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## ELASTIČNA KRITIČNA SILA PODUŽNO NEUKRUĆENIH I-NOSAČA USLED LOKALIZOVANOG OPTEREĆENJA

### Rezime:

U radu se razmatra elastična kritična sila izbočavanja podužno neukrućenih čeličnih I-nosača usled dejstva lokalizovanog opterećenja promenljive dužine, koje deluje u ravni rebra. U aktuelnim propisima, za proračun elastične kritične sile nije uzet doprinos dužine lokalizovanog opterećenja i debljine pojasa. Prikazan je uticaj graničnih uslova na vertikalnim ivicama opterećenog panela na vrednost koeficijenta izbočavanja. Predložen je poboljšani izraz za proračun koeficijenata izbočavanja podužno neukrućenih I-nosača.

*Ključne reči: elastična kritična sila, lokalizovano opterećenje, puni limeni nosači*

## ELASTIC CRITICAL LOAD OF LONGITUDINALLY UNSTIFFENED I-GIRDERS SUBJECTED TO PATCH LOADING

### Summary:

The paper considers the elastic critical load of longitudinally unstiffened I-girders subjected to a localized load of variable length in the plane of the web. In the current regulations, the contribution of the patch load length and the flange thickness is not considered for calculating the elastic critical load. The influence of the boundary conditions on the vertical edges of the loaded panel on the value of the buckling coefficient is presented. An improved expression for the calculation of the buckling coefficients of longitudinally unstiffened I-girders is proposed.

*Key words: elastic critical load, patch loading, steel plate girders*

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## 1. INTRODUCTION

The buckling problem of steel plate girders subjected to patch loading has been a research topic for decades. The subject of research has been isolated web plates and I-girders loaded with a localized load in the plane of the web. Furthermore, the influence of the patch load (or partially distributed load) on the behavior of I-girders without vertical stiffener in the zone of load introduction has been specially investigated. This problem has got importance with a general trend to avoid vertical stiffeners, except at supports and in the case of moving loads, e.g., crane girders loaded by crane wheels and launching phase of multi-span steel plate girder bridges during construction over temporary or permanent supports. In everyday engineering practice, standards for calculating steel plate structures (Eurocode EN1993-1-5 [1]) control the ultimate capacity of structural elements subjected to patch loading. There is a constant tendency to make the standards easy to use while still being accurate enough to describe the problem. Eurocode EN1993-1-5 [1] often leads to conservative values for the patch loading resistance. One of the reasons for that is a simplified expression for buckling coefficient, as indicated in [2-5]. Therefore, an attempt is made in this paper to provide improvements to that procedure and to pay attention to parameters that have been neglected, and their impact is significant.

Critical load for a steel plate girder under localized load acting on one flange and between two vertical stiffeners at a distance  $a$ , should be obtained according to EN1993-1-5 [1]:

$$F_{cr} = 0.9k_F E \frac{t_w^3}{h_w}, \quad (1)$$

with buckling coefficient

$$k_F = 6 + 2 \left( \frac{h_w}{a} \right)^2. \quad (2)$$

Equation (2) was obtained by simplifying the expression proposed by Lagerquist [6].

$$k_F = \left( 1 + \frac{s_s}{2h_w} \right) \left( 5.3 + 1.9 \left( \frac{h_w}{a} \right)^2 + 0.4 \sqrt{\beta} \right), \quad \beta = \frac{b_f \cdot t_f^3}{h_w \cdot t_w^3} \quad (3)$$

As it could be observed, Eq. (2) does not take into account the length of an applied patch load, flange to web stiffness ratio, and ratio between the load length and web depth.

This paper considers the determination of the buckling coefficients for elastic critical loads of unstiffened I-girders using finite element (FE) analysis. The commercial software Abaqus [7] was used in this research as one of the most popular computation tools for application in this field. Numerically obtained elastic critical loads are used to get the buckling coefficient. The aim is to formulate an improved expression for calculation of the critical load for I-girders and make the procedure for determination of patch loading resistance in the current standard EN1993-1-5 [1] more accurate.

The following notations are used in this paper to describe the problem of patch loading, compare the results, and derive conclusions:  $h_w$  - web depth,  $t_w$  - web thickness,  $t_f$  - flange thickness,  $b_f$  - flange width,  $a$  - distance between vertical stiffeners (web width),  $s_s$  - patch load length,  $\nu$  - Poisson's ratio,  $E$  - modulus of elasticity,  $k_F$  - buckling coefficient. The influence of the patch load length, aspect ratio ( $a/h_w$ ), and flange thickness is investigated in this paper.



Recently completed research [8, 9], considered the influence of longitudinal stiffeners on the increase of the carrying capacity of longitudinally unstiffened girders. Numerical parametric analysis provided a large database for longitudinally unstiffened and stiffened girders (162 unstiffened and 486 longitudinally stiffened girders). The main goal of the research was the determination of ultimate loads using nonlinear analysis. Initial imperfections of considered girders were defined as first buckling mode and obtained through elastic numerical analysis. Elastic critical loads for 162 unstiffened girders obtained in that research are used here.

The numerical model is described in Section 2. Details of the parametric analysis are described in Section 3. Detailed analysis of the results and improved expression for buckling coefficient  $k_F$  is proposed in Section 4. The statistical analysis of solutions for the ultimate load obtained with the procedure given in EN1993-1-5 by application of the  $k_F$  in the current standard,  $k_F$  given by Lagerqvist and  $k_F$  here proposed, compared with numerically obtained values of ultimate loads is presented in Part 5. Conclusions are given in Section 6.

## 2. NUMERICAL MODELS

The isolated web steel plate and I-girder with a web panel aspect ratio  $a/h_w = 1,2,3$  (schematically presented in Fig. 1), were investigated. Patch load length  $s_s$ , flange thickness  $t_f$  and web thickness  $t_w$  were varied. The boundary conditions for the plate were set according to the clamped plate - that is, degrees of freedom 2 and 5 are only constrained in the vertical edges, degree of freedom 4 in the horizontal edges, while degree of freedom 3 is constrained in all edges. The considered material is homogenous with an elastic modulus of  $E = 205$  GPa and Poisson's ratio of  $\nu = 0.3$ .

For the FE analysis, a general-purpose four-node quadrilateral shell element with reduced integration and six degrees of freedom per node S4R from the Abaqus element library was used. The finite element size is adopted to 5 mm for all numerical runs. The girders are modeled with two rigid vertical stiffeners, preventing rotation. The load is introduced through a rigid block element, which prevents the rotation of the flange around the longitudinal axis in the zone of load application. The finite element model was validated on experimentally tested girders [10, 11], so the selected girders dimensions in the parametric analysis are grounded on that basis.

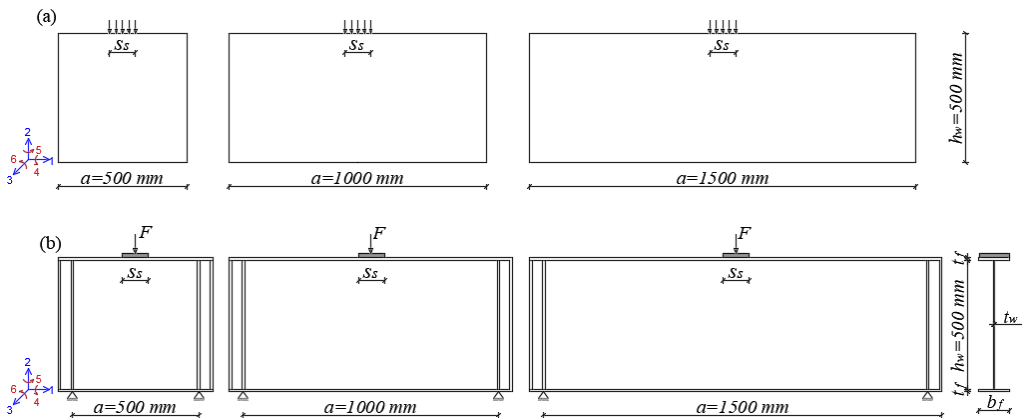


Figure 1 - (a) Plate under patch load; (b) I-girders under patch load

By comparing numerically obtained values of the buckling coefficient of I-girders and using Eq. (3), there were differences in the value of  $k_F$  for the aspect ratio  $a/h_w$  less than 1.5 that should not be neglected. In order to emphasize the way of modeling the boundary conditions on the vertical edges of the girders, the buckling coefficients for the models with rigid vertical stiffeners (corresponding to analysis in this paper) and simply supported vertical edges (as applied by Laguerqvist) were compared, as shown in Figure 2. For both numerical models, the flange rotation is prevented in the zone of load introduction. The analysis of the obtained results is presented in the paper and commented below.

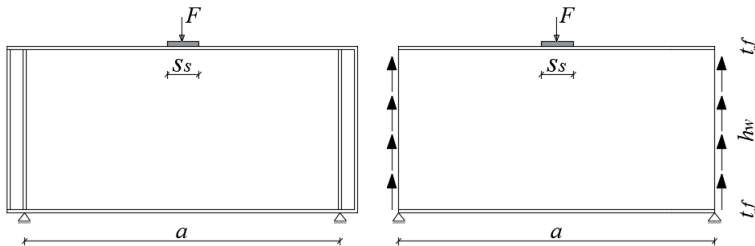


Figure 2 - Boundary conditions on vertical edges

### 3. PARAMETRIC ANALYSIS

This paper shows how the accuracy of the calculation of the buckling coefficient, obtained from the expression for the elastic critical load, can affect the ultimate load capacity and possibly improve it compared to the procedures in EN1993-1-5 [1]. In order to compare the results of the patch loading resistance obtained using the proposed expression for  $k_F$  and the numerically obtained ultimate load, the procedure was carried out on the girders treated in Refs. [8, 9]. The numerical values of the ultimate loads of those girders were given there.

The dimensions of girders employed in the numerical simulations [8, 9] were carefully chosen. They were selected such that characteristic ratios (e.g.,  $a/h_w$ ,  $h_w/t_w$ ,  $t_f/t_w$ ) include limiting and average values frequently used in experimental and computational studies. The following geometric parameters were not varied:  $b_f = 120$  mm, yield strength of the web  $f_{yw} = 323$  MPa and flange  $f_{yf} = 323$  MPa. For each parameter that was varied in the numerical simulations, three values were selected such that they give for characteristic ratios ( $a/h_w$ ,  $h_w/t_w$ ,  $t_f/t_w$ ) approximately (i) lower limit, (ii) average value between lower and upper limit, and (iii) upper limit (of those frequently used in experimental and numerical studies).

The following geometric parameters were varied for longitudinally unstiffened girders:

- Web panel width  $a = 500, 1000, 1500$  mm (i.e., web aspect ratio  $\alpha = 1, 2$  and  $3$ ).
- Three web panel thicknesses  $t_w = 1.25, 2, 4$  mm for each web panel width  $a$  (web slenderness  $h_w/t_w = 400, 250, 125$ ).
- For each of these nine cases, three values of the flange thickness were applied such that  $t_f/t_w = 2, 4$ , and  $6$  (for  $t_w = 1.25 \rightarrow t_f = 2.5, 5, 7.5$ , for  $t_w = 2 \rightarrow t_f = 4, 8, 12$  and for  $t_w = 4 \rightarrow t_f = 8, 16, 24$ ).
- For each of these twenty-seven cases, six patch load lengths were applied:  $s_s = 0$  (concentrated force),  $s_s = 50, 100, 150, 200, 250$  mm ( $s_s/h_w = 0 - 0.50$ ).

Such choice enabled detailed analysis for a much larger range of parameters than applied in Refs. [2-5]. Total number of longitudinally unstiffened models (numerical simulations) was 162.

It was necessary to form additional I-girders models for web aspect ratios  $\alpha = 1.25$  and  $1.5$  and web thickness  $t_w = 4$  mm to find the expression for buckling coefficient by correcting Eq. (2). For those models, the elastic critical load was found without performing a nonlinear analysis for calculating ultimate loads. For all girders mentioned in the parametric study, the elastic critical loads of the isolated web plates were also found to obtain the buckling coefficients. Those values are presented in this paper and compared with the buckling coefficients of I-girders.

#### 4. RESULTS AND DISCUSSION

The buckling coefficient  $k_F$  of the girder subjected to patch loading affects the value of the elastic critical load and patch loading resistance. Based on the analysis given in this paper, it was shown that the buckling coefficient does not depend significantly on the web thickness. Therefore, additional analyses for the aspect ratio  $\alpha = 1.25$  and  $1.5$  were performed only for the web thickness  $t_w = 4$  mm. Table 1 shows the values of  $k_F$  for aspect ratio  $\alpha = 2$ , for different web thicknesses and flange thickness  $2t_w$ , as a function of patch load length  $s_s$ . The buckling coefficients numerically obtained for isolated web plate -  $k_F^{CC}$  and I-girder -  $k_F^{FEA}$ , and using Eq. (3) -  $k_F^L$  are given. The expression proposed by Lagerqvist - Eq. (3) does not take into account the influence of the web thickness on the value of the buckling coefficient, but only the ratio between the flange and web stiffness.

Table 1 - Buckling coefficients of I-girders for different web thicknesses

$t_f = 2t_w$	$\alpha = 2, t_w = 4$ mm			$\alpha = 2, t_w = 2$ mm			$\alpha = 2, t_w = 1.25$ mm		
$s_s/h_w$	$k_F^{CC}$	$k_F^{FEA}$	$k_F^L$	$k_F^{CC}$	$k_F^{FEA}$	$k_F^L$	$k_F^{CC}$	$k_F^{FEA}$	$k_F^L$
0.00	6.17	6.42	6.25	6.21	6.42	6.25	6.18	6.42	6.25
0.10	6.22	6.57	6.56	6.22	6.56	6.56	6.22	6.56	6.56
0.20	6.34	6.81	6.87	6.34	6.79	6.87	6.36	6.78	6.87
0.30	6.51	7.10	7.18	6.53	7.08	7.18	6.53	7.06	7.18
0.40	6.75	7.45	7.50	6.75	7.41	7.50	6.74	7.39	7.50
0.50	7.02	7.86	7.81	7.00	7.81	7.81	7.01	7.78	7.81

Table 2 - Buckling coefficients of I-girders for different boundary conditions at the vertical edges for  $t_w = 4$  mm

$\alpha = 1$	$t_f = 8$ mm		$t_f = 16$ mm		$t_f = 24$ mm	
$s_s/h_w$	$k_F^{FEA}_{SS}$	$k_F^{FEA}_{CC}$	$k_F^{FEA}_{SS}$	$k_F^{FEA}_{CC}$	$k_F^{FEA}_{SS}$	$k_F^{FEA}_{CC}$
0.00	8.98	7.56	10.10	8.02	11.00	8.48
0.10	9.23	7.72	10.40	8.30	11.34	8.65
0.20	9.70	8.10	11.01	8.74	12.11	9.18
0.30	10.37	8.67	11.86	9.36	13.18	9.87
0.40	11.25	9.41	13.00	10.21	14.65	10.88
0.50	12.37	10.40	14.52	11.35	16.68	12.21

Furthermore, it was shown that the coefficient  $k_F$  depends on the boundary conditions on the vertical edges of girder. Table 2 indicates the importance of modeling methods for  $\alpha = 1$ . The notation *SS* refers to the simply supported edges, and *CC* to the rigid vertical stiffeners at the ends preventing rotation. Significant differences occur for a small aspect ratio - especially for  $\alpha = 1-1.5$ , for all analyzed models.

In this paper, an expression for the calculation of the buckling coefficient is proposed - Eq. (4), derived from all the analyzed girders described within the parametric analysis, and which have rigid vertical stiffeners on the edges. The expression represents a modified proposal of Lagerqvist, primarily for the aspect ratio smaller than 1.5.

$$k_F = \left( 1 + f(\alpha_1) \cdot \frac{s_s}{h_w} \right) \cdot \left( 5.30 + f(\alpha_2) \cdot \left( \frac{h_w}{a} \right)^2 + f(\alpha_3) \cdot \sqrt[4]{\bar{\beta}} \right)$$

$$f(\alpha_1) = 1.2\alpha^2 - 3.5\alpha + 3.05$$

$$f(\alpha_2) = 4.6 - 2.4\alpha$$

$$f(\alpha_3) = 1.2\alpha^2 - 3.5\alpha + 3.2$$

For  $\alpha > 1.5$  take  $\alpha = 1.5$

$$\bar{\beta} = \frac{b_f t_f^3}{\left( \frac{h_w + a}{2} \right) t_w^3} \quad (4)$$

Table 3 shows the values of the buckling coefficient  $k_F$  for the considered models of I-girders whose web thickness is set to be  $t_w = 4$  mm: obtained numerically -  $k_F^{\text{FEA}}$ , using Lagerqvist's proposal [6] -  $k_F^{\text{L}}$ , and using Eq. (4) proposed in this paper -  $k_F^{\text{F}}$ . Results are shown for three flange thicknesses  $t_f = 8, 16, 24$  mm and aspect ratio  $\alpha = 1, 1.25, 1.5, 2, 3$ . Furthermore, Table 3 presents the values of the buckling coefficients calculated using the expressions from EN1993-1-5 [1] -  $k_F^{\text{EC}}$  and  $k_F^{\text{CC}}$  - numerically obtained values for isolated web plates.

The buckling coefficient obtained by Eq. (4) corresponds best to the numerical values. However, for an aspect ratio less than 1.5, the improvement achieved using the proposed expression - Eq. (4) compared to Lagerqvist's proposal is more significant. Lagerqvist's expression agrees well with the numerical results for larger aspect ratios, but an improvement is observed for larger flange thicknesses using the expression proposed in this paper - Eq. (4).

Table 3 - Comparison of buckling coefficients of CC plate and I-girders for  $t_w = 4$  mm.

$\alpha = 1$		$t_f = 8$ mm			$t_f = 16$ mm			$t_f = 24$ mm			
$s_s/h_w$	$k_F^{\text{CC}}$	$k_F^{\text{EC}}$	$k_F^{\text{FEA}}$	$k_F^{\text{F}}$	$k_F^{\text{L}}$	$k_F^{\text{FEA}}$	$k_F^{\text{F}}$	$k_F^{\text{L}}$	$k_F^{\text{FEA}}$	$k_F^{\text{F}}$	$k_F^{\text{L}}$
0.00	8.00	8.00	8.98	8.56	7.67	10.10	9.28	7.99	11.00	9.91	8.27
0.10	8.17	8.00	9.23	9.20	8.05	10.40	9.98	8.39	11.34	10.66	8.69
0.20	8.35	8.00	9.70	9.84	8.44	11.01	10.67	8.79	12.11	11.40	9.10
0.30	8.74	8.00	10.37	10.49	8.82	11.86	11.37	9.19	13.18	12.15	9.51
0.40	9.23	8.00	11.25	11.13	9.21	13.00	12.07	9.59	14.65	12.89	9.93
0.50	9.91	8.00	12.37	11.77	9.59	14.52	12.76	9.99	16.68	13.63	10.34

$\alpha = 1.25$			$t_f = 8 \text{ mm}$			$t_f = 16 \text{ mm}$			$t_f = 24 \text{ mm}$		
$s_s/h_w$	$k_F^{CC}$	$k_F^{EC}$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$
0.00	6.89	7.28	7.46	7.12	6.99	8.06	7.67	7.31	8.40	8.15	7.59
0.10	6.96	7.28	7.57	7.52	7.34	8.20	8.09	7.67	8.55	8.60	7.97
0.20	7.12	7.28	7.88	7.91	7.69	8.56	8.51	8.04	8.98	9.04	8.35
0.30	7.37	7.28	8.30	8.30	8.03	9.04	8.94	8.40	9.55	9.49	8.73
0.40	7.69	7.28	8.82	8.69	8.38	9.67	9.36	8.77	10.28	9.94	9.11
0.50	8.09	7.28	9.49	9.08	8.73	10.46	9.78	9.13	11.22	10.39	9.49

$\alpha = 1.5$			$t_f = 8 \text{ mm}$			$t_f = 16 \text{ mm}$			$t_f = 24 \text{ mm}$		
$s_s/h_w$	$k_F^{CC}$	$k_F^{EC}$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$
0.00	6.46	6.89	6.83	6.47	6.62	7.40	6.96	6.94	7.64	7.39	7.22
0.10	6.51	6.89	6.92	6.79	6.95	7.50	7.31	7.28	7.75	7.76	7.58
0.20	6.64	6.89	7.18	7.11	7.28	8.37	7.66	7.63	8.07	8.13	7.94
0.30	6.84	6.89	7.51	7.44	7.61	8.13	8.01	7.98	8.47	8.50	8.30
0.40	7.09	6.89	7.91	7.76	7.94	8.58	8.35	8.32	8.98	8.87	8.66
0.50	7.40	6.89	8.41	8.09	8.27	9.13	8.70	8.67	9.60	9.24	9.02

$\alpha = 2$			$t_f = 8 \text{ mm}$			$t_f = 16 \text{ mm}$			$t_f = 24 \text{ mm}$		
$s_s/h_w$	$k_F^{CC}$	$k_F^{EC}$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$
0.00	6.17	6.50	6.42	6.24	6.25	7.28	6.71	6.57	7.61	7.13	6.85
0.10	6.22	6.50	6.57	6.55	6.56	7.40	7.05	6.90	7.73	7.48	7.19
0.20	6.34	6.50	6.81	6.87	6.87	7.63	7.38	7.22	7.98	7.84	7.53
0.30	6.51	6.50	7.10	7.18	7.18	7.92	7.72	7.55	8.29	8.19	7.88
0.40	6.75	6.50	7.45	7.49	7.50	8.28	8.06	7.88	8.68	8.55	8.22
0.50	7.02	6.50	7.86	7.80	7.81	8.70	8.39	8.21	9.13	8.91	8.56

$\alpha = 3$			$t_f = 8 \text{ mm}$			$t_f = 16 \text{ mm}$			$t_f = 24 \text{ mm}$		
$s_s/h_w$	$k_F^{CC}$	$k_F^{EC}$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$	$k_F^{FEA}$	$k_F^F$	$k_F^L$
0.00	5.62	6.22	6.20	6.05	5.98	7.25	6.49	6.30	7.64	6.88	6.58
0.10	5.67	6.22	6.37	6.36	6.28	7.38	6.82	6.62	7.76	7.22	6.91
0.20	5.78	6.22	6.62	6.66	6.58	7.61	7.14	6.93	8.01	7.57	7.24
0.30	5.95	6.22	6.93	6.96	6.88	7.91	7.47	7.25	8.33	7.91	7.57
0.40	6.15	6.22	7.28	7.27	7.18	8.28	7.79	7.56	8.72	8.25	7.90
0.50	6.41	6.22	7.69	7.57	7.48	8.72	8.12	7.88	9.18	8.60	8.23

Figure 3 shows the diagrams of the buckling coefficient as a function of the patch load length for different aspect ratios. For longer girder widths, i.e., larger aspect ratios, the changes in the values of the buckling coefficient are small. For example, in Figure 3 (a), it can be observed that for the aspect ratio of 1.25, the value of the buckling coefficients drop significantly. Regardless

of the web thickness, for  $\alpha$  greater than 1.5, the influence of the girder length, and therefore the boundary conditions on the edges, is significantly reduced. Figure 4 shows the dependence of the buckling coefficient on the patch load length  $s_s$ , for different calculation methods: obtained numerically -  $k_F^{FEA}$ , using Lagerqvist's proposal [6] -  $k_F^L$ , using Eq. (4) proposed in this paper -  $k_F^F$ , obtained by the procedure given in EN1993-1-5 [1] -  $k_F^{EC}$ , and for isolated web plate  $k_F^{CC}$ . For the considered girder and plate models, EN1993-1-5 [1] gives conservative results, while the proposed expression - Eq. (4) best agrees with the numerically obtained values.

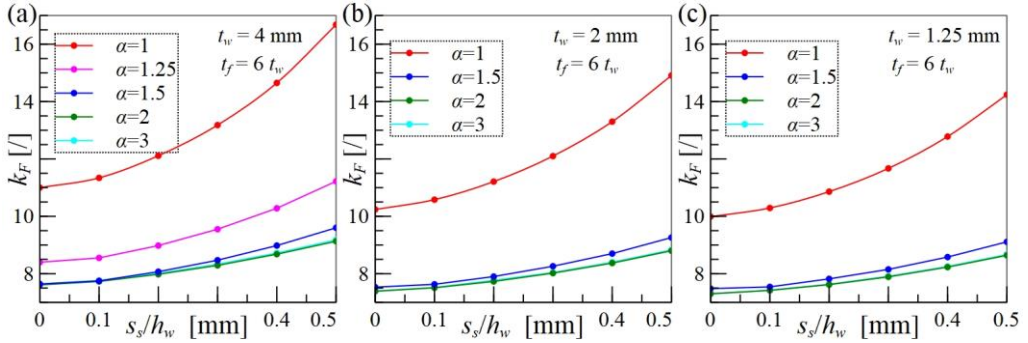


Figure 3 - Comparison of the numerically obtained buckling coefficients for different aspect ratios.

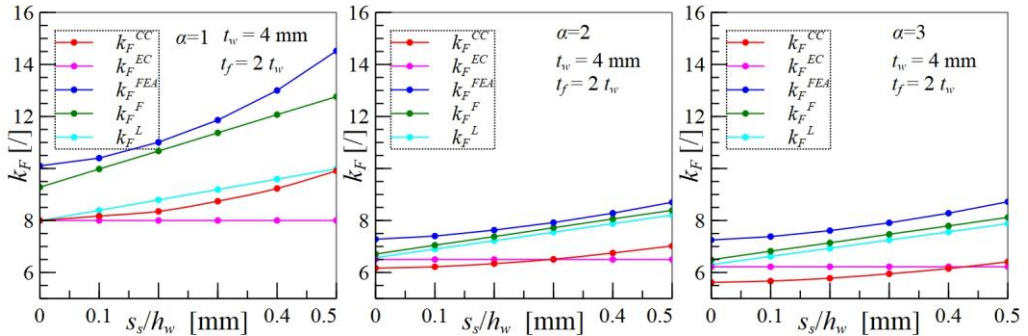


Figure 4 - Comparison of the buckling coefficients for various calculation models.

## 5. COMPARISON OF THE ULTIMATE LOADS

In the procedure given in EN1993-1-5 [1] for calculating the patch loading resistance, the ultimate load depends on the value of the buckling coefficient  $k_F$ . Figure 5 compares the ultimate load values for I-girders described in the parametric analysis, for aspect ratio 1 and web thickness  $t_w = 4$  mm and  $t_f = 4 t_w$ , numerically obtained and using the expression given in EN1993-1-5 [1].  $F_{Rd}^{EC}$  presents the value of the ultimate load obtained using the buckling coefficient calculated from Eq. (2) -  $k_F^{EC}$ .  $F_{Rd}^L$  refers to the patch loading resistance obtained using buckling coefficient calculated from Lagerqvist proposal -  $k_F^L$ , Eq. (3).  $F_{Rd}^F$  presents the ultimate load calculated using buckling coefficient obtained from the expression proposed in this paper -  $k_F^F$ ,

Eq. (4). The ultimate loads of I-girders obtained using Eq. (4) correspond best to the numerically obtained ultimate loads  $F_{un}$ .

		$F_{Rd}^{EC}/F_{un}$	$F_{Rd}^L/F_{un}$	$F_{Rd}^F/F_{un}$
<i>all</i>	$\bar{X}$	0.53	0.56	0.59
	$S_x$	0.08	0.08	0.08
	$CV$	0.15	0.14	0.13
$\alpha = 1$	$\bar{X}$	0.50	0.53	0.58
	$S_x$	0.09	0.08	0.08
	$CV$	0.18	0.16	0.14
$\alpha = 2$	$\bar{X}$	0.53	0.56	0.57
	$S_x$	0.07	0.07	0.07
	$CV$	0.14	0.12	0.12
$\alpha = 3$	$\bar{X}$	0.56	0.60	0.60
	$S_x$	0.07	0.07	0.07
	$CV$	0.13	0.12	0.12

Table 4 - Comparison of the analysed statistical parameters.

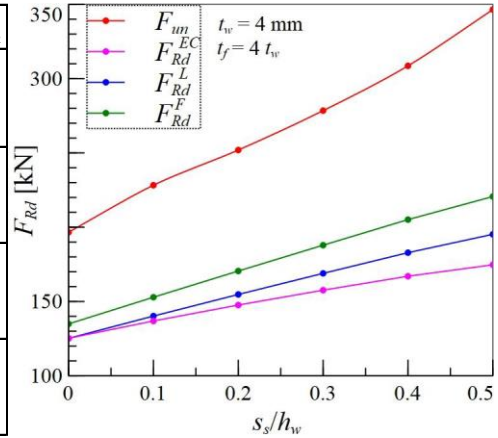


Figure 5 - Comparison of the ultimate loads of I-girders.

Table 4 compares the values for 162 girders of the ultimate loads  $F_{Rd}^{EC}$ ,  $F_{Rd}^L$ ,  $F_{Rd}^F$  with the numerically obtained values  $F_{un}$  using [7]. The basic statistical quantities were compared for each group of girders (aspect ratio  $\alpha = 1, 2, 3$ ) and ultimate loads obtained through different calculations of buckling coefficients: mean value ( $\bar{X}$ ), standard deviation ( $S_x$ ) and coefficient of variation ( $CV$ ). It has been shown that statistical parameters are the most favorable in the case of the calculation of the ultimate load obtained by using the buckling coefficient calculated from the expression proposed in this paper - Eq. (4). The greatest improvement in the patch loading resistance was obtained for the small aspect ratio  $\alpha = 1$ , up to 31%. As the aspect ratio increases, the percentage of ultimate load improvement decreases compared to the ultimate load calculated using the procedures given in EN1993-1-5 [1].

## 6. CONCLUSIONS

The paper presents the elastic critical load of longitudinally unstiffened I-girders subjected to patch loading. The girders described in the parametric analyses were considered. For those girders, the numerical values of the buckling coefficient were found based on the elastic critical load. The influence of the boundary conditions on the vertical edges of the loaded panel on the value of the elastic critical load is shown. A limit is given in the aspect ratio when these conditions become negligible. It was pointed out that the thickness of the web does not significantly affect the value of the buckling coefficient. The design standard EN1993-1-5 gives conservative results for elastic critical load because the influence of flange stiffness and patch load length is neglected. An improved expression for calculating the buckling coefficient of I-girders that considers the flange thickness, the patch load length and aspect ratio is proposed. The ultimate load depends on the buckling coefficient. The patch loading resistance calculation has improved by applying the expression for calculating the coefficient  $k_F$  proposed in this paper. The most significant improvement was obtained for aspect ratio 1 and thicker flange, up to 31%.

More favorable values of statistical quantities were also obtained. The mean value is increased by 10%, and the coefficient of variation is decreased compared to the current standard. All conclusions in the paper refer to precisely defined examples described in Section 2 and Section 3.

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