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**SINARG 2023**

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**PROCEEDINGS**

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**VOLUME 2**



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ИНЖЕЊЕРСКА КОМОРА  
СРБИЈЕ

## PREISACH MODEL FOR INNER HYSTERESIS LOOPS OF CYCLICLY LOADED MILD STEEL ELEMENTS

**Petar Knežević<sup>1</sup>, Aleksandar Radaković<sup>2</sup>, Nikola Velimirović<sup>3</sup>, Zoran Perović<sup>4</sup>, Nenad Stojković<sup>5</sup>**

### **Abstract**

*In this paper, the new type of Preisach model, that describes the inner hysteresis loops of structural mild steel under cyclic loading, is developed. It is the multilinear mechanical model, that describes closed hysteresis loops typical for structural steel. The analytical model and appropriate Preisach triangle, suitable for engineering practice, were defined. This model was verified by comparison with the experimental results. The experiment has been carried out on structural steel S275 specimens, cyclicly loaded.*

**Key words:** *structural mild steel, cyclic loading, hysteresis loop, Preisach model*

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## 1. INTRODUCTION

Hysteretic response of ductile materials subjected to cyclic loading has been long standing topic of research interest. The model today known as Preisach model is first defined by Preisach in 1935 [1] to describe hysteretical phenomena which occur in magnetism. Shortly is found its application in describing the hysteretic response of ductile materials under cyclic loading. The response of rigid materials with linear hardening, elastic perfectly-plastic materials, and elastic materials with linear hardening has been shown via adequate Preisach models in papers [2] and [3]. Preisach model also found application in defining problems such as cyclic bending [4] and damage modeling under cyclic loading [5]. Problems such as material behavior under monotonic loads also are defined [6], where phenomena like yield plateau and damage growth has been explained.

Generally, Preisach model is using hysteretic operator for defining the cyclic behavior of ductile materials. It is a pure mathematical operator [7], which maps input function  $u(t)$  into output function  $f(t)$  in integral form:

$$f(t) = \hat{\Gamma}u(t) = \iint_{\alpha \geq \beta} G_{\alpha,\beta} u(t) \mu(\alpha, \beta) d\alpha d\beta \quad (1)$$

Where  $G_{\alpha,\beta}$  is an elementary hysteresis operator given in Figure 1. Parameters  $\alpha$  and  $\beta$  are up and down switching values of the input, while  $\mu(\alpha, \beta)$  is the Preisach function. Although  $G_{\alpha,\beta}$  operators have a qualitative feature of local memory, the consequence of joining a large (infinite) number of these operators with the same qualitative characteristics is the formation of the Preisach model with the non-local memory.

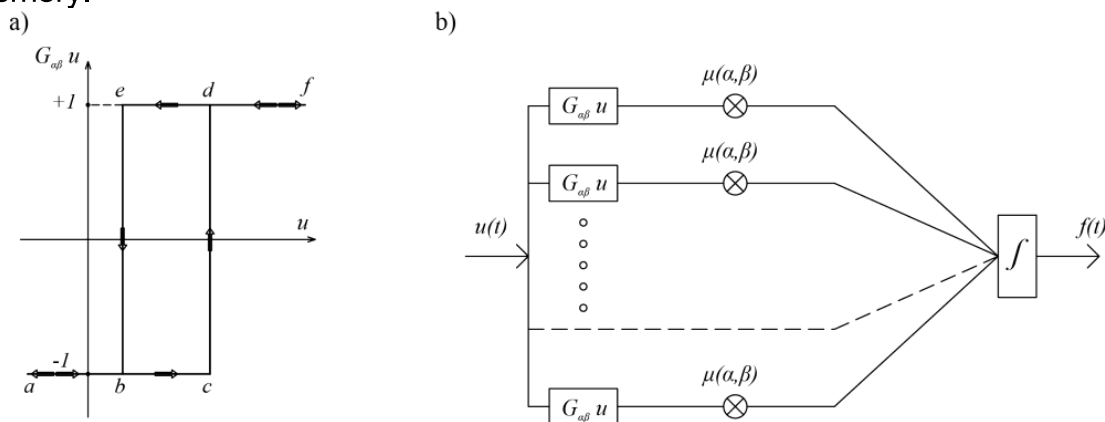


Figure 1- a) Elementary hysteresis operator  $G_{\alpha,\beta}$ ; b) Formation of the final answer in the Preisach model of hysteresis by superposition of elementary hysteresis operators

Existing models are based on bilinear working diagrams and are used for mapping the input function ( $\epsilon$  or  $\sigma$ ) into the output function ( $\sigma$  or  $\epsilon$ ). In this paper, more complex model will be used.

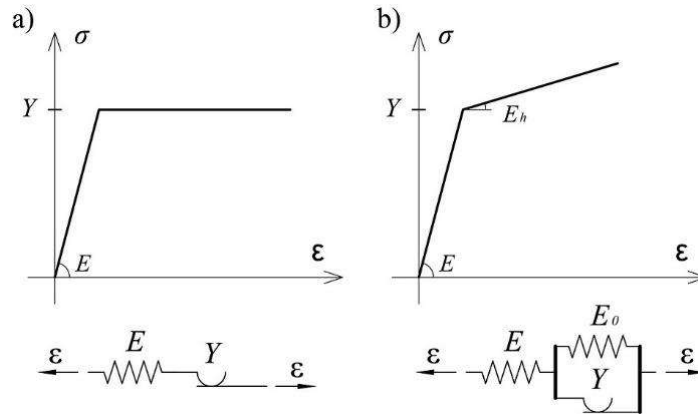


Figure 2 - Working diagrams and mechanical models for: a) ideally elastoplastic material; b) ideally elastoplastic material with hardening

The application of the proposed model is verified by comparison with experimental results obtained by testing cylindrical samples made of one of the most used European structural steel type S275. The first part of the paper represents an introduction to the problem that is analyzed. In the second section of this paper, basic expressions and considerations of the Preisach model for the single crystal under cyclic uniaxial load are shown. In the third section, the polycrystalline model is introduced and finally, verification of the model and comparison with experimental results are presented in section 4.

## 2. SINGLE CRYSTAL PREISACH MODEL FOR INNER HYSTERESIS LOOPS OF CYCLICLY LOADED MILD STEEL ELEMENTS

A new type of Preisach model for response of structural mild steel under constant cyclic loading will be developed in this paper. Its basis is a model that describes the behavior of this type of steel under monotonic loading [7].

The cyclic behavior of the examined steel types is characterized by the formation of regular hysteresis loops, with no yield plateau. A five-element model with a trilinear working diagram, shown in Figure 3, was used in this paper.

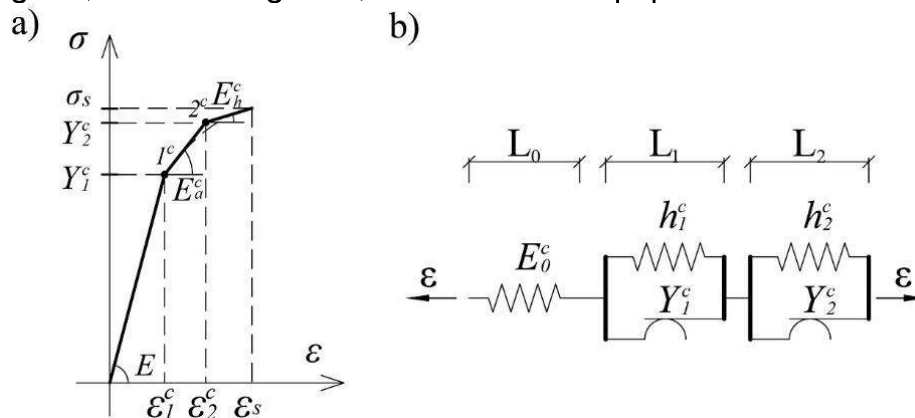


Figure 3. (a) The stress–strain diagram of structural mild steel single crystal under cyclic loading and (b) mechanical single crystal model.

The material properties of the mechanical model, shown in Figure 3b), are defined by Equation (2).

$$\begin{aligned}
 E &= E_0^c(L_0 + L_1 + L_2)/L_0 \\
 E_a &= E_0^c \cdot E_1^c / (E_0^c + E_1^c) \\
 E_1^c &= h_1^c(L_0 + L_1 + L_2)/L_1 \\
 E_h^c &= E_0^c \cdot E_1^c \cdot E_2^c / (E_0^c + E_1^c + E_2^c) \\
 E_2^c &= h_2^c(L_0 + L_1 + L_2)/L_2
 \end{aligned}
 \tag{2}$$

It is possible to define a new hysteresis mechanical model based on the working diagram shown in Figure 3a), with single integrals.

$$\begin{aligned}
 \sigma^c(t) &= \frac{E}{2} \int_{-\varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha} \varepsilon(t) d\alpha \\
 &+ \frac{E_a^c - E}{2} \int_{2\varepsilon_1^c - \varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha - 2\varepsilon_1^c} \varepsilon(t) d\alpha \\
 &+ \frac{E_h^c - E_a^c}{2} \int_{2\varepsilon_2^c - \varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha - 2\varepsilon_2^c} \varepsilon(t) d\alpha
 \end{aligned}
 \tag{3}$$

The first integral of Equation (3) represents the stress due to elastic deformation. The second and third addends describe the behavior of the material after reaching the yield strengths  $Y_1^c$  and  $Y_2^c$ , respectively. The shown solution presents the response of one single crystal due to the cyclic axial load.

### 3. POLYCRYSTALLINE PREISACH MODEL FOR INNER HYSTERESIS LOOPS OF CYCLICLY LOADED MILD STEEL ELEMENTS

According to Iwan [8], a new model represented by a parallel connection of an infinite number of models, where the output of a single model is defined by the expression (3), can be obtained.

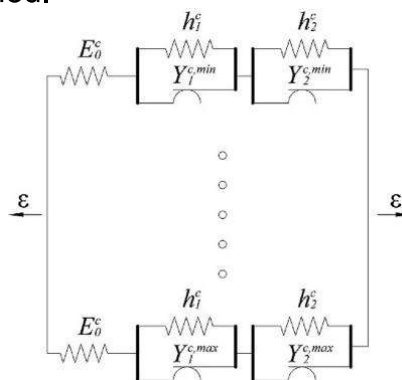


Figure 4 - Parallel connection of infinitely many unit models with different yield limits  $Y_i^{c,min} \leq Y_i \leq Y_i^{c,max}$

The parallel connection of the model is possible due to the deformation  $\varepsilon(t)$  as the input. For a system of infinitely parallel connected models (Figure 4) with different yield limits  $Y_i^{c,min} \leq Y_i \leq Y_i^{c,max}$ , the expression for the total stress becomes:

$$\sigma(t) = \Sigma \int_{Y_i^{c,min}}^{Y_i^{c,max}} p(Y_i^c) \sigma(Y_i^c, t) dY_i^c
 \tag{4}$$

In the expression (4),  $\sigma(Y_i^c, t)$  represents stress corresponding to the single element of the yield limit  $Y_i^c$ , and  $p(Y_i^c)$  is the distribution function of the yield limit. In this way, the material is defined as a polycrystalline, consisting of crystals with different yield limits  $Y_i^c$ , but of the same Young's modulus  $E$ , and the hardening modulus  $E_a^c$  and  $E_h^c$ . The distribution function of all  $Y_i^c$  values is uniform, as in papers [2]–[4], [6], [9], and [10]:

$$p(Y_i) = \frac{1}{Y_i^{c,max} - Y_i^{c,min}} = const \quad (5)$$

the total stress, due to strain as an input, becomes:  $\sigma^c(t) = \sigma^c(t) =$

$$\begin{aligned} & \frac{E}{2} \left[ \int_{-\varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha} \varepsilon(t) d\alpha \right. \\ & + \frac{E_a^c - E}{E} p(Y_1^c) \int_{Y_1^{c,min}}^{Y_1^{c,max}} \int_{2\varepsilon_1^c - \varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha - 2\varepsilon_1^c} \varepsilon(t) d\alpha dY_1^c + \\ & \left. \frac{E_h^c - E_a^c}{E} p(Y_2^c) \int_{Y_2^{c,min}}^{Y_2^{c,max}} \int_{2\varepsilon_2^c - \varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha - 2\varepsilon_2^c} \varepsilon(t) d\alpha dY_2^c \right] \quad (6) \end{aligned}$$

It can be seen that the first part of the expression (6) does not depend on  $Y_i^c$ , and based on the second and third parts, following equalities hold respectively:  $\alpha - \beta = 2\varepsilon_1^c$ ,  $\alpha - \beta = 2\varepsilon_2^c$ .

Based on expressions (2),  $\beta$  can be introduced again, with the change -  $d\beta \cdot (E/2) = dY_1^c$  and  $-d\beta \cdot (E_a^c/2) = dY_2^c$ , where the negative sign of the change is lost to the shift of the integration boundaries:

$$\begin{aligned} \sigma^c(t) = & \frac{E}{2} \int_{-\varepsilon_s}^{\varepsilon_s} G_{\alpha,\alpha} \varepsilon(t) d\alpha + \frac{E(E_a^c - E)}{4} p(Y_1^c) \iint_{A'} G_{\alpha,\beta} \varepsilon(t) d\alpha d\beta \\ & + \frac{E_a^c(E_h^c - E_a^c)}{4} p(Y_2^c) \iint_{B'} G_{\alpha,\beta} \varepsilon(t) d\alpha d\beta \quad (7) \end{aligned}$$

The first part of the expression is the elastic stress, while the integration domains represent the areas of the bands between the corresponding lines in a bounded triangle (Figure 5):

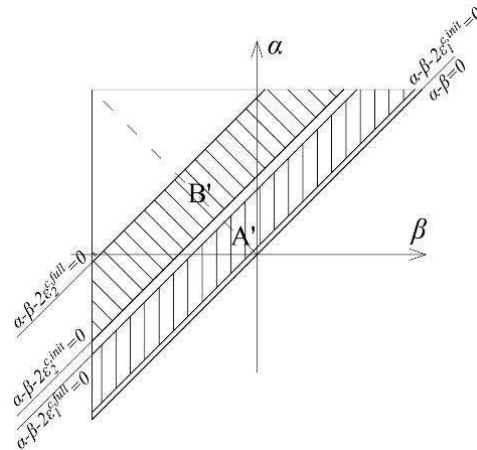


Figure 5 - The Preisach triangle for the material model defined by the expression (7)

#### 4. EXPERIMENTAL RESULTS AND MODEL VERIFICATION

In order to determine the parameters that define the material model correctly, monotonic axial tests are conducted on S275 structural mild steel. The all test



samples are made according to [11]. dimensions of test samples are shown in Figure 6.

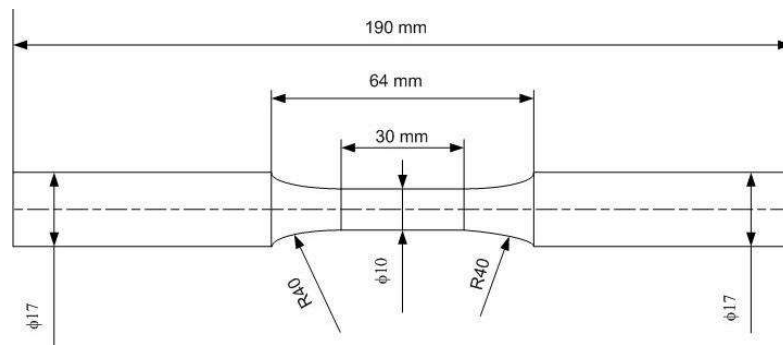


Figure 6 - Specimens dimensions according to [11]

Four samples made of S275 grade structural steel were subjected to symmetrical cyclic stress and a symmetrical load history of the same range  $\varepsilon = \pm 1.5$ . The loading device is SHIMADZU ServoPulser (Figure 7) which is universal tension and compression fatigue testing machine, providing stocky configuration, fine alignment, and restraint of lateral movement of cross-heads.



Figure 7 – Loading device

The extensometer SHOWA-SOKI TCK-1-IF (Figure 8), with gauge length of 25mm, is used for strain measurement. The strain amplitudes are applied with a constant frequency of 1Hz.



Figure 8 – Strain measurement equipment

A comparison between average test results and analytical model results are shown in Figure 9 [10].

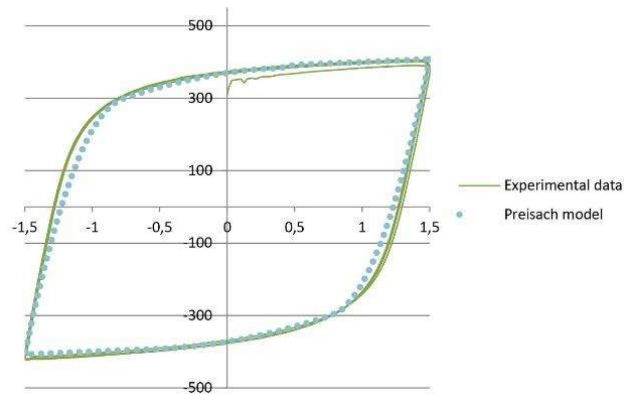


Figure 9- Comparison of experimental and analytical data

## 5. CONCLUSION

The new analytical model that describes the formation of inner hysteresis loops of cyclic loaded structural mild steel elements has been introduced in this paper. Existing Preisach models owing to their simplicity, can not accurately describe the hysteresis loops shape of cyclic-loaded structural steel.

The analytical model developed in this paper belongs to a group of Preisach models, with a very suitable capability of modeling the behavior of structural steel under cyclic loading, which is shown through comparison with experimental results.

Besides the ability to provide excellent agreement with experimental results, the main advantage of this model lies in its convenience for use in engineering practice, as a result of its geometrical interpretation in the form of Preisach triangle.

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