ELASTOPLASTIC DAMAGE ANALYSIS OF TRUSSES SUBJECTED TO CYCLIC LOADING

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Summary: *In the present paper, implememtation of Preisach model of hysteresis to elastoplastic analysis of trusses subjected to cyclic is shown. It is also shown that damage effects can be included in presented analysis by introducing basic concepts of continuum damage mechanics. Using finite element method, equilibrium equations are obtained and algorithm for numerical solution is defined. Some advantages of this approach are underlined through several numerical examples.*

Keywords: *Cyclic plasticity, Preisach model, trusses, damage*

1. INTRODUCTION

In the present paper, the Preisach model of hysteresis [1], which was already successfully implemented for solving problems of cyclic plasticity of axially loaded bar [3] and cyclic bending of elastoplastic beam [4] and [5], is extended to structural analysis of trusses subjected to cyclic loading. Application of the Preisach model to cyclic behavior of elasto-plastic material was introduced in 1993 by ([Lubarda, Sumarac and Krajcinovic [2],[3]). One of the most important properties of the Preisach operator is the so-called memory map [9], but in addition it is shown in [2] that suggested (Preisach) model also possesses congruency and wiping out property, which makes this model [2],[3] appropriate to describe hysteretic behavior of elasto-plastic material. This model has advantage in comparison with classical approach [8], [11] because of simplicity and strict mathematical rigorous procedure.

2. THE PREISACH MODEL OF HYSTERESIS FOR CYCLIC BEHAVIOR OF DUCTILE MATERIALS

Definition and the most comprehensive analysis of Preisach model of hysteresis can be found in [9]. One dimensional hysteretic behavior of elasto-plastic material can be successfully described by the Preisach model [2] and [3]. Ductile material can be represented in various ways by a series or parallel connections of elastic (spring) and

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plastic (slip) elements, Lubarda, at al. [2]. In this paper, three-element units are used to model elastioplastic material with linear hardening [2]. Therefore, the Preisach function can be determined from the hysteresis nonlinearity as shown in [2] and the expression for stress as a function of applied strain is, consequently,

$$\sigma(t)=\frac{E}{2}\left[\int_{-\varepsilon_0}^{\varepsilon_0} G_{\alpha,\alpha}\varepsilon(t)\,d\alpha-\frac{(E-E_h)}{E}\int_{2Y/E-\varepsilon_0}^{\varepsilon_0} G_{\alpha,\alpha-2Y/E}\varepsilon(t)\,d\alpha\right] \tag{1}$$

The first and second term on the right-hand side of (1) are elastic and plastic stress, respectively. For a system consisting of infinitely many of three-element units, connected in a parallel and with uniform yield strength distribution within the range $Y_{min}\leq Y\leq Y_{max}$, the total stress is

$$\sigma(t)=\frac{E}{2}\left[\int_{-\varepsilon_0}^{\varepsilon_0} G_{\alpha,\alpha}\varepsilon(t)\,d\alpha-\frac{E-E_h}{2}\frac{1}{Y_{max}-Y_{min}}\iint_A G_{\alpha,\beta}\varepsilon(t)\,d\alpha\, d\beta\right] \tag{2}$$

In (2) the integration domain A is the area of the band contained between the lines $\alpha-\beta=2Y_{min}/E$ and $\alpha-\beta=2Y_{max}/E$ in the limiting triangle, shown in [2] and [3]. For results obtained in experiment of cyclic loading of material in stable cycle loop, published in the paper [10], analytical solution was determined based on model of parallel connection of infinitely many elements [2],[3] presented in this paper. In this experiment, sample of Titanium alloy was subjected to strain controlled cyclic loadings $\varepsilon=\pm$ 1.2% and stable hysteretic curves were obtained. By analyzing shape of this hysteresis, parameters for material behavior defined in (2) could be determined by considering geometry of experimental curve [2]. If Preisach triangle is analyzed [2], it can be seen that elastic part of curve's reloading segment always defines constant strain value of $2Y_{min}/E$, while the elastic and nonlinear plastic part of curve's reloading segment give constant strain value of $2Y_{max}/E$. Hence, stress limits Y_{min}=450MPa and Y_{max}=999MPa are defined. Experimentally obtained stable cycle loop was in excellent agreement with one obtained using described model, as it is shown on Fig.1.(a)

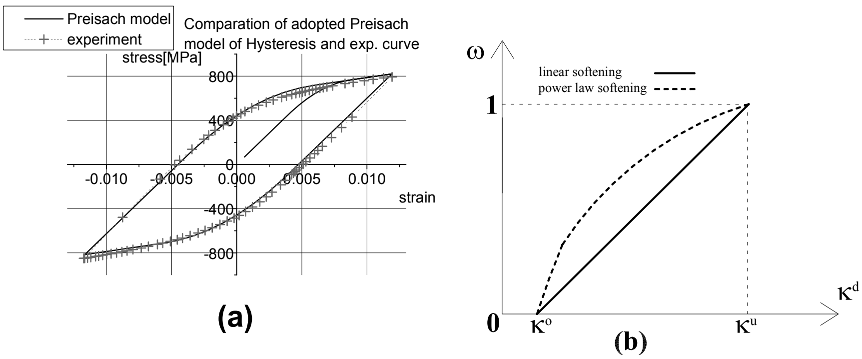

Fig. 1. (a) Results of experiment of cyclic loading of alloy of Tittanium published in [10] and corresponding numerical model ; (b) Damage evolution law for damage variable ω

3. FINITE ELEMENT EQUATIONS FOR TRUSSES SUBJECTED TO CYCLIC LOADING AND DAMAGE

Using principle of virtual displacements, equations for finite element analysis of trusses can be obtained:

$$\sum_{m} \int_{V^{(m)}} \overline{\varepsilon}^{(m)T} \sigma^{(m)} dV^{(m)} = \sum_{i} \overline{u}^{iT} R_C^i \tag{3}$$

where σ represents stresses in equilibrium with applied loads, R_C^i denotes concentrated forces on point i of applied loads, $\overline{u}^i$ denotes virtual displacements, $\overline{\varepsilon}$ corresponding virtual strains and $m= 1,2...k,$ where k is the number of elements (bars). Detailed formulation of algorithm for numerical analysis of trusses subjected to cyclic loading is shown in [6]. If only one element m of structure is analyzed, it is shown in [6] that equation (3) becomes:

$$\overline{u}^{(m)T} \left[\int_{V^{(m)}} B^{(m)T} E B^{(m)} dV^{(m)} \right] \overline{u}^{(m)} - \overline{u}^{(m)T} \left[\int_{V^{(m)}} B^{(m)T} \frac{E\left(E - E_h\right)}{4\left(Y_{\max} - Y_{\min}\right)} \frac{1}{\left(L^{(m)}\right)^2} dV^{(m)} \right] \cdot u_{pl}^{(m)} = \overline{u}^{(m)T} R_C^i \tag{4}$$

It is considered that this problem would not require large displacement and large strain analysis, and if strain displacement matrix B is introduced, expressions in brackets of first and second part of (4) are actually defining elastic stiffness matrix and plastic stiffness matrix respectively. For the finite element assemblage, expression in Eq.(4) becomes

$$K_{el} U - K_{pl} \cdot U_{pl} = R \tag{5}$$

It is important to emphasize that elements of vectors U represent nodal displacements of the global system while elements of vector U_{pl} represent differences of positive and negative sets in corresponding Preisach triangle, transformed in global system [6]. For solving problem of nonlinear static analysis, iterative procedure using Newton-Raphson initial stress method can be applied with appropriate convergence criterion. In presented analysis basic concepts of macroscopic damage is introduced. Simple isotropic damage theory is implemented by introducing scalar damage measure in form of scalar variable ω that evolves from 0 (undamaged material) to 1 (fully damaged material):

$$\sigma = \left(1 - \omega\right) \hat{\sigma} \tag{6}$$

where $\hat{\sigma}$ represents effective stress of undamaged body (in case of elastic or elastoplastic analysis) and σ represents actual stress caused by damage. Effective strain of undamaged body $\hat{\varepsilon}$ is considered to be equal to effective strain of damaged body ε. In this approach for including damage into analysis, the plasticity formulation remains standard. Algorithm for elastoplastic analysis including damage can be defined as explained in [14]. Ductile damage variable ω can be defined as function of damage history parameter κ^d and and it grows from zero to one as the parameter κ^d grows from threshold κ_o to its ultimate value κ_u. Damage evolution can be defined as function that limits elastoplastic behavior in stress space and determines initiation of damage:

$$f^d = \bar{\varepsilon} - \kappa^d \tag{7}$$

where measure $\bar{\varepsilon}$ can be adopted as equivalent plastic strain. The damage growth function governs damage variable evolution and it can be determined experimentally [15] in linear, power law or other form as shown on Fig.1.b.

4. NUMERICAL EXAMPLES

In all numerical examples, material properties for all truss bars are taken from experimental results [10], shown in paragraph 2.

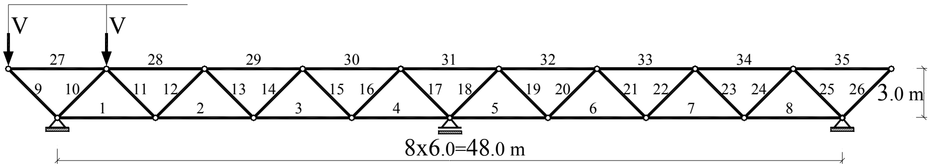

Fig. 2. Geometry and loading of truss structure for all numerical examples

Truss structure shown in Fig.2. is analyzed under moving load pattern of two concentrated forces *2xV (V=6000kN)*. Structure consists of two types of bars. Horizontal bars with length of *6m*, and cross section areas $A_{hor}= 0.02m^2$ and diagonal bars with cross section areas $A_{dia}= 0.015m^2$. Although the applied moving load pattern *2xV* doesn't have cyclic character, bars will be subjected to load reversals, since these concentrate forces move across two span of continuous truss structure. Structure is subjected to five consecutive cycles of loading according to pattern on Fig.2, and three different material models are analyzed and compared. In the first model, only elastoplastic behavior of material is defined, while in second and third model, linear and power law damage evolution for damage variable is coupled with plasticity, respectively.

Parameters for damage variable ω are adopted as follows: κ_o = 0.004, κ_u = 0.3. By analyzing change of the tangent modulus on stress-strain curves, degradation of elastic and hardening modulus can be observed in models that included damage (Fig.3.b.) There is also stabilization of deformation occurred in all three different models of material (Fig.3.a.). It can be seen that resulting behavior is dependent from damage evolution law, so appropriate attention should be made for determination the nature of damage process. Summary and comparison of results according to three different analyzed models is presented on Table 1.

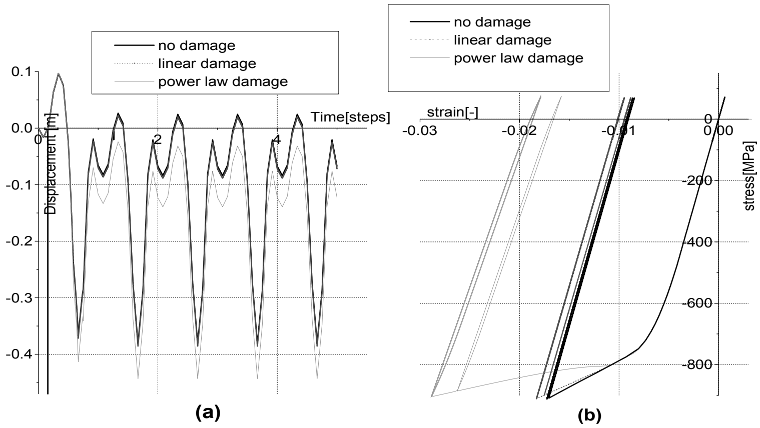

Fig. 3 (a) Vertical displacement of right midspan according to three analyzed models; (b) Stress - strain hysteresis curves for bar 18 according to three analyzed models

Table 1. Results obtained in numerical analysis

	max damage–bar 17 ω	max displacement [cm]	max stress–bar 17 [Mpa]
ElastoPlastic analysis	-	38.10	912
EP with linear damage evol.	0.021	38.58	911
EP with pow.law damage evol.	0.190	44.30	905

If the applied load is increased, damage variables change, according to corresponding evolution law. Consequently significant difference in structural response is obtained, as shown on Fig.4. where truss structure is analyzed under different leveles of applied load.

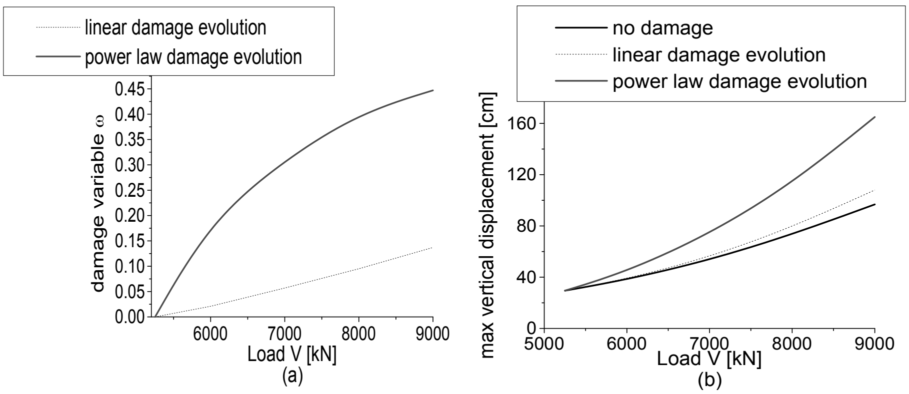

Fig. 4.(a) Maximum damage variable ω (bar 17) vs. levels of applied load on analyzed structure;(b) Maximum vertical displacement (right midspan) vs. levels of applied load on analyzed structure.

5. CONCLUSIONS

In the present paper it is shown that the Preisach model of hysteresis can be successfully applied in structural analysis of trusses subjected to cyclic loading in the plastic range. Presented model is adequate in representing uniaxial material behavior in cyclic plasticity. Damage can be included in presented algorithm by introducing scalar damage variable and basic concepts of continuum damage mechanics. Damage evolution law has high influence on structural responce after damage initiation. It is also shown that the Preisach model can be defined in purely geometric terms, without any reference to analytical definition which is less atractive approach to engineers. Obvious advantage of presented approach reflects in analytical solution in closed form that provides mathematical rigor of the Preisach model, while its absolute equivalent geometric interpretation enables numerical effective solution and less computational cost. Considering all possibilities that Preisach model poses, this type of analysis in finite element procedures is yet to be applied.

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ЕЛАСТОПЛАСТИЧНА АНАЛИЗА ОШТЕЋЕЊА РЕШЕТКАСТИХ НОСАЧА ПРИ ЦИКЛИЧНОМ ОПТЕРЕЋЕЊУ

Резиме *У овом раду, прказана је примена Прајзаковог модела хистерезиса, у структурној анализи решеткастих носача који су изложени цикличном опетрећењу. Такође је приказано да се ефекти оштећења могу укључити у приказану анализу увођењем основних принципа механике оштећења у континууму. Користећи методу коначних елемената, једначине равнотеже и алгоритам за нумеричко решавање је дефинисан. Неке предности оваквог приступа су наглашене кроз неколико нумеричких примера.*

Кључне речи: *Циклична пластичност, Прајзаков модел, решеткасти носачи, оштећење*