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**MODAL ANALYSIS OF THE SUSPENSION FOOTBRIDGE OVER  
RIVER VRBAS IN BANJA LUKA**

***Abstract***

When evaluating modal properties of lively structures as footbridges, numerical modelling should be used with caution. This is due to model uncertainties mostly related to material parameters, boundary conditions, the effects of the non-structural elements, and also other inevitable differences between as-built and designed structures. Differences between the dynamic behaviour of the as-built structure and the numerical finite element model are best visible after performing vibration testing and modal properties identification based on experimentally obtained results. Conclusions regarding reasons for these differences can be useful for further understanding of vibrational behaviour of footbridge structures and can help designers in better predicting dynamic properties of such structures in the design stage.

This paper describes a lively full-scale footbridge over the Vrbas river in Banja Luka, Prijecani sub-area, Serbian Republic, Bosnia and Herzegovina, its modal properties estimation based on ambient vibration measurements and numerical modelling.

Identification of modal properties was performed using Ambient Modal Testing applying Multiple Test Setup Measurement Procedure and Frequency Domain Decomposition method. The ambient vibration measurement technique for the subject footbridge, which gave a good quality of the results obtained, is presented.

Analysis of the obtained experimental results is given, as well as their comparison with numerically estimated values in the finite element model. Despite some irregular features of the subject footbridge, correlation is obtained, and conclusions are made regarding the effects of numerical model parameters on structural dynamic properties.

***Keywords***

Suspension footbridge, ambient testing, modal properties identification, FE modelling

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## 1. INTRODUCTION

The study presents an analysis of the modal properties of a lively suspension footbridge over the Vrbas river in Banja Luka, in Priječani sub-area. Experimental modal analysis is performed under ambient conditions where only the response of the structure is measured – *Output-only Modal Analysis* or *Ambient Modal Analysis*. Results presented herein - footbridge natural frequencies and mode shapes are obtained from acceleration measurements using *Frequency Domain Decomposition* method in *ARTEMIS* software [1]. These experimentally obtained results are compared with the numerically estimated modal properties of the structure modelled in software *SAP2000*.



Figure 1. Suspension footbridge over the Vrbas river in Banja Luka (Priječani sub-area)

## 2. FOOTBRIDGE STRUCTURAL DATA

Footbridge structural data is obtained from the main repair design, in-situ visual inspection, and dimensional control. The bridge is constructed in 2016, but due to design mistakes, considering pedestrian load cases, it had to be reconstructed, and the works on the reconstruction were finished in 2018. Originally, according to the *Rulebook on technical standards for determination of bridge loads, "Sl. list SFRJ", no. 1/91*, the bridge was designed for  $4 \text{ kN/m}^2$  pedestrian load, distributed on the pedestrian area, but for this type of bridge – suspension bridge, this was not a critical load case for all structural elements [2], [3], which was detected too late. In the main repair design, new elements were added, and bridge vibrations were checked, which conditioned pedestrian load restrictions.

The span of the footbridge is 108.00 m, and the total length, with transitions from both sides, is 120 m. It has typical elements of suspension bridges – the main deck hung by vertical suspenders on two main curved cables in tension which transfer the load on two diamond-shaped pylons, and which are anchored in concrete blocks onshore. The main deck with overhung transitions is supported on concrete blocks at the ends. The bridge layout is shown in Figure 2.

Longitudinal beams of the main deck are placed on the transverse beams, which are hung on suspenders. Longitudinal beams are on 1,8 m distance, and transverse beams are placed on every 4 m. Steel pylons are diamond-shaped and 15,5 m high. The main cable is steel rope  $\varnothing 46$

mm and suspenders are of inox profiles  $\varnothing 18$ , placed on every 4 meters, except around pylons, where are placed on every 2 m. There are also 4 side ropes  $\varnothing 16$ , and 2 side ropes  $\varnothing 10$  on pylons. Bridge details and cross-section are shown in Figure 3.

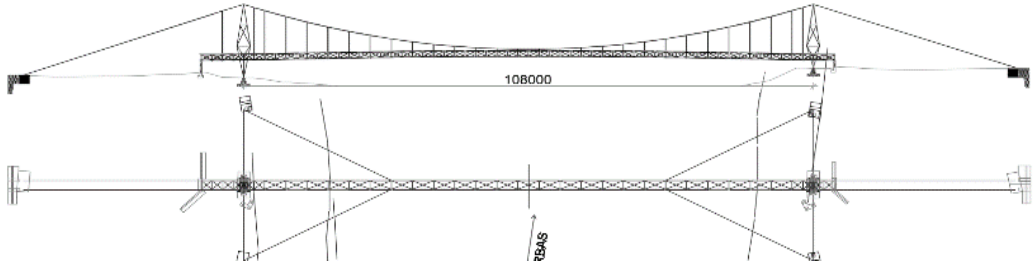


Figure 2. Footbridge layout

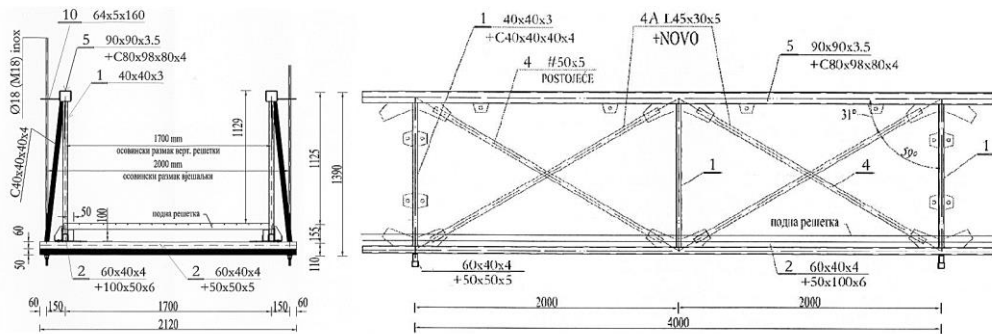


Figure 3. Elements of reconstructed footbridge structure

### 3. MODAL PROPERTIES ESTIMATED BY AMBIENT MODAL ANALYSIS

In the case of large structures, *Ambient Modal Analysis* is a practical solution, since excitation is given from ambient conditions and there is no need for excitation measurement, only the response during everyday use is measured. The traffic does not have to be disturbed, which allows measurements long enough to gain quality estimation of modal properties.

For post-processing of the recorded accelerations, a software package for analysis of vibration measurement – *ARTEMIS* [1] was used, and to define measuring locations and sampling frequencies, a preliminary structural FE model in software package *SAP2000* for structural analysis was used.

The ambient modal analysis procedure was similar to one presented in [4], where it also gave good quality results.

#### 3.1. EQUIPMENT USED FOR VIBRATION MEASURING

Vibration responses were measured using high-sensitive accelerometers with a range of  $\pm 2g$ , produced by the company *Silicon Designs*, model-2240. They were connected to the universal measuring amplifier of *QuantumX* series - MX840A, produced by *Hottinger Baldwin*

*Messtechnik* – HBM with 24-bit resolution and simultaneous sampling. For data visualization, *Catman Easy* acquisition software was used.

### 3.2. TEST SETUP

*Multiple Test Setups Measurement Procedure* [1] was used for ambient vibration measurements, which implies moving sensors from one set of positions to another. To estimate a large number of natural modes and to get clear mode shapes, dynamic behaviour was recorded with 8 accelerometers, differently arranged in 5 different test setups. In every test setup accelerometers were placed on 5 measurement locations, 3 locations with 2 accelerometers measuring accelerations in vertical and horizontal (lateral) direction, and 2 locations with only vertical acceleration measurements.

One reference location is chosen where the modes of interest were predicted to have a significant response level, which was at the quarter of the bridge span. In reference position, two sensors were placed measuring accelerations in the vertical and lateral direction. These reference sensors were kept at the same position during all setups. Their purpose is to adjust the mode shape values obtained in different setups.

Test setups layout is shown in Figure 4.

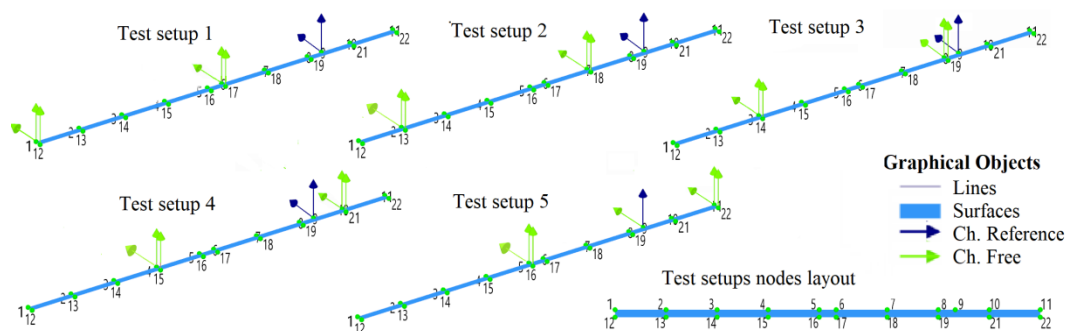


Figure 4. Test setups layout

### 3.3. DATA PROCESSING

As shown, a set of five measurements series were performed, and every measurement series lasted about 1 hour, which results in quite adequate frequency resolution. The sampling frequency was 600 Hz, and the *Butterworth* low pass filter was applied, with a cut-off frequency of 50 Hz.

In *ARTEMIS* software package the *Frequency Domain Decomposition* (FDD) technique was performed. Singular values of the spectral density matrices were obtained and corresponding curves of all test setups are averaged [1]. After inspection of averaged singular values, candidate modes were estimated by the *Peak Picking* method, since there were closely spaced mode frequencies.

### 3.4. MEASUREMENT RESULTS AND ANALYSIS

Singular values of spectral densities for all test setups, obtained in *ARTEMIS* software, are shown in Figure 5, for frequencies in the range 0-10 Hz.

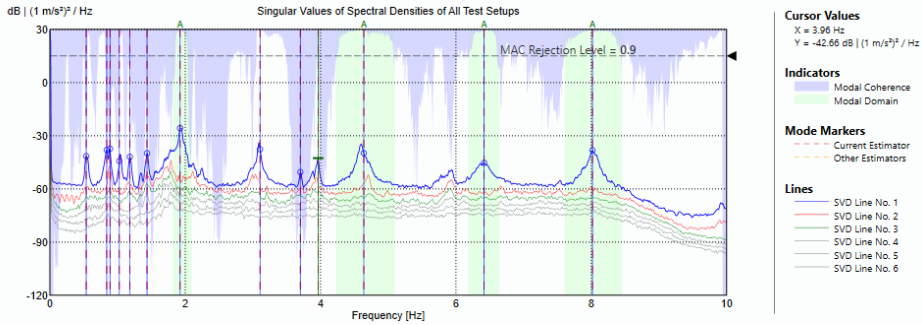


Figure 5. Singular values of spectral densities

Also obtained in ARTeMIS software, graphical presentation of the experimentally estimated mode shapes of the bridge deck for the first twelve modes are given in Figure 6.

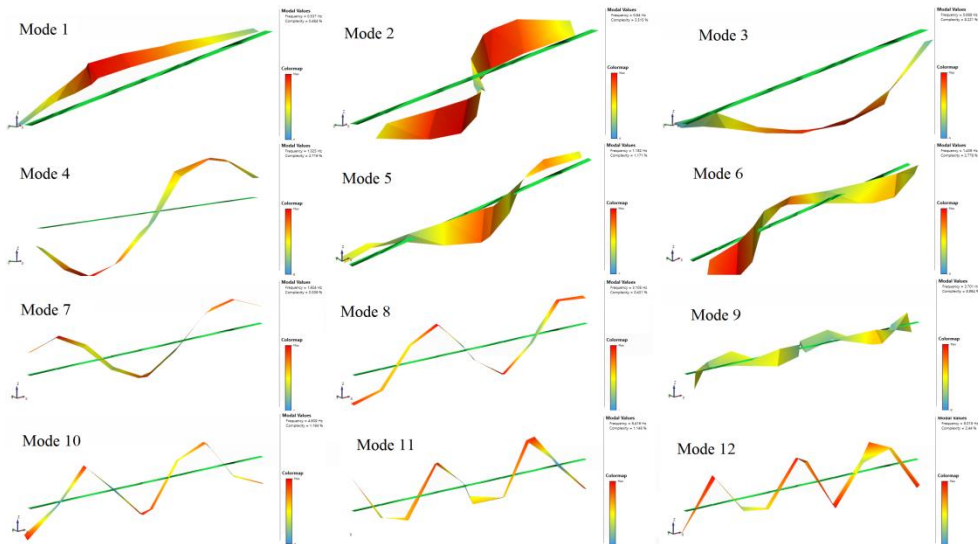


Figure 6. First twelve mode shapes of vibration extracted from recorded data

As presented, clear modal properties of a high number of modes are obtained. To confirm the uniqueness of the mode shapes at extracted modal frequencies, mode shapes were compared using the statistical indicator - *Modal Assurance Criteria* (MAC) procedure, and using such criteria all extracted mode shapes are clearly unique, as all MAC values were close to 1.

In Table 1, the summary of obtained experimental results is shown - estimated modal frequencies with corresponded mode shape characters.

Table 1. Frequency values and mode shape character gained based on measurements, where L marks predominantly lateral and V marks vertical modes

Mode	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Identified frequency (Hz) / mode character	0,537 (1L)	0,840 (2L)	0,889 (1V)	1,045 (2V)	1,182 (3L)	1,436 (4L)	1,924 (3V)	3,105 (4V)	3,701 (5L)	4,639 (5V)	6,416 (6V)	8,018 (7V)

The first two modes are lateral, followed by two vertical modes. It can be noted that obtained lateral mode shapes have torsion admixtures, because of the main cables. Predominantly torsional modes were not noted in results, as they were not recognized precisely, or such modes may be coupled with horizontal modes.

#### 4. NUMERICALLY ESTIMATED MODAL PROPERTIES

Numerical estimation is based on a frame FE model in software package *SAP2000*, where cable element type was used for cable, frame element type for beams, and area element type for the deck.

The preliminary numerical model, also used for experimental test planning, was based just on design documentation and on-site checks. Along with this initial model, a manually updated model is presented hereafter. Manual tuning was based on the comparison between numerically and experimentally obtained frequency values for several first vertical and lateral modes.

As in the experimentally obtained results, because of the main cables, lateral modes in FE model have torsional admixtures, but they can be characterised as predominantly lateral. Unlike in experimentally obtained results, predominant torsional modes in model can be discerned.

Figure 7 shows mode shapes of the first 12 modes of the updated model and Table 2 shows modal frequencies and mode shape types for the initial and manually tuned FE model, also for the first 12 modes.

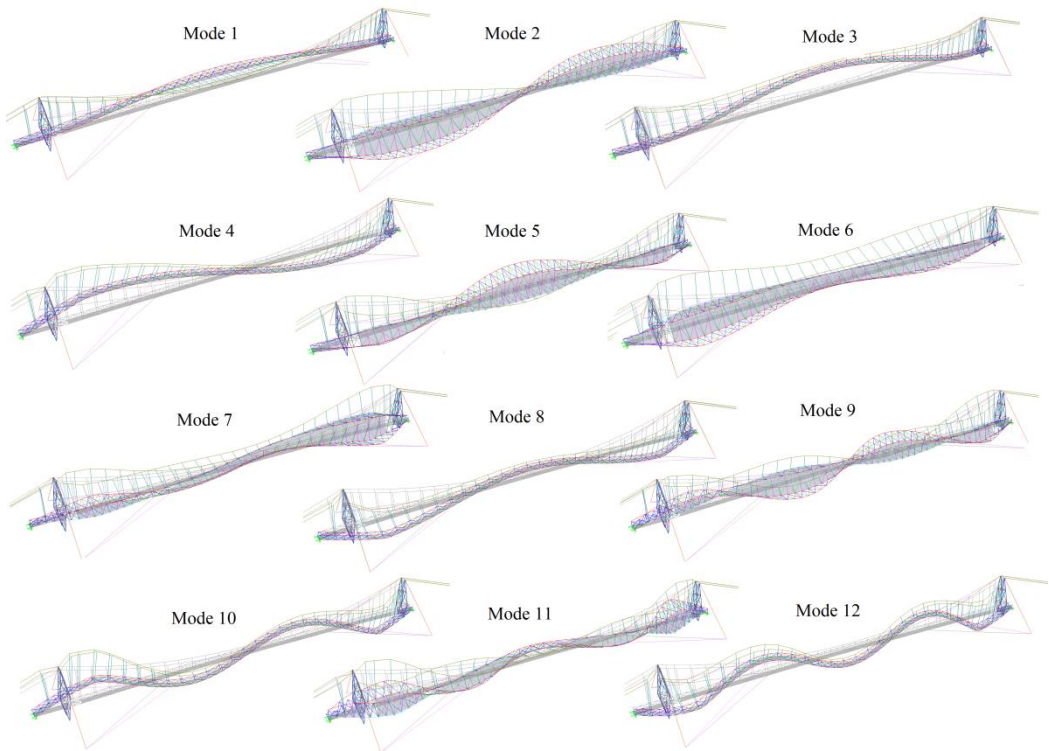


Figure 7. Mode shapes for the first 12 modes numerically determined in *SAP2000*



Table 2. Frequency values and mode shape character obtained in numerical analysis, where L marks predominantly lateral, V vertical and T predominantly torsional modes

Mode	1	2	3	4	5	6	7	8	9	10	11	12
Computed frequencies of initial FE model (Hz) /mode character	0,589 (1L)	0,740 (2V)	0,776 (2L)	1,218 (1V)	1,366 (3L)	1,517 (1T)	1,595 (3V)	1,617 (2T)	2,387 (4L)	2,588 (4V)	2,743 (5L)	3,876 (3T)
Computed frequencies of updated FE model (Hz) /mode character	0,542 (1L)	0,807 (2L)	0,883 (1V)	1,305 (2V)	1,411 (3L)	1,699 (1T)	1,805 (2T)	1,838 (3V)	2,670 (4L)	3,122 (4V)	3,227 (5L)	5,119 (5V)

## 5. COMPARISON OF EXPERIMENTALLY AND NUMERICALLY ESTIMATED MODAL PROPERTIES

The bridge was very slender, its execution was not precise and overall it was later reinforced, so it was hard to define all bridge parameters of significance for modal analysis, even after performing measurements, and obtaining experimentally estimated modal properties. Figure 8 presents some bridge details which parameters can be taken as unknown in FE numerical model.



Figure 8. Characteristic details of the footbridge – deck end supports, pedestrian walk board and side ropes, respectively

In the initial FE model, based on the design documentation, all supports were rigid, and the steel mesh walk board was not modelled as a structural element, but only as load.

In manually tuned FE model, steel mesh walk board was modelled as a structural membrane element with reduced stiffness, and the supports in the longitudinal direction of the main deck were modelled as spring elements. After these modifications in the initial FE model, parameters that were recognized as the ones with the greatest impact on modal properties and which were chosen for tuning were the stiffness in the longitudinal direction of end supports of the main deck, the stiffness of steel mesh walk board, force in the side ropes, and modulus of elasticity of steel. So, in the updated FE model, longitudinal stiffness of supports was tuned, walk board was also given some longitudinal stiffness (along the bridge axis), force in side ropes was decreased up to 50%, and modulus of elasticity of steel was slightly increased.

Table 3 shows comparison of experimentally and numerically estimated frequency values for first five lateral and horizontal modes of vibration. As presented in Table 3, better concurrence is obtained for vertical modes of vibration, compared to the lateral modes. More bridge parameters which affect lateral movements are unknown, compared to the parameters that affect vertical movements.



Table 3. Comparison of experimentally and numerically estimated frequency values

Mode	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
	<i>predominantly lateral</i>					<i>vertical</i>				
Experimentally identified frequencies (Hz)	<b>0.537</b>	<b>0.840</b>	<b>1.182</b>	<b>1.436</b>	<b>3.701</b>	<b>0.889</b>	<b>1.045</b>	<b>1.924</b>	<b>3.105</b>	<b>4.639</b>
Computed frequencies of <b>initial</b> FE model (Hz)	0.589	0.776	1.366	2.387	2.743	0.740	1.218	1.595	2.588	4.209
Computed frequencies of <b>updated</b> FE model (Hz)	<b>0.542</b>	<b>0.807</b>	<b>1.411</b>	<b>2.670</b>	<b>3.227</b>	<b>0.883</b>	<b>1.305</b>	<b>1.838</b>	<b>3.122</b>	<b>5.119</b>

## 6. CONCLUSIONS

The paper shows a method for ambient modal testing of lively footbridge using *Multiple Test Setups Measurement Procedure* based on acceleration measurements and *Frequency Domain Decomposition*. The presented method successfully identified a high number of vertical and lateral modes of vibration. The structure possesses quite low and closely spaced natural frequencies.

Experimental results were compared to the preliminary FE model, based on bridge design documentation and visual and dimensional control. The initial FE model modal properties showed significant differences compared to experimentally obtained properties. Discussion of possible model parameters which cause such differences is given. These parameters are mainly such that they could not be known in the phase of initial modelling, but due to some irregular features of bridge structure and number of unknown properties, they were hard to define even after performing measurements and obtaining quality data estimated experimentally. Based on experimentally obtained results, manual model tuning was performed, and modal properties of the updated model are presented. For this structure, the parameters which influence most strongly the modal frequencies were the stiffness of non-structural elements – pedestrian walk board, force in the side ropes, and stiffness in the longitudinal direction of end supports of the main deck. Better concurrence to experimental results is obtained for vertical modes of vibration, compared to the lateral modes, which is because more parameters that affect lateral movements are unknown.

Manual updating gave somehow improved FE model and pointed to the relevant parameters which have a high influence on modal properties, but there is still divergence in compared data. This is a starting point for further analysis, and eventually some improved model updating method, for more precise determination of model parameters used in tuning.

## ACKNOWLEDGEMENT

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## LITERATURE

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