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LOW-CYCLE FATIGUE DAMAGE MODEL FOR DUCTILE MATERIALS

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ABSTRACT: In this paper, a uniaxial material model with the ability to describe progressive damage growth, but also to provide a reliable estimation of fatigue life in the low-cycle regime of loading, is presented. The determination of damage in the material is based on two levels of modeling mechanical behavior. The element on the micro level establishes an elastoplastic damage model that depends on the maximum strain. The second level of modeling is defined by the connection of microelements with different values of total energy dissipated at failure. Hysteretic energy dissipated in heat during cyclic loading is determined for each micro element based on the analytical expression provided by a hysteretic operator. Different distributions of values of maximum dissipated energy can thus provide various fatigue damage evolution laws. The analytical expression for hysteretic energy loss for one element and its numerical implementation is enabled by the computational model whose parameters can be defined by monotonic and cyclic loading experimental tests. Validation of the introduced material model can additionally be concluded by constant and variable strain-controlled experiments, as well as with the comparison of constructed failure curves with existing methods for assessment of mean stress effect in fatigue analysis.

Key words: fatigue, damage variable, hysteretic energy loss, uniaxial stress state

Introduction

In structural analysis, appearance of total failure in element is undesired effect, but in order to evaluate degradation of mechanical properties in material of structure properly, it is necessary to have suitable model for initiation and progressive increase of damage. Although, the material under cyclic loading can exhibit various type of failure modes, the following research is concentrated on ductile materials in uniaxial stress state. Evaluating damage parameter in fatigue analysis based on various continuum damage approach is developed by [1][2][3] , and correlation of specific energy-based approaches, in multiaxial loading cases, is presented in [4]. Cumulative fatigue damage theories in [5] grouped models in six categories, although no clear boundaries exist among some of them, In energy based approach, [6] proposed model takes into account change of hysteresis loop parameters *n, K*. for new approach for PM method in cyclically unstable materials - nonlinear damage accumulation model. In this paper, new energy-based model will be discussed, where the main parameter for fatigue damage is hysteretic energy loss. Presented material model is based on Preisach hysteretic operator [7] and implemented in mechanical model for cyclically stable material [8][9]. This approach for modeling material is to be expanded to the second modeling level by parallel connection of the elements (units) with ability to model elastoplastic damage behavior of the material, based on the strain input.

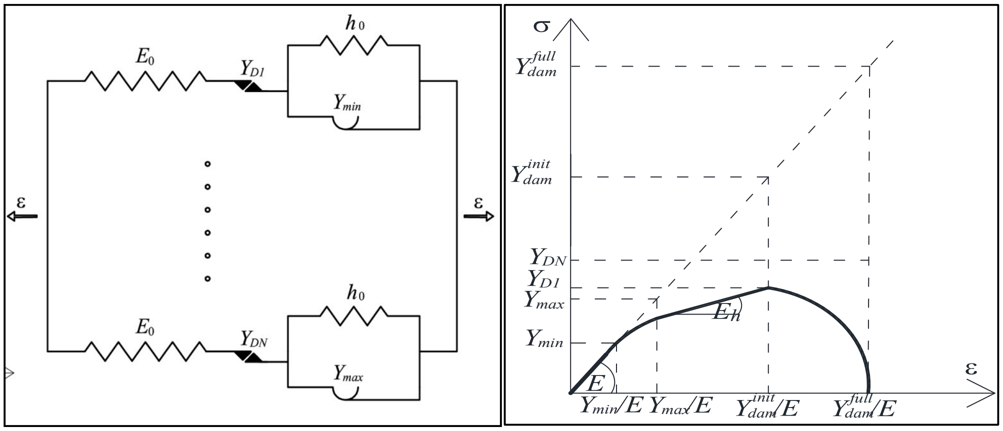
Enhancing elastoplastic damage model with fatigue

One unit of the micro model represents the infinitely many parallel connected cells (as represented in Fig.1) of mechanical model. Starting from analytical expression for stress, based on Preisach hysteretic operator, modified model that accounts effect of fatigue is defined in Fig.2, on the basis of maximum hysteretic energy loss for each unit. Units marked as Pr_1 to Pr_N are defined according to Preisach function and subsequent expression for stress is presented in Eq. (1). This type of function for stress is reproducing mechanical model depicted in Fig.1. Since this approach is based on an analytical expression in closed-form, its implementation enables very efficient numerical analysis.in cyclic loading regime [10].

$$\begin{aligned}
&\overline{\sigma}(t)=\frac{E}{2}\iint_{A1} G_{\alpha,\beta}\varepsilon(t)\,d\alpha d\beta- \\
&\frac{E\cdot(E-E_h)}{4(Y_{max}-Y_{min})}\iint_{A2} G_{\alpha,\beta}\varepsilon(t)\,d\alpha d\beta-\frac{{E_h}^2}{2(Y_{DN}-Y_{D1})}\iint_{A3} G_{\alpha,\beta}\varepsilon(t)\,d\alpha d\beta- \\
&\frac{E_h}{2(Y_{dam}^{full}/E-Y_{dam}^{init}/E)}\iint_{A4} G_{\alpha,\beta}\varepsilon(t)\,d\alpha d\beta \\
&-\frac{Y_{D1}}{2(Y_{dam}^{full}/E-Y_{dam}^{init}/E)}\iint_{A5} G_{\alpha,\beta}\varepsilon(t)\,d\alpha d\beta\,.
\end{aligned} \tag{1}$$

The physical meaning of stress parameters in Eq.(1), is explained in [10].$G_{\alpha,\beta}$ is a hysteretic (switch) operator [11] determined by the input parameters α and β, and *A1-A5* are the corresponding areas in the Preisach triangle described in [8]. Parameter *D* is regarded as scalar damage parameter, and in this case, the linear evolution function of strain for this parameter is imposed by the definition of the model. *D* takes values from 0 to 1 when strain ε grows from the initial value for damage initiation to ultimate value (fracture) of the corresponding mechanical model presented in Figure 1(b).

$$D=\frac{\varepsilon_{max}-Y_{dam}^{init}/E}{Y_{dam}^{full}/E-Y_{dam}^{init}/E} \tag{2}$$



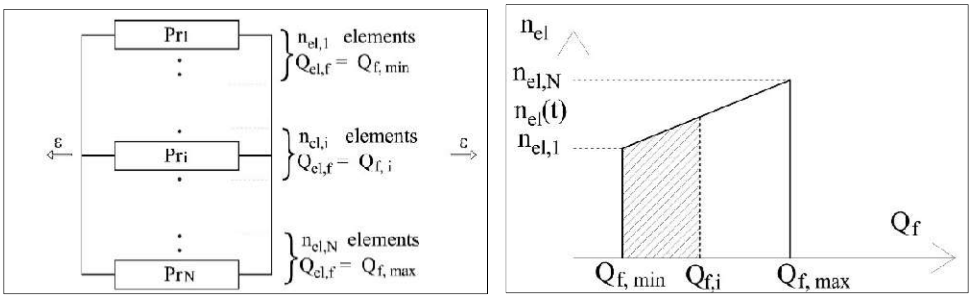
(a) (b)

Fig.1. (a) Mechanical model; (b) Stress strain relation based on the model (a)

Model for determination energy loss in one cycle for one cell (operator) is calculated as volume with base between limit values [8][11]. For presented unit mechanical model (Fig.1), consisting of infinitely many operators-cells, hysteretic energy loss dissipated into heat is calculated:

$$Q_{hys} = \iint_{\Omega(t)} P(\alpha,\beta)\cdot(\alpha-\beta)\,d\alpha\, d\beta \qquad \mathbf{(3)}$$

Since each unit has identical parameters, the hysteretic energy loss is evaluated numerically only once, but the different fracture energy of particular elements govern their elimination from macro model.



(a) (b)

Fig.2. (a) Material model defined by parallel connection of unit elements (Fig.1) ; (b) Linear distribution of number of elements with different maximum heat energy loss (fracture energy)

Macro model is formed by parallel connection of individual elements where elements are grouped according to identical maximum hysteretic energy loss. In Fig.2(a), elements Pr_1, Pr_2,…Pr_j (total $n_{el,1}$ elements) have lowest fracture energy $Q_{el,f}= Q_{f,\min}$, and elements Pr_m, Pr_{m+1},…Pr_N (total $n_{el,N}$ elements) have highest fracture energy $Q_{el,f}= Q_{f,\max}$. In Fig.2(b) distribution of different fracture energy is linear, but in general, it can be in different form: exponential, Weibull, normal distribution, etc. These various types of distribution enable different type of damage evolution laws. In Fig.2(b) shaded area is related to fractured (eliminated) unit elements from macro model, therefore expression for strain can be defined:

$$\sigma = N_{tot}\cdot\overline{\sigma}\left(1-D_{fat}\right) \qquad \mathbf{(4)}$$

where evolution of damage parameter D_{fat} is defined as the relation of number of fractured elements to total number of elements N_{tot} (total area in Fig.2(b). The value $\overline{\sigma}$ is defined in Eq.(1), and it is identical for unit elements. As it was stated, evaluation of hysteretic energy loss from Eq.(2) is also identical for all unit elements, but their failure occurs when total hysteretic loss reach different limit. The total elastic and hardening moduli of macro model are also evaluated in the same manner as stress in Eq.(4). Total hysteretic loss of macro element represents sum of the hysteretic loss of all active unit elements. Based on the experimental results and numerical models for evolution of damage parameter presented in [12][13], parameter D_{fat} needs to be adjusted for specific type of damage growth in low-cycle fatigue loading, which, in this case, can be achieved by different distribution of number of unit elements and corresponding fracture limit values. Since total plastic energy at failure is not constant for most materials [5], the evolution of damage parameter (and fracture energy) could be further modified by taking into account the effect of different levels of strain on fatigue damage. Again, different distributions of this relation (defined by coefficient of distribution c_d) can also enable various material behavior during cyclic loading in fatigue analysis. This coefficient represents influence of maximum strain reached in loading history. Fatigue damage and total hysteretic loss $Q_{\text{tot}}(t)$ at particular point of loading is defined by the corresponding value of distribution function $Q_{\text{el}}(t) = Q_{f,i}$ (i.e. Fig.2(b)), as presented in Eq.(5) and Eq.(6).

$$Q_{el}(t) = c_d\left(\varepsilon(t)\right)\cdot Q_{hys}(t) \quad ; \quad Q_{tot}(t) = \sum_{n_{el,i}} Q_{el}(t) \tag{5}$$

$$D_{fat}(t) = 1 - \frac{N_{el,act}(t)}{N_{tot}} = \frac{CDF(Q_{el}(t))}{CDF(Q_{f,max})} \tag{6}$$

Where for c_d=1, the resulting total fracture energy is constant for every loading history, and in this case evolution of parameter D_{fat} is governed by distribution of maximum fracture energy of unit elements only. Distribution function for parameter c_d (values distributed between 0 and 1) can be defined similar as the fracture energy distribution, although they are independent. However, this distribution influences on total energy of material and number of cycles to failure as well. Therefore, different fracture energy $Q_{\text{tot,f}}$ for different number of cycles to failure N_f can be obtained. Matching experimental results can be achieved with substantial accuracy in both damage evolution (stress-strain curves) and fatigue life estimation. Energy dissipation in once cycle is determined according to the Eq.(3), although it may also be approximated with the area of hysteresis loop [14].

Comparison to experimental results.

Based on the considerable data of fatigue tests of constructional steel, an appropriate model for monotonic loading to failure is determined. Parameters of material plasticity and damage (for defining expression in Eq.(1) and Eq.(2)) are the basis of resolving the parameters of D_{fat}, corresponding distribution of fracture energy limits and parameter c_d. Damage evolution observed in the tests from [15][16] , through degraded stress amplitude results, indicated that the distribution function or fracture energy limits of unit elements (Fig.2(b)) could be approximated with Weibull distribution function. On the other hand, parameter c_d is determined according to total fracture for different strain levels Resulting dissipated energy according to adopted model from Eq.(5) can also be evaluated. Comparison of the experimental results and results obtained in the proposed model is presented in Table 1.

Table 1. Comparison of experimental data (number of cycles to failure N_f) and results of numerical models from variable and constant strain amplitude tests

		S275		S355	
		exper.	num.model	exper.	num.model
N_f [15]	const. ampl. 1%	1216	1303	1140	1109
	const. ampl. 3%	115	100	121	93
	const. ampl. 5%	34	32	30	30
	const. ampl. 7%	12	15	12	15
	variable ampl.	30	32	29	30

		S355		S690	
		exper.	num.model	exper.	num.model
N_f [16]	const. ampl. 1%	4805	4105	1920	1850
	const. ampl. 2%,1.5%	336	475	410	486
	const. ampl. 2%	542	475	160	151

Conclusion

In the presented paper, fatigue damage model is proposed. On the micro level, an elastoplastic damage model is used, where the main input parameter is strain. This unit element is based on hysteretic operator that enables analytical solution (and corresponding numerical implementation) for calculating heat loss. These unit elements with different fracture energy limit can be connected parallel, thus forming material model on macro level. Based on the varying distribution of the total heat loss (fracture energy) limits, different material behavior can be defined. On the macro level, the damage variable for fatigue directly depends on the type of distribution of elements. Evaluation of hysteretic energy loss for different strain levels can additionally be modified, enabling different total fracture energy for materials with different strain histories. It is shown that the proposed model can adequately model stress-strain behavior and estimate fatigue life of ductile materials.

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