when the modulus of elasticity E has a constant value which is given in the input file.

After that, it is possible to perform "inelastic" stability analysis. When the critical load in the "elastic" domain is obtained, it should be calculated the critical stress as the ratio of the critical normal force and the cross-sectional area of the analyzed element. When the obtained critical stress is greater than the proportionality limit (σ_p), calculation continues as it is previously presented. It means that it is necessary to change stiffness for such columns, and a new tangent modulus (E_t) should be taken in the form (2). Thus, the new stiffness matrix in the local coordinate system has to be formed for those columns. Columns with critical stress which did not reach proportionality limit should keep "old characteristics". So, the stiffness matrix for this element is the same as in the first part of this calculation. Then, all matrices should be transformed from the local to the global coordinate system and the stiffness matrix of elements is formed. The iterative calculation should be again performed in the same way as when determining the critical load in the elastic range. As a result of this procedure, corresponding critical load factor and the value of tangent modulus for all elements buckling in the inelastic domain are obtained.

The detailed description of the program is given in [18], [20].

4. Numerical examples

In order to illustrate the proposed procedure, a three-bay plane frame of six stories is considered. The sway and non-sway frames will be analyzed separately. Figure 2 presents the geometry and the loading of a frame without lateral bracing which will be considered first. As it can be seen, rigid connections for columns ends including their supports at the base are assumed. Two load cases are considered: when the axial forces are applied at the top of the frame (Fig.2a) and when these forces are applied on each column at each story in the frame (Fig.2b).



(a) (b)

Figure 2. Six-story three-bay plane sway frame

Considered frame is made of steel with modulus of elasticity E = 210,000,000 kN/m^2 and yield stress σ_v = 240,000 kN/m^2. So, proportional limit can be calculated as $\sigma p = 0.5 \cdot \sigma v$ = 120,000 kN/m^2, according to the Eq.(3). In this analysis several different section types are used for the columns (IPB100, IPB140, IPB180, IPB220, IPB260 and IPB300). The sections of the girders in all stories are assumed as IPE300.

The results of the performed stability analysis for the frame given in Figure 2 (a) are given in Table 1.

Profile	Elastic analysis	Inelastic analysis	
	$P_{cr,el}$	$P_{cr,inel}$	E_t
IPB100	525.25	445.48	171,564,770
IPB140	1433.09	869.19	111,653,565
IPB180	2597.58	1399.81	80,135,580
IPB220	3613.95	2003.42	63,771,472
IPB260	4351.28	2631.47	55,267,926

Table 1. Values of P_{cr} (kN) and E_t (kN/m^2) for the frame given in Figure 2(a)

In the load case (b) the axial load in the columns is increasing from the upper stories to the basic level. It means that conducted elasto-plastic analysis of stability leads to the different behaviour of the columns in the different stories. The calculated values of the critical load and tangent moduli for the characteristic floors are shown in Table 2.

Profile	Elastic analysis	Inelastic analysis			
	$P_{cr,el}$	$P_{cr,inel}$	E_t – 6th floor	E_t – 3rd floor	E_t – 1st floor
IPB100	95.93	76.08	210,000,000	209,873,261	164,953,336
IPB140	282.91	148.16	210,000,000	205,366,416	100,283,751
IPB180	557.34	238.89	210,000,000	199,888,758	65,629,798
IPB220	829.79	342.77	210,000,000	196,285,251	46,142,006
IPB260	1069.86	451.92	210,000,000	193,931,804	34,213,200

Table 2. Values of P_{cr} (kN) and E_t (kN/m^2) for the frame given in Figure 2(b)

From these results it is obvious that frames with stiffer cross sections of the columns can be exposed to larger critical load. It means that the application of the stability analysis in inelastic domain is even more justified for such frames. The similar conclusion was observed and for other types of steel cross-sections, as it is shown in [20]. Also, it is clear that overall buckling of such multi-story frame is governed by the behaviour of the columns in the first, most loaded floor. That is the reason why the stress in columns in upper floors does not exceed the proportionality limit of the material. So their characteristics remain unchanged, i.e. their tangent module remains the same as the Young's modulus of elasticity.

Comparison of the results for both load cases is presented in Figure 3.

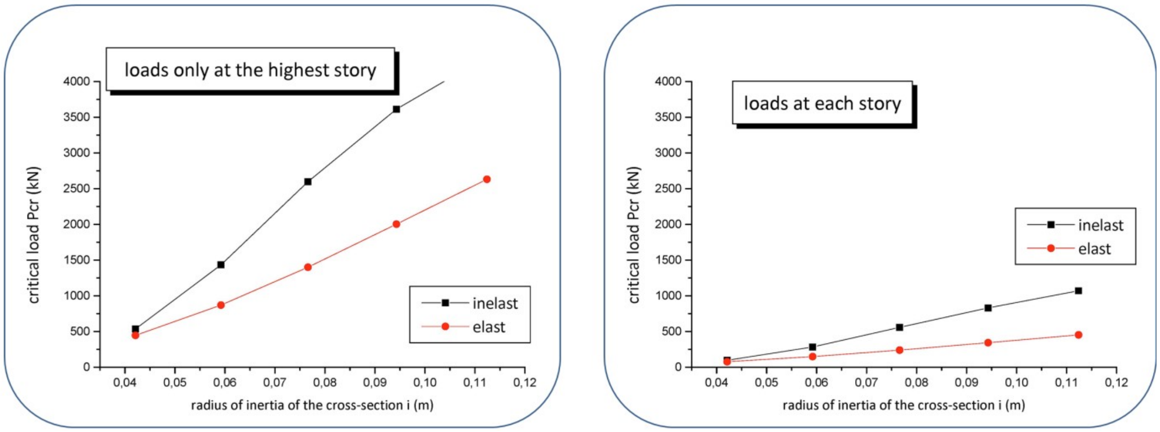
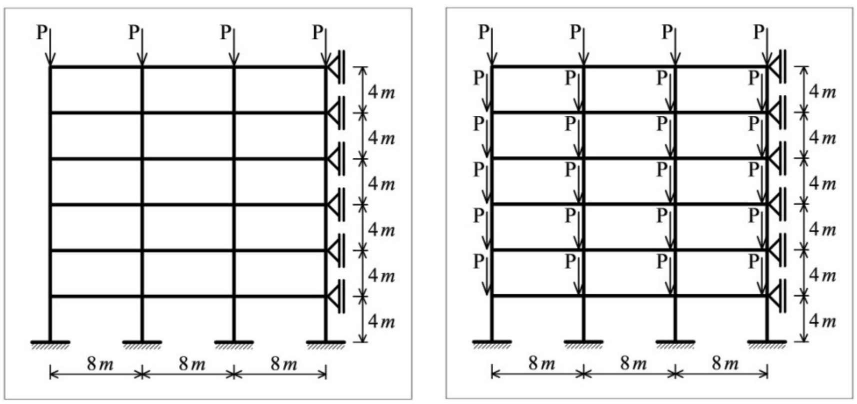


Figure 3. Critical load vs. radius of inertia of analysed cross-section for sway frames

Considered six-story three-bay non-sway (braced) frames with two load cases are shown in Figure 4. The geometry and the member characteristics are the same as in the previous example.



(a) (b)

Figure 4. Six-story three-bay plane non-sway frame

The results obtained by ALIN program are given in Tables 3 and 4.

Profile	Elastic analysis	Inelastic analysis	
	$P_{cr,el}$	$P_{cr,inel}$	E_t
IPB100	1829.24	579.89	55,181,756
IPB140	4228.50	992.97	30,564,565
IPB180	7940.69	1528.75	20,104,996
IPB220	13958.22	2152.20	12,050,779
IPB260	23264.87	2792.69	11,469,690

Table 3. Values of P_{cr} (kN) and E_t (kN/m^2) for the frame given in Figure 4(a)

Profile	Elastic analysis	Inelastic analysis			
	$P_{cr,el}$	$P_{cr,inel}$	E_t – 3rd floor	E_t – 2nd floor	E_t – 1st floor
IPB100	358.87	97.12	197,377,066	144,972,868	51,869,283
IPB140	918.32	166.26	192,481,190	131,592,892	27,101,155
IPB180	1843.17	255.99	190,242,250	125,744,940	16,424,481
IPB220	3379.18	359.22	189,054,395	122,696,386	10,889,735
IPB260	5753.00	467.64	188,359,143	120,927,723	7,687,822

Table 4. Values of P_{cr} (kN) and E_t (kN/m^2) for the frame given in Figure 4(b)

These results lead to a similar outcome as in the case of sway frames. Also, it is shown how important is to use inelastic stability analysis for such examples, especially in the case of rigid structures. Namely, the application of traditional elastic buckling analysis may lead to substantial errors. This numerical example also illustrates the difference in the stability analysis of sway and non-sway frames in the elastic-plastic field. It can be noted in Figure 5, which shows the value of elastic modulus and tangent modules at the time of buckling.

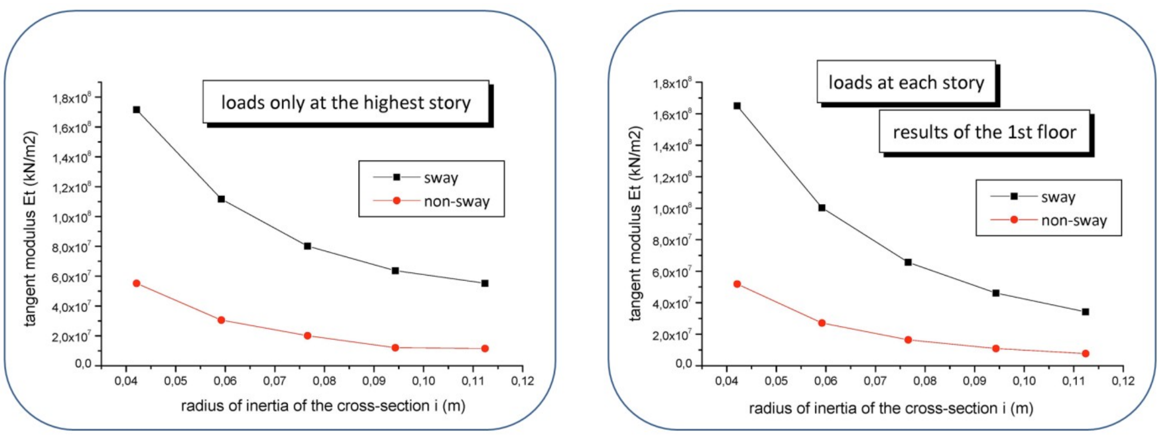


Figure 5. Comparison of tangent modulus values for the sway and non-sway frames

5. Conclusions

In this paper, it is presented a procedure for the global stability analysis of the steel frame structures. Applied matrix analysis is based on the use of trigonometric shape functions. The proposed method is not unknown, but the way how it is formulated and implemented here hasn't been applied in any of the commercial programs or design procedure that deal with the stability of frames.

Presented analysis shows the advantages of performed inelastic stability calculation when compared with traditional elastic stability analysis. In this case, corresponding stiffness matrices have been derived using the tangent modulus approach. These matrices have been implemented in the self-developed computer code ALIN.

Some of the possibilities of such a calculation algorithm are presented on the example of one multi-story frame. It has been shown that for rigid frames, the calculation in the inelastic domain has greater application. The stiffness of the system is varied through the different cross-sections of the axially loaded columns and the consideration of the sway and non-sway frames separately.

So, obtained results confirm that the proposed inelastic buckling approach is convenient for determination of the stability analysis of the steel frame structures. Therefore it can be used as a good alternative for estimation of the load-bearing capacity of the axially loaded elements in the design of steel frames.

In the end, it should be emphasized again that presented calculated algorithm brings more precise stability calculation both in the elastic and in the inelastic domain. It enables monitoring of the loss of structural stability in the plastic range and determination of the critical load and tangent modulus of the axially loaded elements when the frame structure buckles.

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