UDK: 519.816 : 624.21
Izvorni naučni članak

PRIMENA FUZZY TOPSIS METODE ZA VIŠEKRITERIJUMSKI IZBOR OBJEKATA ZA REKONSTRUKCIJU I ODRŽAVANJE

Živojin Praščević, Nataša Praščević

Građevinski fakultet Univreziteta u Beogradu

REZIME

U ovom radu je dat prikaz predloženog postuka za višekritejirumsko rangiranje alternativa Fuzzy TOPSIS koje je primenjen za određivanje optimalne raspodela investicionih sredstava za održavanje građevinskih objekata i višekriterijumski izbor objekata za rekonstrukciju. Prema ovom postupku napisan je odgovarajući kompjuterski program i prikazan jaden ilustrativan primer ocene rizika i rangiranja za održavanje mostovskih konstrukcija.

KLJUČNE REČI: Fuzzy TOPSIS, održavanje objekata, raspodela investicija

APPLICATION OF FUZZY TOPSIS METHOD FOR MULTIPLE CRITERIA CHOICE OF OBJECTS FOR RECOTSNRUCTION AND MAINTENANCE

ABSTRACT

A survey of proposed procedure for multiple criteria ranking of alternatives Fuzzy TOPSIS is presented in this paper This procedure is applied for determination of the optimal distribution of investments for the maintenance of civil engineering objects and their multiple criteria choice for reconstruction. According to this procedure corresponding computer program has been written out and one illustrative example of the bridge risk assessment and their ranking for maintenance is presented in the paper.

KEY WORDS: fuzzy TOPSIS, maintenance, distribution of investments

INTRODUCTION

TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) for solving multiple criteria decision problem (MCDMP) with several alternatives was proposed and developed by Hwang and Yoon (1981). The method is based on the fact that the chosen or most appropriate alternative should have the shortest distance from positive ideal solution (PIS) and the longest distance from the negative ideal (anti ideal) solution (NIS). This alternative has the maximum similarity with positive ideal solution and minimum similarity with negative ideal solution. Chen and Hwang (1992) have transformed this method with the crisp (nonfuzzy) data to the method with the fuzzy data. In last twenty years a lot of authors take part in development of this method and proposed numerous modifications. The method was applied usefully in the practice as a help to the decision makers to solve many problems in different fields. Opricović (1998) proposed method, named VIKOR, for multiple criteria optimization of complex systems. This method focuses on ranking and selecting alternatives in the presence of the conflicting criteria. He introduced the multiple criteria ranking index based on the particular measure of closeness to the ideal solution. Opricović and Tzeng (2004) compared main fetures of VIKOR and TOPSIS in all steps of a problem solution: procedural basis, normalization, aggregation and final solution. Opricović later extended his VICOR method for soving fuzzy multiple criteria problems with conflicting and non conflicting criteria and developed VIKOR-F (Opricović, 2007). VIKOR method has been used many times for multiple criteria ranking of alternatives for solution of many problems in civil, hydrotechnical and transportation engineering and other branches of practice as well. Wang and Elhang (2006) proposed fuzzy TOPSIS method based on alfa level sets with application to the bridge risk management. For every alternative and chosen alfa level, they formulated nonlinear programs (NLP) with lower and upper value of relative closeness to NIS as the objective functions and with prescribed lower and upper values as the constrains. In such a way these relative closeness are obtained as fuzzy numbers and then after defuzzification the alternatives are ranked.

The risk assessment of an object (bridge, building, etc) is usually performed to determine the optimal scheme or rank order of the object maintnance. This problem has been investigated by many autthors and in the literature exist differnt methods for the risk assessment. For instance, Adey, Hajdin and Brühwiler (2003) presented risk-based approach to the determination of optimal interventions for bridges affected by multiple hazards. Wang and Ehlang (2007) proposed a fuzzy group decision making approach for the risk assesment using fuzzy TOPSIS method.

In this paper is considered a problem of multiple criteria ranking of objects for reconstruction against prescribed criteria using modified fuzzy TOPSIS procedure proposed by authors (Prascevic and Prascevic, 2010). In this method all input data are presented as triangular fuzzy numbers as probabilistic fuzzy input data. For these fuzzy numbers and their products are found generalized expected values, variances, standard deviations and coefficients of variations. These values are used in the mathematical formulas for relative distances to PIS and NIS to rank chosen alternatives. This procedure is more general than the procedure based on crisp data and gives to the decision maker more important data which are relevant to make an optimal decision.

DEFINITION OF THE PROBLEM

In this problem is assumed some firm or institution (owner) which is responsible for the maintenance of n objects (buildings, bridges or other objects) $A_1, A_2, \ldots, A_m$. To reduce consequences of a risk that influence on safety, functionality, sustainability, availability, environmental and other important factors, a corresponding amount of money should be invested in the maintenance of these objects. The available amount of money usually is not sufficient for all objects or projects, so that they should be ranked according to the risk rating, and the money should be invested in the objects according to this rank list. The mentioned factors are named as criteria denoted by $C_1, C_2, \ldots, C_n$, while the objects represent alternatives for multi-criteria decision making (MCDM). Each alternative A_i is numerically evaluated by experts with respect to the criterion C_j by values f_{ij} ($i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$). These values are elements of a decision matrix denoted by $\mathbf{F} = [f_{ij}]_{m \times n}$

The set of criteria Ω contains two disjunct subsets Ω_b and Ω_c, i.e

$$\Omega = (C_1, C_2, \ldots, C_n) = (\Omega_b \cup \Omega_c), \quad (\Omega_c \cap \Omega_b) = \varnothing \qquad (1)$$

The subset of criteriteria Ω_b represents benefits or criteria with favourable effects that should be maximised, while subset of criteria Ω_c represents costs or criteria with unfavourable effects that should be minimized in the procedure.

Every criterion C_j is assessed by experts with relative weight values w_j (j = 1,2,...,n). These values form the *vector of weights* w = $[w_j]_{1\times n}$. The problem is to find the most preferable or the best (compromise) alternative A_c that satisfies all criteria together and which is closest to the *ideal positive solution* and farthest to the *negative ideal solution*, and to rank alternatives according to this rule.

The ideal positive solution F^* is formed by the values f_{ij} that are maximal for the benefit criteria and minimal for the cost criteria, i.e

$$F^* = \{f_1^*, .. f_i^* ... f_n^*\} = \{(\max_i f_{ij}, i \in \Omega_b), (\min_i f_{ij}, i \in \Omega_c)\}.$$
(2)

The ideal negative solution F^- is formed by the values f_{ij} that are minimal for the benefit criteria and maximal for the cost criteria, i.e

$$F^- = \{f_1^-, .. f_i^- ... f_n^-\} = \{(\min_i f_{ij}, i \in \Omega_b), (\max_i f_{ij}, i \in \Omega_c)\}.$$
(3)

In many real situations elements of decision matrix f_{ij} and vector of weights w_j can not be assessed precisely and expressed by crisp numbers. Some of these elements sometimes may be quantified by linguistic values "good", "bad", "high", "low" and in other similar way. For these reasons, the fuzzy numbers for input data should be used, and the problem transformed to the fuzzy multiple criteria decision making problem (FMCDMP). In the literature exist many methods and its modifications to solve this problem with fuzzy and nonfuzzy (crisp) data. In this paper is used the triangular fuzzy number $\tilde{A}$, which is shown on Fig. 1, described with three characteristic values a_l, a_m and a_u i.. e. $\tilde{A} = (a_l, a_m, a_u)$.

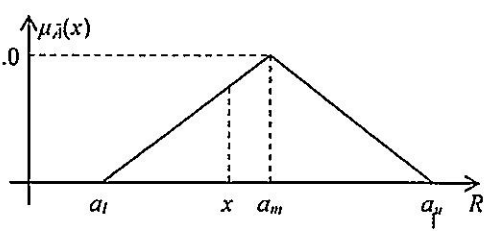

Fig. 1 Triangular fuzzy number

Sliaka 1 Trouglast fazi broj

FUZZY TOPSIS PROCEDURE

Elements of the fuzzy decision matrix $\tilde{\mathbf{F}}$ are triangular fuzzy numbers $\tilde{f}_{ij} = (f_{ij}^{(l)}, f_{ij}^{(m)}, f_{ij}^{(u)})$, so that this matrix can be expressed by three crisp matrices $\tilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u)$. Fuzzy TOPSIS procedure performs in several steps which will be explained in this work with proposed modification. These steps are normalization, calculation of generalized expected values and standard deviations, ranking alternatives and choice of the best alternative.

Normalization

Since criteria of the decision making problem have different nature and meaning, and thus are expressed by the values which usually have different dimensions and scale, it should to perform normalization of their values and obtain dimensionless values of the decision matrix. In the literature exist several methods for this normalization (Wang and Elhang, 2006), and here will be given method used by Ertugrud and Karakasagly (2008). Normalized values of elements $\tilde{f}_{ij}$ of the fuzzy decision matrix $\tilde{\mathbf{F}}$ are denoted as $\tilde{a}_{ij}$, which consist the normalized fuzzy matrix $\tilde{\mathbf{A}}$ and are calculated by the next formula

$$\tilde{a}_{ij} = (f_{ij}^{(l)} / f_i^{*(u)}, f_{ij}^{m} / f_i^{*(u)}, f_{ij}^{u} / f_i^{*(u)}) ; \; i = 1,2,...,m; j = 1,2,...n; \tag{4}$$

where for every criterion i

$$f_i^{*(u)} = \max_j f_{ij}^{(u)}, \; i = 1,2,...,m.$$

(5)

Determination of expected values, dispersions (variances) and standard deviation of fuzzy elements of the weighted normalized decision matrix $\tilde{\mathbf{V}}$

Elements $\tilde{v}_{ij}$ of a weighted decision matrix $\tilde{\mathbf{V}}$ are calculated as a product of two fuzzy numbers $\tilde{a}_{ij}$ and weight $\tilde{w}_i$, which in many cases represents coefficient *of significance* of the alternative A_i

$$\tilde{v}_{ij} = \tilde{a}_{ij}\tilde{w}_i; \quad i = 1,2,...,m; j = 1,2,...,n$$

(6)

Some authors (Ates *at al.*, 2006) calculate elements of the fuzzy weighted matrix $\tilde{\mathbf{V}}$ by the formula

$$\tilde{v}_{ij} = (a_{ij}^{(l)} w_i^{(l)}, a_{ij}^{(m)} w_i^{(m)}, a_{ij}^{(u)} w_i^{(u)}) \tag{7}$$

In the authors earlier paper (Prascevic and Prascevic, 2010) is proposed procedure with the generalized expected values e_{ij} and dispersions dij of the fuzzy numbers products

$$e_{ij} = x_c(\tilde{a}_{ij}\tilde{w}_i), \quad d_{ij} = D(\tilde{a}_{ij}\tilde{w}_i); \quad i = 1,2,...,m; \; j = 1,2,...,n. \tag{8}$$

These values are elements of matrices **E** and **D** respectively and are calculated by the formulae that are given in the paper (Prascevic and Prascevic, 2010) depending on the chosen probability distribution of fuzzy events, which may be uniform or tirangular one.

Calculation of the expected ideal positive and ideal negative solutions

For every criterion C_i are found the best expected ideal positive solution e_i^* and the worst ideal negative solution e_i^- in the columns of the matrix of expected values **E** by the next formulae

$$e_i^* = \{\max_i e_{ij} : j \in \Omega_b \text{ or } \min_i e_{ij} : j \in \Omega_c\}. \tag{9}$$

$$e_i^- = \{\min_i e_{ij} : j \in \Omega_b \text{ or } \max_i e_{ij} : j \in \Omega_c\}. \tag{10}$$

These values are elements of vectors of expected ideal positive A^* and expected ideal negative A^- solution

$$A^* = [e_1^*, e_2^*, \ldots, e_n^*], \quad A^- = [e_1^-, e_2^-, \ldots, e_n^-] \qquad (11)$$

Dispersions that corresponds to these expected values are denoted as d_i^* and d_i^- and they constitute vectors

$$D^* = [d_1^*, d_2^*, \ldots, d_n^*], \quad D^- = [d_1^-, d_2^-, \ldots, d_n^-] \qquad (12)$$

Calculation of the expected Euclidean distances and dispersion from ideal positive and ideal negative solution

The expected Euclidean distances for every alternative A_i from the expected positive ideal solution A^* and from expected negative ideal solution A^- are calculated by formulae

$$ED_i^* = [\sum_{j=1}^{n} (e_{ij} - e_j^*)^2]^{1/2}, \quad i = 1,2,\ldots,m; \qquad (13)$$

$$ED_i^- = [\sum_{j=1}^{n} (e_{ij} - e_j^-)^2]^{1/2}, \quad i = 1,2,\ldots,m. \qquad (14)$$

Variance V_j^* of the distances of alternative A_j from the positive ideal solution A^* and variance V_j^- from the negative ideal solution A^-, are calculated by the next formulae, taking into account rule for summation and subtraction of variances for the mutually independent variables

$$V_i^* = \sum_{i=1}^{n} (d_{ij} + d_j^*), \quad i = 1,2,\ldots,m;$$

(15)

$$V_i^- = \sum_{i=1}^{n} (d_{ij} + d_j^-), \quad i = 1,2,\ldots,n. \qquad (16)$$

Corresponding standard deviation σ_i^* of the distance of each alternative A_i from the ideal positive solution A^* and standard deviation σ_i^- of each alternative A_i from negative ideal solution A^- are

$$\sigma_i^* = [V_i^*]^{1/2}, \quad \sigma_i^- = [V_i^-]^{1/2}; \; i=1,2,\ldots,n. \qquad (17)$$

These characterstic values of distances of each alternative A_i from ideal positive and ideal negative solution are further used to formulate rules for the alternative ranking and choice of best alternative. The distances from positive and negative ideal solutions are assumed as the fuzzy numbers, or probabilistic fuzzy events, characterzed by these values.

Expected relative closeness and relative standard deviation to ideal pozitive and ideal negative solution and ranking alternatives

Like in the TOPSIS method with crisp data, expected relative closeness of each alternative A_i to the positive ideal solution $RC_{e,i}^*$, and negative ideal solution $RC_{e,i}^-$, are important indicators for ranking alternatives. These values are calculated by next formulae

$$ERC_i^{*} = ED_i^{*}/(ED_i^{*} + ED_i^{-}), \quad i = 1,2,\ldots,m; \tag{18}$$
$$ERC_i^{-} = ED_i^{-}/(ED_i^{*} + ED_i^{-}), \quad i = 1,2,\ldots,m. \tag{19}$$

Alternative with smaller ERC_i^{*} and bigger ERC_i^{-} are better ranked.

Cheng (1998) proposed CV index to improve Lee and Li's method (Lee and Li, 1988) of ranking fuzzy numbers. This index represents the coefficient of variation which is calculated for the distance of alternative A_i from ideal positive solution CV_i^{*} and ideal negative solution CV_i^{-}, respectively

$$CV_i^{*} = \sigma_i^{*}/ED_i^{*}, \quad CV_i^{-} = \sigma_i^{-}/ED_i^{-}, \quad i = 1,2,\ldots,m. \tag{20}$$

Alternative with bigger CV_i^{*} and smaller CV_i^{-} has the better rank on the rank list. Ranking alternatives in this way is simple, but sometimes has some disadvantage. It is possible such a case when comparing two alternatives A_i and A_k which have expected distances from positive ideal solutions $ED_i^{*} > ED_k^{*}$ and $CV_i^{*} < CV_k^{*}$. According this ranking rule, alternative A_k is better ranked then alternative A_i. This conclusion will not be accepted by the decision maker if differences between CV_i^{*} and CV_k^{*} are small. In such a case alternative A_k will be ranged better then alternative A_i, especially when alternative A_k has smaller expected relative closeness then alternative A_i, i.e. $RC_k^{*} < RC_i^{*}$.

Ranking according to expected relative closeness have advantage over other rules. But in practice should to apply all the rules and then analyze obtained results and propose to the decision maker that alternative which satisfies maximally these rules.

If an amount of money Q, which is determined for the maintenance of considered objects, then it be delivered according to the obtained rank list by the next formulae

$Q_{ci} = (KIC)_i Q$ for the rank list according to ERC_i^{*}, (21)

$Q_{vi} = (KIV)_i Q$ for the rank list according to CV_i^{*}, (22)

where KIC_i and KIV_i coefficients of distribution of the amount of money Q

$$KIC_i = ERC_i^{-}/\sum_{i=1}^{m} ERC_i^{-}, \quad KIV_i = CV_i^{*}/\sum_{i=1}^{m} CV_i^{*}, \quad ERC_i^{-} = 1 - ERC_i^{*} \tag{23}$$

According to this procedure, the authors have written corresponding computer program FUZZY_TOPSIS in MATLAB programming system.

EXAMPLE

This example, which is related to bridge risk assessment, is taken from papers written by Wang and Ehlang (2003,2007), where this problem is solved in quite different way. According to British Highway Agency (2004), bridge risk is defined as any event or hazard that could hinder the achievement of business goals or the delivery of stakeholder expectations and is defined as product of the likelihood (probability) and consequence of the event occurred.

In the example are considered five bridge structures BS_1, BS_2,..., BS_5 which represent alternatives A_1, A_2,..., A_5. All consequences and probabilities of the risk events are assessed on the base of evidence and engineering judgment by three experts against four criteria: *safety* (C_1), *functionality* (C_2), *sustainability* (C_3) and *environment* (C_4). The coeficients of significance of alternatives are as assessed by eexperts. These values are assessed as linguistic and numeric variables that are finally transformed into triangular fuzzy numbers. These values are elements of the fuzzy decision matrix $\tilde{F} = (F_l, F_m, F_u)$ and denotes levels of risk of bridge structure BS_i against criterion C_j (i=1,2,...,5; j=1,2,...,4). The task is to determine optimal scheme (rank order) and coefficients distribution.

$$F_l = \begin{bmatrix} 73 & 38 & 62 & 15 \\ 62 & 62 & 38 & 22 \\ 27 & 73 & 10 & 15 \\ 0 & 62 & 62 & 27 \\ 0 & 0 & 62 & 73 \end{bmatrix}, \quad F_m = \begin{bmatrix} 85 & 73 & 85 & 50 \\ 85 & 85 & 73 & 50 \\ 62 & 85 & 38 & 50 \\ 0 & 85 & 85 & 62 \\ 0 & 0 & 85 & 85 \end{bmatrix}, \quad F_u = \begin{bmatrix} 100 & 95 & 100 & 85 \\ 100 & 100 & 95 & 78 \\ 90 & 100 & 73 & 85 \\ 5 & 100 & 100 & 90 \\ 5 & 10 & 100 & 100 \end{bmatrix}.$$

$$w_l = [0.77 \;\; 0.50 \;\; 0.30 \;\; 0.13], \quad w_m = [0.93 \;\; 0.70 \;\; 0.50 \;\; 0.30], \quad w_u = [1.00 \;\; 0.87 \;\; 0.70 \;\; 0.50].$$

Since the rank order is calculated according to high level of risk, the subsets Ω_h and Ω_c are

$\Omega_h = (C_1, C_2, C_3, C_4)$, $\Omega_c = \emptyset$.

Using computer program FUZZY TOPSIS, developed by authors of this work, corresponding results are obtained that are summarized in the next table.

Table 1. Sumarized results
Tabela 1. Sumarni rezulatti

Rank of altt.	Exp.Rel. Alternat.	Distance ED_i^*	Exp.Rel. Alternat.	Clossen. ERC_i^*	KIC_i %	Coeff of Alternat.	Variation CV_i^*	KIV_i %
1	$A_2=BS_2$	0.1203	$A_2=BS_2$	0.1142	28.7	$A_2=BS_2$	0.9089	28.6
2	$A_1=BS_1$	0.1402	$A_1=BS_1$	0.1322	28.1	$A_1=BS_1$	0.8455	26.5
3	$A_3=BS_3$	0.3141	$A_3=BS_3$	0.2846	23.1	$A_3=BS_3$	0.7510	23.5
4	$A_4=BS_4$	0.7684	$A_4=BS_4$	0.5651	14.1	$A_4=BS_4$	0.4795	15.0
5	$A_5=BS_5$	0.9584	$A_5=BS_5$	0.8129	6.0	$A_5=BS_5$	0.2028	6.4

From this table can be concluded:

- Bridge structure BS_2 (alternative A_2) has the smallest value of the distance form ideal positive solution, i.e solution with highest values of degree of risk;
- Bridge structure BS_1 (alternative A_1) has all characteristic values that are very close to BS_2, so that these two structures have practically the same degree of risk and require the same amount of money for the maintenance;
- Bridge structures BS_4 and BS_5 have smaller characteristic values and smaller level of risk, so that they require smaller amount of money for the maintenance then structures BS_1 and BS_2;
- Rank list made by the expected relative closeness ERC_i^* and by the generalized coefficient of variation CV_i^* in this case are the same;
- Coefficients of investment distribution KIC_i and KIV_i are very close in this case for all bridge structures.

CONCLUSION

Fuzzy TOPSIS method, enables more comlete and flexible modeling of the multiple criteria decision making problems then crisp TOPSIS method. In Fuzzy TOPSIS method can be introduced imprecise input data for the decision matrix and weights of criteria. Proposed method gives to the decision maker more relevant output data than clasic TOPSIS method, which is important to make suitable decisions. This method may be successfuly used for rannikg alternatives and optimally deliver investments on projects, optimal risk assessment of different type of objects, optimal choice of objects for reconstruction, choice of amoost appropriate conatractor on tendering procedure and in many other casaes of multiple criteria decision making.

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