A PRACTICAL METHOD FOR STRUT-AND-TIE MODELLING OF THE BRIDGE PILE CAP

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A PRACTICAL METHOD FOR STRUT-AND-TIE MODELLING OF THE BRIDGE PILE CAP

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Summary:

Pile cap is a typical example of a discontinuity region, where the classical beam theory does not apply. Therefore, strut-and-tie (STM) model is considered appropriate for the analysis of pile caps. Although, the strut-and-tie method is a powerful tool, one of its shortcomings is the need for guidelines for the standardized models. Otherwise, it is left to the designer to come up subjectively with the STM model, which is a sensitive task. In literature, usually simple loading (axial compression or compression with bending only in one direction) on a four-pile cap is discussed. In practice, for real bridge structure, usual loading is alternate biaxial bending, along with compression, on much larger number of piles, making the typical four-pile caps models useless. A practical methodology for creating pile cap model with large number of piles and with complex loading is presented step-by-step. It is further illustrated for the cases with six piles and eight piles from the actual bridge structures, where the method was applied in practice.

Key words: pile cap, strut-and-tie

PRAKTIČAN METOD DIMENZIONISANJA NAGLAVICE MOSTA METODOM PRITISNUTIH ŠTAPOVA I ZATEGA

Rezime:

Naglavica šipova je tipičan primer zone diskontinuiteta, gde klasična teorija grede nije primenjiva. Stoga se metoda pritisnutih štapova i zatega smatra odgovarajućom za analizu naglavice. Iako je ova metoda vrlo efikasan način proračuna, jedna od njenih mana je neophodnost smernica za standardizovane modele. U suprotnom se ostavlja projektantu da subjektivno napravi model, što je dosta osetljiv zadatak. U literaturi je obično razmatran samo slučaj jednostavnog opterećenja (aksijalni pritisak ili pritisak sa savijanjem oko jedne ose) na naglavici sa četiri šipa. Medjutim, za uobičajeni most iz prakse, uobičajeno opterećenje je aksijalni pritisak sa savijanjem oko obe ose, što pravi ovaj tipičan model sa četiri šipa beskoristan. U ovom radu je data praktična metodologija, korak po korak, za analizu naglavice pomoću metode pritisnutih štapova i zatega sa bilo kojim brojem šipova i sa kompleksnim opterećenjem. Metoda je ilustrovana sa slučajevima iz prakse gde je bila primenjena, sa šest i osam šipova.

Ključne reči: naglavica, metoda pritisnutih štapova i zatega

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1. INTRODUCTION

Pile cap is a typical example of a discontinuity region, where the classical beam theory does not apply. Therefore, strut-and-tie (STM) model is considered appropriate for the analysis of pile caps. Although, the strut-and-tie method is a powerful tool, one of its shortcomings is the need for guidelines for standardized models, since it is left to the designer to come up subjectively with the STM model, which is a sensitive task. In literature, usually simple loading (axial compression or compression with bending only in one direction) on a four-pile cap is discussed. In practice, for real bridge structure, usual loading is alternate biaxial bending, along with compression, on much larger number of piles, making the typical four-pile caps models useless. A practical methodology for creating pile cap with large number of piles and with complex, but realistic loading is presented step-by-step. The method is further illustrated for the cases with six piles and eight piles from the actual bridge structures, where the method was applied in practice. The intent is to make the unique model for one pile cap where numerous load combinations can be easily applied with complex alternating loading.

Strut-and-tie method, in general, is a practical application of the static theorem of plasticity [1]. The consequence is that the solution of the STM model is safe as long as the forces are in equilibrium. The structure will eventually come to the assumed strut-and-tie model force equilibrium, but if it is not the appropriate model, a severe unacceptable cracking and redistribution of forces will occur first. Therefore, a standardized procedure for constructing the STM model for each application (pile cap in this case) is needed, for its accurate and fast application in practice. The strut-and-tie method, when applicable, in general provides usually less required reinforcement than the classical beam theory, and moreover the reinforcement will be on the more appropriate (accurate) locations[2]. The intent of the paper is to be of the immediate benefit to the practicing engineers. The presented method can be used for the design of new bridge structures, as well as for evaluation of the existing bridges. Its application, however, is possible only for pile caps with two or more rows of piles; the single-row pile caps strut-and-tie models are inherently unstable for bending moment around pile-cap longitudinal direction.

2. LITERATURE REVIEW

A brief literature review is presented. In the fib Bulletin 100 [3], a STM model for four-pile cap loaded with only axial compression is discussed. Williams et al [4] studied the STM analysis of the four-pile cap with two load cases, each being compression with bending in one direction. Model from [4] is shown in Fig. 1, where red solid lines represent ties and green dashed lines represent struts; compression and one-directional bending is resolved in four forces applied on the top of the truss elements. Different model is needed for each load case, as the model become unstable once the loading is changed. This model becomes unstable also, for example, when bending in the other direction is added to the shown load combination, to obtain biaxial bending, what is the load case almost always encountered in practice.

Similarly, Schlaich et al [5] proposed a simple STM model for pile cap on four piles, supporting the wall (Fig. 2). Numerous other research has been published, regarding the pile cap STM modelling, but almost always a four-pile pile cap, with simple loading, is considered [6]–[15].

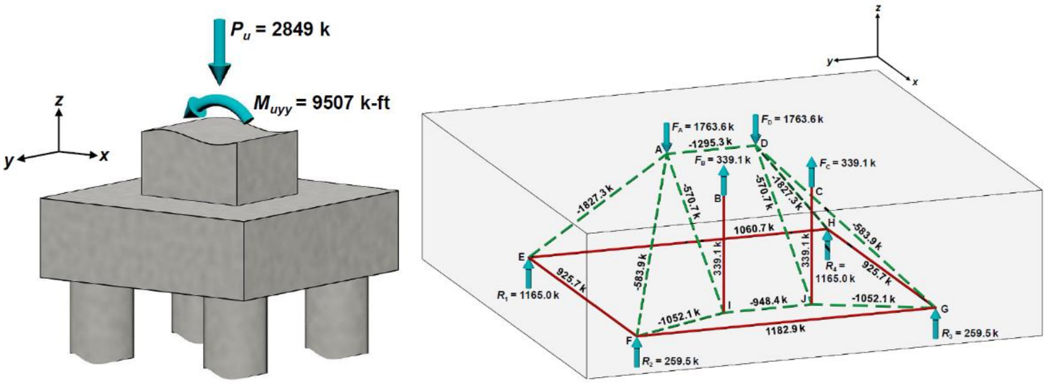


Fig. 1 - A strut-and-tie model for pile cap with four bored piles and one-directional bending load case [4]

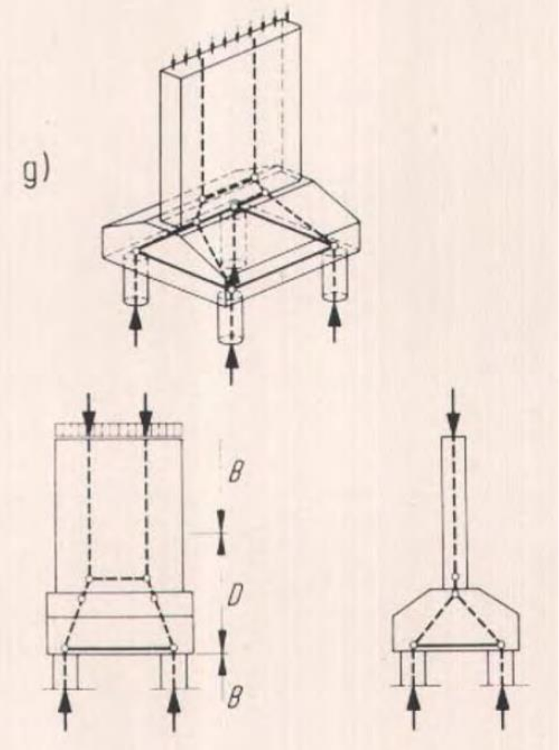


Fig. 2 - Strut-and-tie model for four-pile cap supporting the wall [5]

3. METHOD OVERVIEW

In this chapter, the step-by-step methodology for creating the STM truss models, using a typical finite-element software, for a pile cap with large number of piles and with complex realistic loading will be presented. The final goal is fast creation of safe and accurate STM model of the pile cap with any number of piles and any pile arrangement. The intent is to make the unique model, where numerous load combinations can be applied with alternating biaxial bending and compression force. The design checks for ties and struts according to the applicable code is considered determined and known to the reader/user of the method; they are not the focus of the paper.

However, as the strut-and-tie models are a simplified analogy with the stress field method, 3D node regions should be studied, since several 3D space members intersect in one point, which is not usually the case in strut-and-tie applications. This topic, however, is not the focus of this paper, and it will not be considered herein, although it should be a topic of future research.

The step-by-step procedure is presented below, followed by two examples of pile caps with six and with eight piles on two actual bridge structures, where the method had been applied in the practice.

3.1. STEP 1 – CREATE THE PILES

At the location of piles, create the beam elements with the actual geometric dimensions and concrete material properties of the piles. These are the only beam elements in the model. Appropriate spring stiffnesses representing the soil horizontal stiffness needs to be applied along the piles. Determination of the spring stiffnesses is considered to be outside of the scope of the paper.

3.2. STEP 2 – CREATE MAIN TIES (REINFORCEMENT)

Connect the top of each pile with truss element in an orthogonal grid. These truss elements represent the steel reinforcement connecting the piles at the lower face of the pile cap. The main rule is that the ties need to be in the orthogonal directions to match direction of the usual reinforcement layout. The forces in these ties determine the reinforcement quantity in the strips above the piles (connecting the piles) at the lower face of the pile cap. This is the main output of the analysis.

3.3. STEP 3 – CREATE TOP STIFF PLATE ELEMENT, WHERE THE CONCENTRATED LOAD IS APPLIED

Create the plate element at the connection between pile cap and column. Top surface elevation of the top plate element should correspond to the top surface elevation of the pile cap. The layout dimensions of the plate element should be equal to the layout dimensions of the column. Plate thickness should be one fifth of the pile cap thickness at the column face, and modulus of elasticity of the plate should be ten times the modulus of elasticity of the pile cap concrete. This stiffness is suggested based on the parametric study discussed later, in order to properly distribute the applied loads. This top stiff plate element is the element where the concentrated forces (compression and bending moment in two directions, obtained by the analysis of the global model of the bridge structure) is applied in the centroid of the plate.

3.4. STEP 4 – CREATE MAIN CONCRETE STRUTS

Divide the top stiff plate in (n-1)*(m-1) equal-area segments, where m and n are the number of piles in two orthogonal directions. Consequently, pile grid will be projected to stiff top plate. Connect the top of the piles and corresponding corners of top stiff plate segments with relatively massive concrete strut, using the compression-only truss element. The angle of approximately 45 to 60 degrees from main concrete strut to the horizontal at the pile cap lower face will usually be constructed for pile caps justifying the D-region assumption. Top plate element is connecting the main struts, making them and entire model stable. Modulus of elasticity should be the same as the pile cap material and dimensions should be the same as pile dimensions or slightly smaller. These main concrete struts follow the main compression flow of forces.

3.5. STEP 5 – CREATE PERIMETER CONCRETE DIAGONAL STRUT

Make one bracing per perimeter (inclined) face of the pile cap model (similar as bracing in building design), by connecting the top of one main concrete strut to the bottom of adjacent main concrete strut with compression-only truss element. If, in one direction of the pile cap, there are more than two piles, create two diagonal struts per face, each in adjacent bay, to form a K-bracing. If, in one direction of the pile cap, there are only two piles, create two diagonal struts (not connected to each other, but crossing each other) to form a X-bracing in that one bay. Create bracing in the mid zone of the perimeter face, as pile-cap corners are expected to be more loaded, with the assumption of the pile cap rotating as a rigid body. The material properties should be the same as the pile cap concrete, and cross sectional area should be two or three times smaller than the main concrete struts. These compression-only truss element represent the key elements for the stability of the model for different load combinations with alternate biaxial bending – they are forming, together with the main strut, the “outer core” of the space truss model.

3.6. STEP 6 – CREATE THE VERTICAL TIES REPRESENTING THE PILE CAP ANCHORS TO THE COLUMN

Create the vertical ties as vertical truss element at the four corners of the top plate element representing the anchors from the pile cap to the columns. These ties are connected at the top with stiff plate, and supported at the bottom by three struts located in 3D orientation. These ties usually attract large tensile forces, but they do not require special proportioning for the reinforcement, since they represent the column reinforcement, with significant amount of the cross-sectional area already determined during the column design. These ties will show forces obtained by the column reaction moment arm and are necessary to introduce the column moment in the pile-cap space-truss model.

3.7. STEP 7 – CREATE AUXILIARY DIAGONAL CONCRETE STRUTS

Create auxiliary diagonal concrete struts (as compression-only truss elements) between lower joint of the vertical column ties and the upper joint of the adjacent vertical column tie, forming four “X” bracing in four directions (two intersecting struts forming the bracing are not connected to each other). Together with the vertical ties, these auxiliary diagonals form the “inner core” of the pile-cap space-truss model. The column bending moment reaction is resolved by the compression in the main struts and the tension in the vertical ties (column anchors) on the opposite side – this tension is finally introduced in the space truss system with the “inner core” created in this step. The material properties should be the same as the pile cap concrete, and cross sectional area should be two or three times smaller than the main concrete struts.

3.8. STEP 8 – CREATE BOTTOM LEVEL DIAGONAL CONCRETE STRUTS

Connect the bottom of column anchor vertical ties (from Step 6) with top of the corner piles, with compression-only truss element in a horizontal brace in the level of reinforcement. The material properties should be the same as the pile cap concrete, and cross sectional area should be two or three times smaller than the main concrete struts. These compression-only truss element represent one of the key elements for the stability of the model for different load combinations with alternate biaxial bending.

4. DISCUSSION

Usually, the main challenge is to make the truss model stable for different complex alternate loading cases, i.e. for introduction of the bending moment in the space truss model of the pile cap. In the presented method, bending moment is introduced in the pile-cap space-truss system by the means of vertical column anchors (ties from the Step 6) and auxiliary compression struts (from Steps 7 and 8). The model obtained by the proposed approach is statically indeterminate, bringing the uncertainty in the analysis, corresponding to the appropriate selection of the truss element stiffness and consequent force distribution. However, the use of the compression-only elements for the struts reduces the statical indeterminacy of the system, as often there are zero-force truss elements or truss elements with small value of compression, for certain load combination. Further, the statical indeterminacy of the system (and related consequences of the element stiffness choice), can be justified by the static theorem of plasticity (the base for the STM), stating that the configuration is safe as long as the forces are in equilibrium. Further, for the STM it stands that the structure will eventually come to the assumed strut-and-tie model force equilibrium, but if it is not the appropriate model, a severe unacceptable cracking and redistribution of forces will occur first. However, the model set in this approach and corresponding flow of forces seems intuitive and appropriate, enabling multiple biaxial load combination. An advantage of the proposed approach is that ties are located only in the direction where the main reinforcement already exists in the conventional pile cap reinforcement layout. These are the locations of the horizontal orthogonal reinforcement and vertical column anchors. The use of compression-only truss elements requires the use of nonlinear finite-element analysis, since the stiffness matrix depends on the displacements (i.e. it is not constant). The iterative process for nonlinear analysis is incorporated in most modern finite-element softwares.

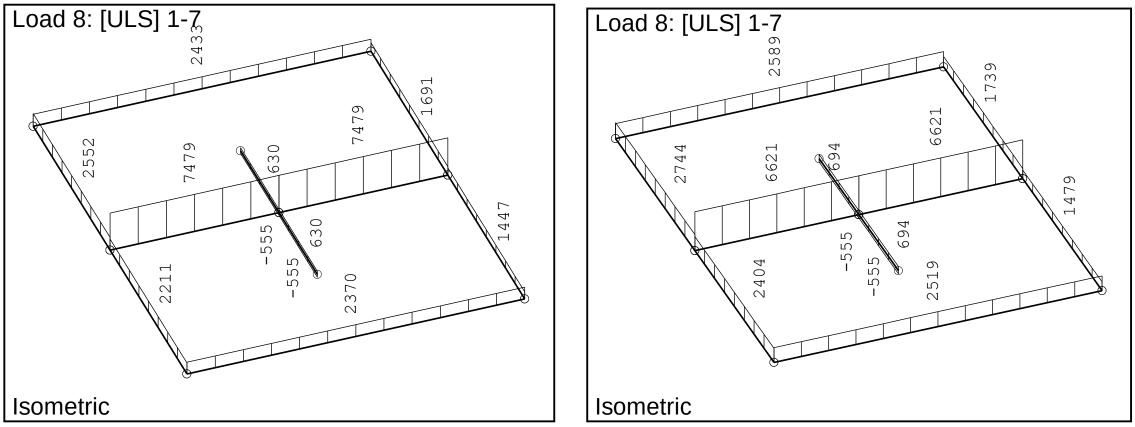
The main output of the analysis is the reinforcement quantity obtained from the tensile forces in the main ties. Compression in the strut usually does not represent a large unacceptable loading (although need to be checked of course), since the pile cap is a massive concrete block, allowing for enough concrete to be activated in the strut. One of the advantages of the proposed method is that the only one model is made, on which different numerous load combinations can be easily applied, allowing for its fast application in practice.

Parametric study of the top plate stiffness, analysing the distribution of the tensile forces in the main ties, was performed. The top stiff plate thickness (t_{plate}) is modelled as one-fifth of the pile cap thickness (t_{pile_cap}) to have geometrically meaningful dimensions, and modulus of elasticity of the plate element (E_{plate}) was varied from the same as the pile cap concrete ($E_{concrete}$), to two, four, eight, ten, and 15 times the pile cap concrete modulus of elasticity, in order to obtain the realistic stiffness. The layout dimensions of the plate element are modeled equal to the layout dimensions of the column. The forces in ties for each case are shown in Fig. 3. It is concluded that relatively soft plate (for example, $t_{plate} = 1/5*t_{pile\_cap}$ and $E_{plate} = E_{concrete}$) does not distribute well the forces between different ties in the horizontal tie network – the most loading goes to the middle ties. However, with increasing the top plate stiffness, the force distribution between different ties is improved, and start to converge at one point, showing well-distributed tie forces. It has been suggested to use one fifth of the pile cap for the top plate thickness and to increase the modulus of elasticity of pile cap concrete by ten times, to obtain the appropriate stiffness of the top plate.

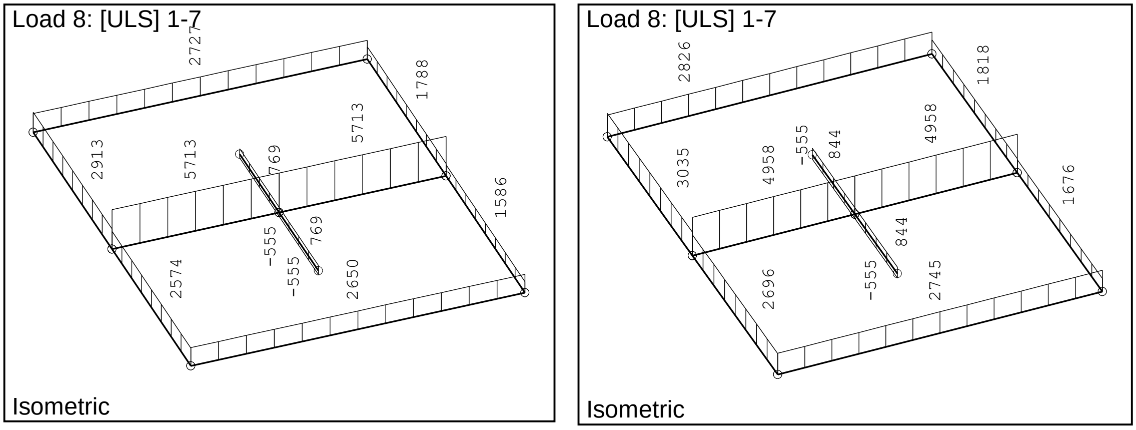
Further, it has been discussed in the literature that zones above the piles (where the ties end) are in highly compressed region (due to pile compression force), so the tie reinforcement can be anchored only by the straight bars extended to the pile cap face [16]. However, from the constructability point of view, it should not be a problem to provide a 90-degree hook. Only one layer of the main bottom-level tie reinforcement is usually necessary. The reinforcement in the main ties is the only reinforcement obtained from the structural calculation – however, reinforcement between strips/main ties, skin reinforcement on side faces, and reinforcement in the top surface of the pile cap should be also provided. It should be in the amount of at least 0.1% of the cross sectional area of the pile cap (a requirement of the Eurocode). For a 2m-deep and 8m-wide pile cap, this corresponds to minimum Ø20/15 for top mat reinforcement and side reinforcement. Often large portion of diagonal struts will not be activated for certain load combination – only those diagonal required for the stability of the model for the specific load case will be activated and will receive the compression force. Its application, however, is possible only for pile caps with two or more rows of piles; the single-row pile caps strut-and-tie models are inherently unstable for bending moment around pile-cap longitudinal direction.

The method can be applied not only to the pile caps with bored piles (drilled shafts), but also to other pile types, such as driven steel HP piles, often used in the U.S.A. [17]. The proposed method is also very useful when additional piles, due to unnapropriate construction of a pile (the case often happened in practice), are added to the pile cap, making pile cap shape irregular.

$t_{pl}=1/5*t_{pile\_cap}$; $E_{plate}=E_{concrete}$ $t_{pl}=1/5*t_{pile\_cap}$; $E_{plate}=2*E_{concrete}$



$t_{pl}=1/5*t_{pile\_cap}$; $E_{plate}=4*E_{concrete}$ $t_{pl}=1/5*t_{pile\_cap}$; $E_{plate}=8*E_{concrete}$



$t_{pl}=1/5*t_{pile\_cap}$; $E_{plate}=10*E_{concrete}$ $t_{pl}=1/5*t_{pile\_cap}$; $E_{plate}=15*E_{concrete}$

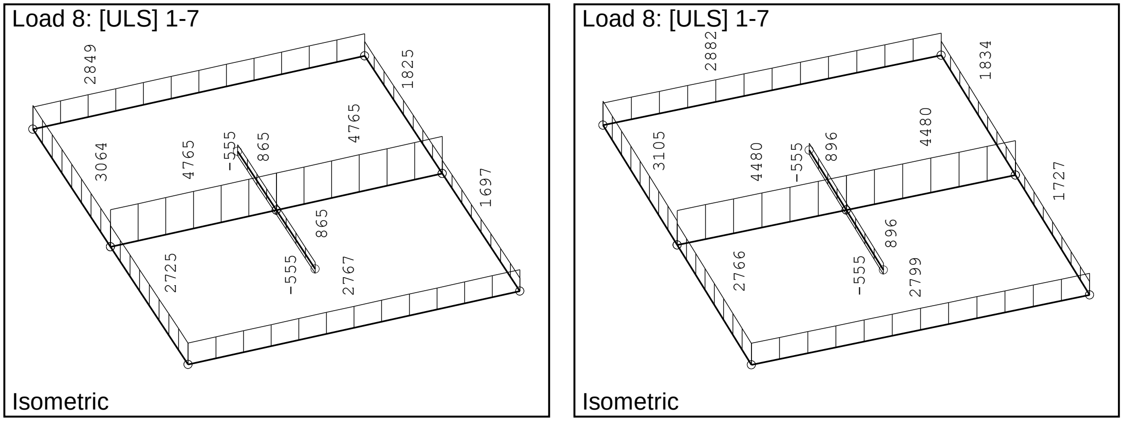


Fig. 3 - Tie forces distribution for different concrete modulus of elasticity and fixed top plate thicknesses

5. CASE STUDY 1 – PILE CAP WITH SIX PILES

The method proposed in this paper has been applied in the practice to the actual bridge structure shown in Fig. 4, for a 2m-thick pile cap with six bored concrete piles with

1.2m diameter. One model is built (Fig. 5), on which concentrated column-base reactions from the global model were applied, for all applicable Eurocode load combinations, including seismic ones.

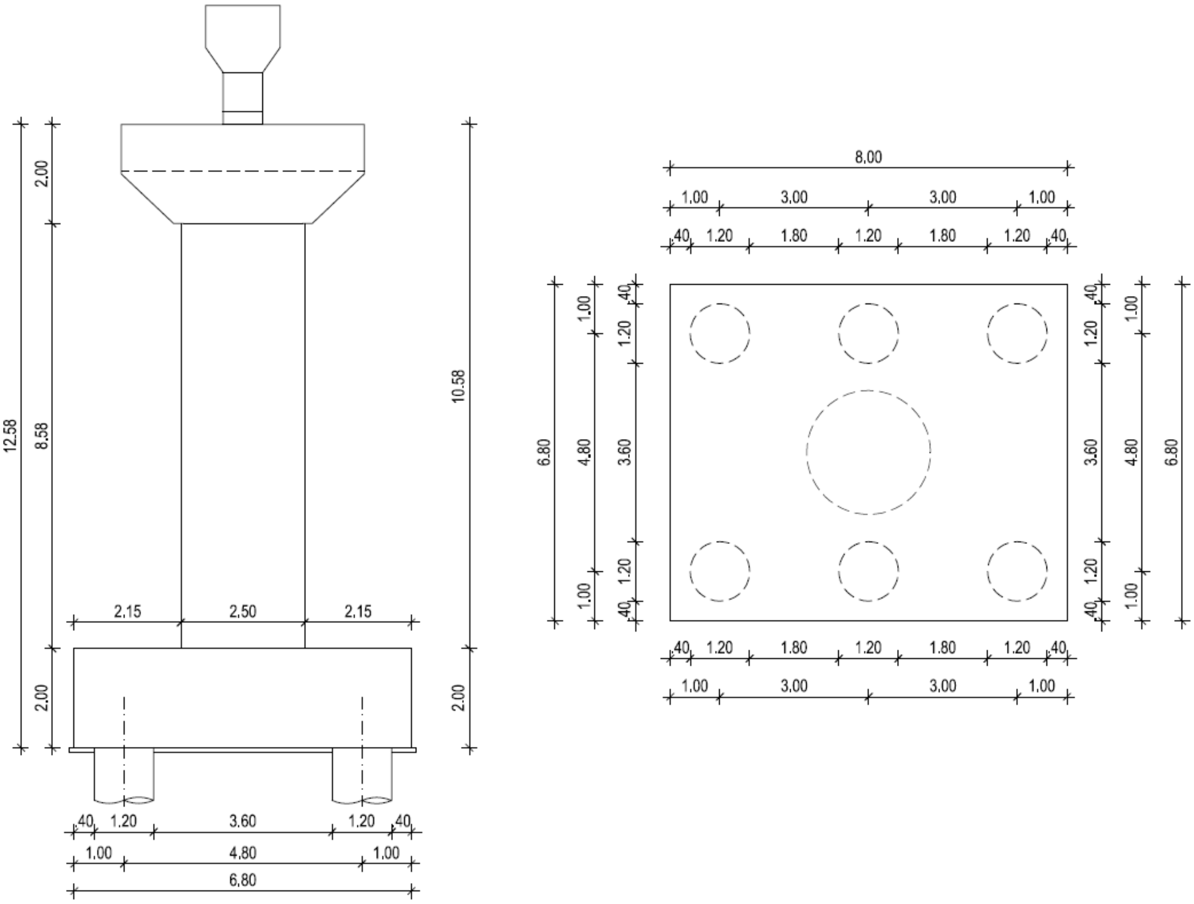


Fig. 4 – Pile cap at bridge at Morava Corridor in Serbia (at km 15+759, near Kruševac)

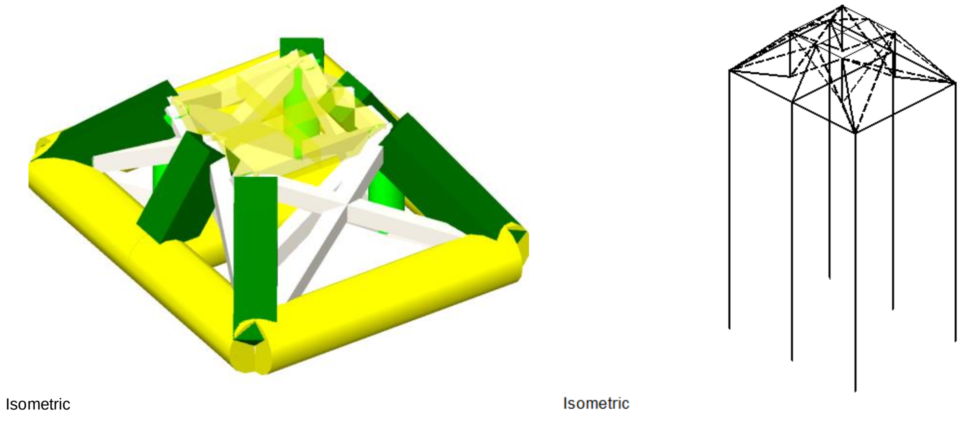


Fig. 5 - 3D rendering of the STM pile cap model with six piles (left full rendering shown without piles present in the model)

In Fig. 6, the decomposition of the model is shown: (1) main ties, in the upper left corner, created in the step 2 of the method; (2) main concrete struts, in the upper right corner, created in the step 4 of the method; (3) concrete perimeter diagonals, in the lower left corner, created in step 5 of the method, forming the “outer core”; and (4) the “inner core”, in the lower right corner, created in the steps 6, 7 and 8 of the method.

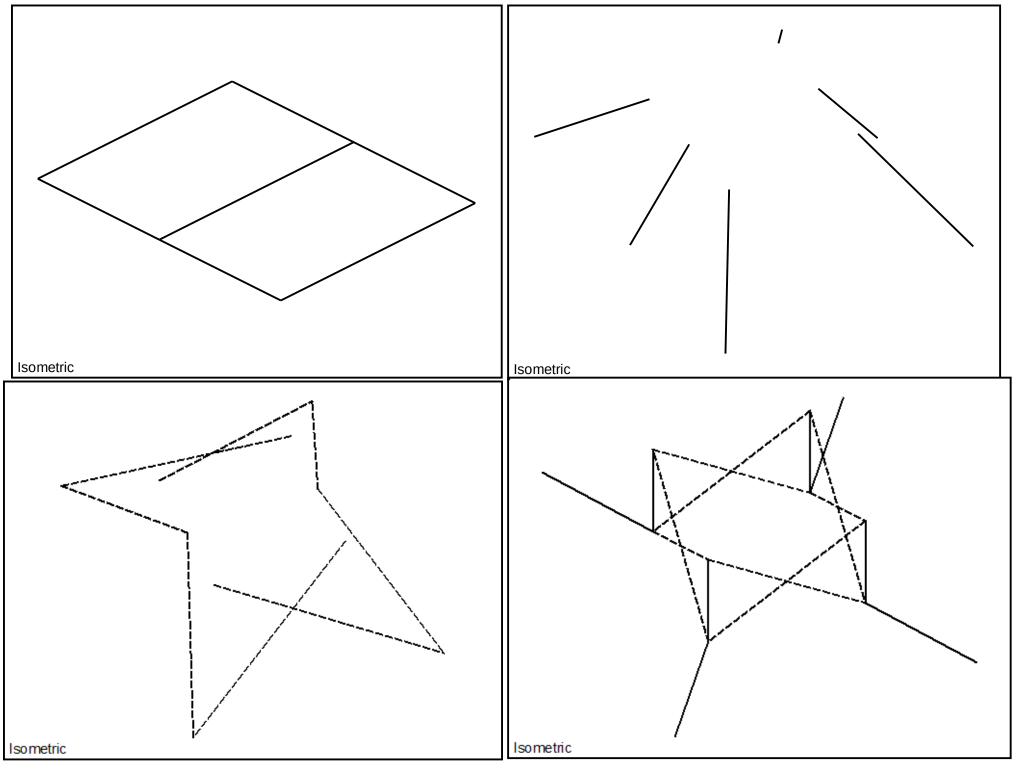


Fig. 6 - Upper left: ties (reinforcement) at strip above the piles; upper right: main concrete struts; lower left: concrete diagonals; lower right: the "inner core"

6. CASE STUDY 2 – PILE CAP WITH EIGHT PILES

The method proposed in this paper has been also applied in the practice to the actual bridge structure shown in Fig. 8, for a 2.2m-thick pile cap with eight bored concrete piles with 1.2m diameter. Again, one model is built (Fig. 7), on which concentrated column-base reactions from the global model were applied, for all applicable Eurocode load combinations, including seismic ones.

In Fig. 9, the decomposition of the model is shown: (1) main ties, in the upper left corner, created in the step 2 of the method; (2) main concrete struts, in the upper right corner, created in the step 4 of the method; (3) concrete perimeter diagonals, in the lower left corner, created in step 5 of the method, forming the "outer core"; and (4) the "inner core", in the lower right corner, created in the steps 6, 7 and 8 of the method.

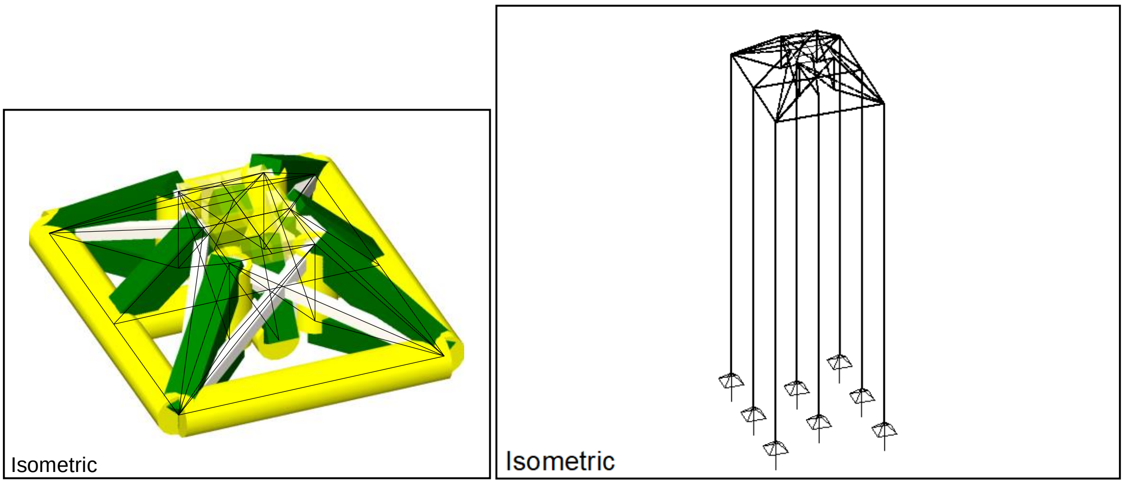


Fig. 7 - 3D rendering of the STM pile cap model with eight piles (left full rendering shown without piles present in the model)

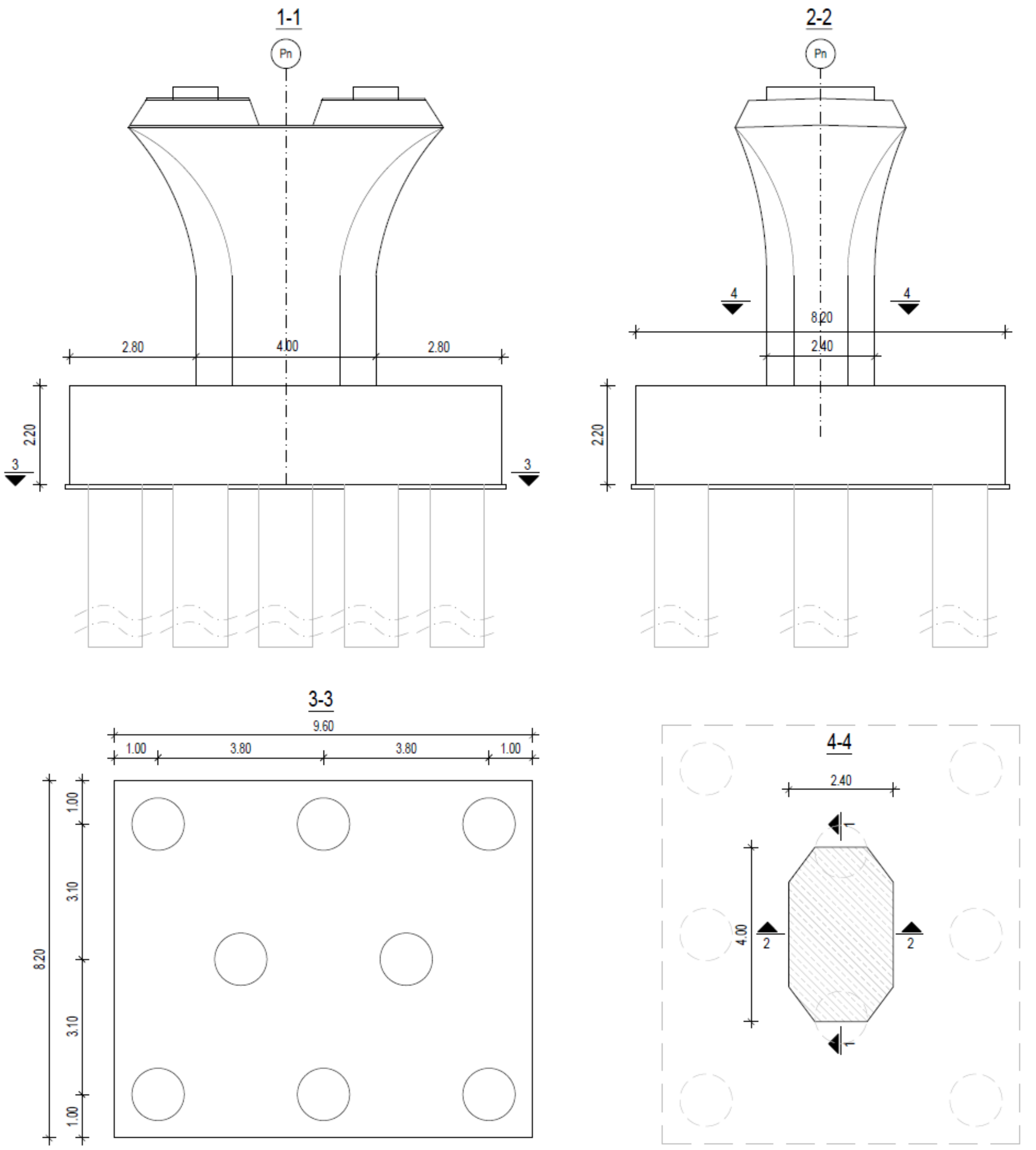


Fig. 8 – Pile cap at bridge at Novi Sad - Subotica - State border high-speed railway in Serbia (viaduct at km 117+155, near Vrbas)

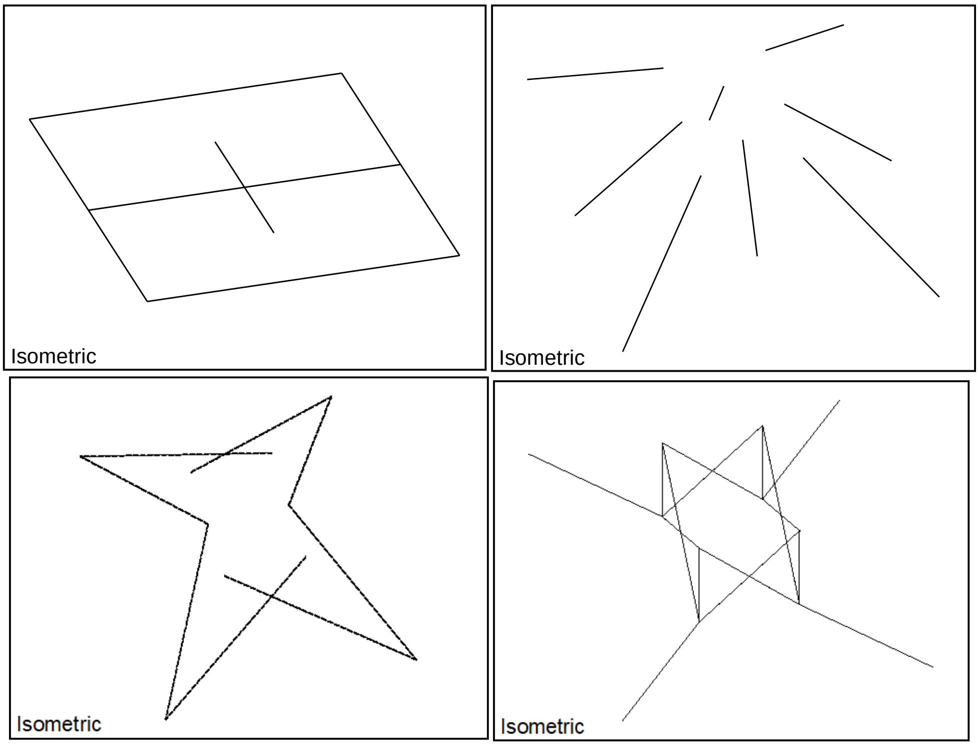


Fig. 9 - Upper left: ties (reinforcement) at strip above the piles; upper right: main concrete struts; lower left: concrete diagonals; lower right: the “inner core”

7. CONCLUSION

A standardized procedure for constructing the STM model for each application (pile cap in this case) is needed, for its safe and fast application in practice. A practical methodology for creating pile cap, usually encountered in practice, with large number of piles and with complex, but realistic loading is presented. The method is illustrated for the cases with six piles and eight piles from the actual bridge structures, where the method was applied in practice. The main advantage of the model is that the unique model for the specific pile cap is made in a fast and accurate manner, where numerous load combinations can be easily applied with realistic alternating loading (compression and biaxial bending). The method can be used for the design of new bridge structures, as well as for evaluation of the existing bridges. The paper is intended to be a practical guideline to the practicing engineers.

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