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APPLICATION OF CURVES AND SURFACES OF HIGHER ORDERS OBTAINED BY INVERSION IN THE PRACTICE OF ARCHITECTURE

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ABSTRACT: Throughout the world, teams of architects are constantly competing to find the most attractive form of future buildings that apart from being functional must meet the criterion of aesthetic beauty. There are numerous architectural objects that are thought to be works of art and their beauty and shape are not only the pride of their authors, they are symbols of the cities and states in which they were built. In order to make the realization of the designed objects possible, geometric shapes (curves and surfaces) must have clearly defined geometry and they must satisfy a number of structural requirements. The principles of their design and construction define the purpose to which they will be used and determine their usability. The major problem that designers face is how to set the mechanism for the design of surfaces and their contours which are curves of different orders
The advance of digital technology in the last twenty years has led to formal freedom in the design of buildings and in their representation in the virtual space. The combination of non-standard geometry and CAD tools has enabled us to express and realize architectural structures in new and modern ways. Transformation of doubly curved surfaces into structural and physical objects always entails huge costs and serious problems both of geometric or static nature.
The paper explains the ways surfaces of higher orders are constructed and used in the practice of architecture. The basic transformation is inversion which is interpreted in two ways: as quadratic transformation in the classical projective geometry and as pure symmetry in the relativistic geometry, with constant comparison of the two geometric systems and their opportunities for explications and generalizations.
NURBS curves are used in the computer graphics to represent various forms needed in engineering, design and animation. NURBS or Non - Uniform Rational B -Splines are groups of curves which are due to its flexibility and accuracy used in all processes of modeling, from artwork to production. NURBS curves have precise and well-known definitions. They are studied in mathematics and there is even a whole new field called NURBS geometry, which is studied by mathematicians and programmers. The main advantage of NURBS lies in their ability to provide an accurate presentation of the standard geometric shapes such as circle, ellipse, spherical surface, or torus, as well as representations of free-forms such as forms of industrial products or a human figure. The surfaces obtained by inversion consist of (one, two, or three) circular cross-sections and therefore they have certain advantages both in the design and in the manufacturing phase compared to the surfaces designed using NURBS geometry.
We cannot talk about the application of various surfaces in architecture without mentioning diagrid. Diagrid is a structural system whose diagonal members are connected in such a way that they act like a weaving or knitting network which can `wrap` around various geometric forms whether they are obtained by the inversion, or in some other way. Diagrid a structural system that has been widely used for newly constructed high-rise buildings made of steel and it consists of triangular structures

with diagonal support beams.

Keywords: Inversion, Axial symmetry, Curves of the 3rd and 4th order, Surfaces of the 3rd and 4th order

1. INTRODUCTION

Construction is the language of architecture, or as Auguste Perret said "Construction is the mother tongue of the architect".

Figure 1: Phoenix International Media Center

Following previous scientific papers (M.Obradovic) [17] the geometry of high-order curves and surfaces obtained by inversion has been compared with the structure of constructed buildings and the possibility of applying these forms in the practice of architecture was analyzed.

One of the best illustrations of the use of high-order surfaces in architecture would be the Phoenix International Media Center (Beijing), which is a surface of the 4th order (Fig.1). This object leaves its mark on the urban landscape of Beijing. Photos were taken from: http://www.-coolhunting.com/design/phoenix-international-media-center.php . The building's sculptural shape originates from the "Mobius Strip" (August Ferdinand Möbius, 1790-1868). This surface is of the 3rd order, but the architects have kept only the outline of the strip, because an architectural object cannot be built as a strip. The building was constructed in the diagrid structural system. This system will be described later in the paper. The main directions of the construction elements are calculated as isolines with the degree of leaning that is the most suitable for a diagrid construction. Parametric engineering has become the standard of building design nowadays. Different geometrical forms, such as surfaces designed using NURBS geometry are feasible. Diagrid, as a new system, seems to be the most suitable for the construction of such buildings.

2. CONSTRUCTION SYSTEMS USED IN THE PRACTICE OF ARCHITECTURE FOR THE SURFACES OBTAINED BY HARMONIC SYMMETRY

The advance of science and technology has permitted the use of complex geometric forms in the practice of civil engineering and architecture. This technological leap has been achieved primarily thanks to the invention of flexible building materials, such as sheet steel used for wall screens and roof cladding on the buildings that are cylindroids in shape. The increasing use of reinforced concrete with pre-stressing presented the major milestone. It allowed the construction of doubly curved surfaces of large spans. Besides these materials, laminated timber is also bendable and as such it can be used for the construction of such surfaces. Plastics and reinforced tarpaulin are among other materials that are used for tents and pneumatic structures. These materials can also be used to design both simple and complex geometric surfaces [18].

Linear construction elements of future structural systems used in the practice of architecture and construction are shaped by the geometry of curves of the second, third, fourth or higher orders [9].

A building can be designed by using parts of surfaces which are then fitted into a whole. They can make complex structural assemblages. In this way, we can make free forms, which are, in practice, still limited by technology and building materials. However, these limitations have become less strict in the recent years with the advent of new materials and new structural systems that are used in architecture and construction. An increasing number of buildings are designed in freeform, and many of them (both the finished ones and the ones that are still in projects) win us with their unusual shape and beauty.

Surfaces obtained by harmonic symmetry consist of systems of circular cross-sections that define the surface (one, two, or three). Circular cross-sections are easy to use (Fig. 2 and Fig. 3) [5].

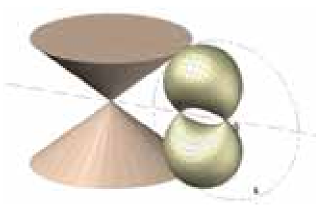

Figure 2: Model of the cone and its harmonic equivalent – spindle cyclide

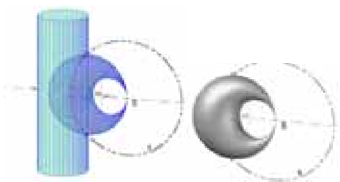

Figure 3: Model of the cylinder and its inverted Dupin Cyclide

On the basis of extensive literature on structural systems the following text provides an overview of structural systems that can use harmonically generated curves and surfaces.

The shell structure has a three-dimensional (stiff) support structure, which carries load primarily with the forces in the plane (uniformly distributed across the thickness of the shell), and sometimes with bending but only in the zone of leaning on or joining the other elements. The shell thickness ranges from less than 1/50 to1/100 of the whole structure and there is only the direct membrane stress (tension and pressure), without bending moments (or they can be ignored), but only if the shell is uniformly loaded and supported. To achieve a state that resembles the state of a membrane stress in the shell, the requirements of the membrane theory must be taken into account when selecting the surface or its parts.

With the proper selection of geometry, shells with small thickness can be extremely efficient elements concerning the cost of materials because thanks to their form and reinforcement they have a large carrying capacity despite their thinness. In general, shells can be formed from a variety of surfaces that are characterized by Gaussian curve measure, the product of the curves of the main directions *K*.

When it comes to the shell structural system, the use of pre-stressed reinforced concrete has produced structures with a longer span, which could not have been even imagined with the use of ordinary concrete.

This paper studies several examples of modern architecture and points to the possibility of their further application. Special attention is focused on doubly curved surfaces obtained by harmonic symmetry - with the positive Gaussian curvature from the harmonic group of cylinders and with the negative Gaussian curvature from the harmonic group of paraboloids. Doubly curved surfaces with the positive Gaussian curvature resulting from the harmonic inversion of the revolving cone - spindle cyclide (Fig. 2) and the revolving cylinder - Dupin cyclide (Fig. 3) are suitable for the shell structural system.

Hanging structures are hung on solid supports installed at great heights, where the main supporting elements are stretched. Supporting elements are small dimensioned, usually steel cables with high tensile strength. This structural system is most commonly used in roofing constructions. The idea of reverse dome has existed since the time of the Incas and the era of ancient China. The surface shown in Fig. 4, which is a harmonic equivalent to the parabolic cylinder, can be used in hanging structures with two systems of pre-stressed high tensile cables[8].

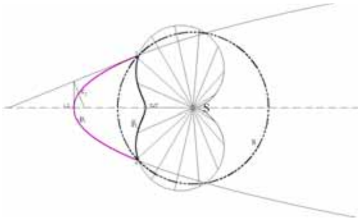

Figure 4: Harmonic equivalent of the parabolic cylinder – the surface of the 4th order with a spike

Harmonic symmetry of the revolving cylinder with the center of inversion ***S*** on the contour generating line of the cylinder and the radius of the sphere of inversion - ***s*** being the same as the diameter of the circular section of the cylinder produces the 3rd order surface which is suitable for hanging structures (Fig. 5) [13].

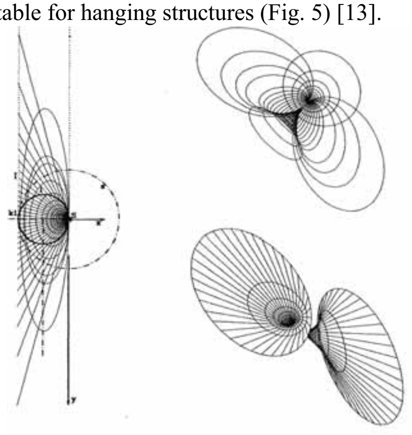

Figure 5: Harmonic equivalent of the revolving cylinder – the surface of the 3rd order

Pneumatic structures – The operation of pneumatic halls and facilities, in general, is based on the vault shaped membrane that is supported by internal air pressure. Pneumatic structures are often said to be airborne. These structures belong to the group of natural structural systems and they are commonly found in zoology and botany. Human muscle tissue and skin are, for instance, stretched under pressure and supported by the solid skeleton.

Pneumatic structures make a special field of textile architecture. Being specific and ingenious in many ways, these structures are in some of their applications simply irreplaceable. They were first used in aeronautics, for the construction of dirigibles and balloons. Although these structures have never come into wide use, they were revived in the form of buildings, during the 80`s of the 20th century. Although some of their properties have often been disputed, pneumatic structures are still famed for their quick installation and deinstallation and their low cost.

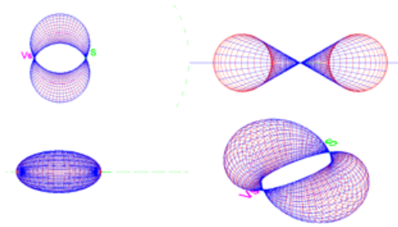

Figure 6: Surface of the 4th order – harmonic equivalent of the revolving cone

Pneumatic structures can use surfaces with two systems of circular cross-sections, which are marked red and blue (Fig. 6). There are three surface projections and one axonometric projection. The main structural feature of this surface is a double curvature with the positive Gaussian curvature (the radii of the double curved surface is on the same side of the tangent plane). A series of spheres can be inscribed into the surface, so that the surface touches the surface of the sphere. This harmonically generated surface satisfies both conditions - circular systems are meridians of individual inscribed spheres [13].

3. APPLICATION OF HIGHER ORDER SURFACES OBTAINED BY HARMONIC SYMMETRY IN LANDSCAPE ARCHITECTURE AND DESIGN OF AMBIENT UNITS

“Cloud Gate” is a public sculpture made by Indian-born British artist Anish Kapoor. The sculpture is located in the Millennium Park in

Chicago in the state of Illinois, USA. It was built between 2004 and 2006 and named "The Bean", because of its legume-like shape. It is composed of 168 welded plates of stainless steel, and there are no visible seams on its highly polished exterior. The sculpture is popular with tourists because of its unique reflective properties.

This paper analyzes the bean curve shown in Figure 7, obtained by harmonic symmetry of an ellipse with the corresponding position of the center and the circle of inversion.

When the center of inversion ***S*** is set beyond the ellipse, its inversion produces a curve of the 4th order – the bean curve[6]. Since the curve has an irregular shape we might think it is asymmetrical. However, it is harmonically symmetrical with the given ellipse and with all the curves that are harmonically symmetrical with the ellipse. This bean curve is centrally symmetrical with respect to the points $\overline{C}_1$ and $\overline{C}_2$= S, and it has two circular axis of autosymmetry ($\bar{a}_E$ and $\bar{b}_E$). The point $\overline{S}$ is an isolated antipodal point of the ellipse, and the point S is an isolated point of the curve of the 4th order[16]. The osculatory circles O_1, O_2, O_3 and O_4 are inscribed into the ellipse apices 1, 2, 3, and 4. These circles are inverted into the osculatory circles of the 4th order curve in the points $\overline{1}$, $\overline{2}$, $\overline{3}$ and $\overline{4}$ [7].

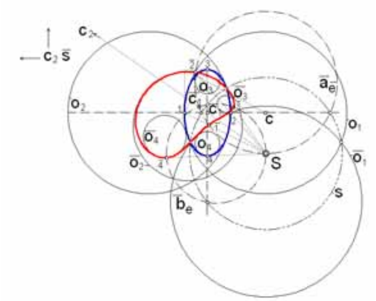

Figure 7: Ellipse and the 4th order curve with two circular axes of harmonic symmetry - bean curve

Photos in Figure 8 and Figure 9 were downloaded from the web site: http://www.anishkapoor.com/484/Studio.html and they are examples of surfaces that can be obtained by harmonic symmetry. In the photo of the sculpture "Cloud Gate" the surface contour is marked red – it is the bean curve of the 4th order which can be obtained as the harmonic equivalent of the ellipse (Fig.9).

Figure 8: Surface of the 4th order `Cloud Gate` designed in Anish Kapoor's project and built in Chicago in the U.S.

Figure 9: The bean curve of the 4th order

4. CONSTRUCTION OF BUILDINGS MODELED BY THE GEOMETRY OF THE CURVES AND SURFACES OF THE 3RD AND 4TH ORDER

There is a link between architecture, nature, and science that can't be broken. The theory of analytical geometry and its practical application on the natural forms in architecture allow us to construct buildings that are not only visually appealing but also admired for their stiffness, hardness and stability. This is best illustrated by objects such as Crescent Moon Tower, designed to be built in Dubai. Some people call it "The Horn". It is shown in Figure 10 [19]. Figure 11 presents the modeled surface obtained by harmonic symmetry of the revolving paraboloid, which is, like the building itself, crescent-shaped and has two systems of circular sections.

Figure 10: Moon Tower designed to be built in Dubai

Figure 11: Surface of the 4th order obtained by inversion of the revolving paraboloid

The revolving paraboloid has the *z* axis. Harmonic symmetry of the parabolid is done using the sphere ***s*** and the center of harmonic symmetry ***S*** [14]. The paraboloid is inverted into the surface of the 4^{th} order with a circular axis and circular generating lines, which is shown in Fig.12. The circles which are in the horizontal planes, perpendicular to the circular axis of the rotating quadric (marked as blue rays (for instance - *β*)), are mapped into circles ($\overline{\square}$) which are in the planes perpendicular to the circular axis of the equivalent quadric. In the frontal plane of projection, they are also in the ray position and concurrent in the point *K.* Inversion mapping uses the contour of he paraboloid in the frontal plane of projection, circles in H-plane (*β*) and axis *a* which is in F-plane. The axis of the cone *a* (*z* axis) is mapped into the circular axis that passes the point ***S***. [15]

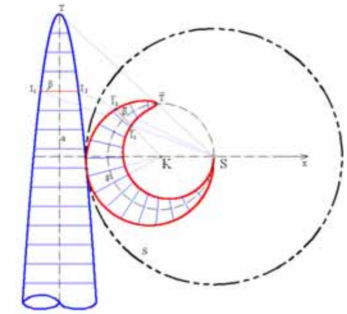

Figure 12: Mapping the revolving paraboloid into the crescent-shaped surface of the 4th order

By selecting different spheres-*s* and centers of harmonic symmetry *S*, we can produce surfaces of attractive shapes that are suitable for constructions because of the system of circular cross-sections that are obtained by the above described mapping. To construct buildings modeled in this way, in a system such as Diagrid, it is necessary to find optimal directions of isolines, which would represent the directions of the supporting beams of the diagrid structural system and optimize its capacity.

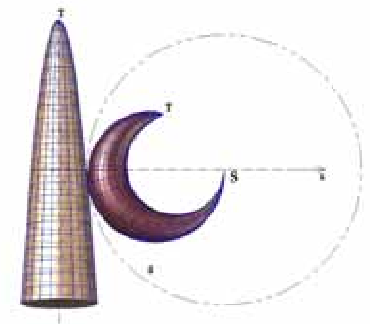

Figure 13: Surface Model of the revolving paraboloid and the crescent-shaped surface of the 4^{th} order

5. DIAGRID STRUCTURAL SYSTEM

Diagrid is a structural system which consists of triangular structures with diagonal support beams. This system has been widely used for newly-constructed high-rise steel buildings because of its exceptional structural efficiency and aesthetic potential. There is no structural system superior to diagrid concerning the efficiency in the distribution of load among all the members of a structure [11]. This structural innovation allows the construction of tall buildings in forms that we previously thought impossible. It uses a number of units that are combined to make a grid with a myriad of different structural possibilities and to maximize the efficiency of the most commonly used steel units. The use of this system requires less structural steel, thereby reducing the weight and cost of buildings, and at the same time allows the transfer of significant vertical load. However, the grid itself is not strong enough to carry the horizontal load, so a ring system has to be added to prevent bending. Diagrid can be used for a variety of geometric shapes and it has enormous potential for the future architecture. This can be seen in Figure 4, which was taken from the site: http://plusmood.com/2008/11/capital-gate-rmjm-architects/. This project used Tekle-structured software which has been used for modeling some of the world's tallest and most popular buildings. This software uses three-dimensional modeling. It calculates three-dimensional coordinates for each component of the diagrid grid, which ensures a flawless integration and assembly of the main frame and the supporting beams.

Figure 14: Capital Gate of the architectural company RMJM in Abu Dhabi

Diagonal structures were invented by Russian engineer Vladimir Shukhov. He was famous for his pioneering work on new analytical methods in different areas. Shukhov left a permanent mark on early constructivism in Soviet Russia. As a leading engineer and mathematician in the late 18[th] and early 19[th] century, he created world's first hyperboloids as well as the thin shell of tensile structures of exquisite sophistication and elegance.

In his paper Chao Huang [4] examines the carrying capacity of the diagrid structure depending on the angle between the diagonals and it varies up to 30%.

There is an increasing number of high-rise buildings around the world [1] that dominate the landscape. Their construction is possible due to the invention of the Diagrid system and rapid development of construction equipment, mechanization and materials [10]. Future research studies should investigate structural systems that are efficient in seismic zones. Advantages of the Diagrid system compared to conventional systems lie in its large load-bearing capacity and in the structure of the shell that allows large openings on the buildings and uses 20% less steel than conventional steel structures. It can be concluded that the diagrid system would be particularly suitable for the surfaces obtained by inversion, because they have two systems of circular cross-sections and the parts of these surfaces can be used to make different shapes and free-forms.

6. CONCLUSIONS

With different appointments of the sphere-s and the center of harmonic symmetry S, we can transform surfaces of lower orders into surfaces of higher order with attractive forms that are suitable for constructions because of their systems of circular cross-sections obtained by mapping. To construct buildings modeled in this way, in a system such as Diagrid, it is necessary to find optimal directions of isolines, which would represent the directions of the supporting beams of the diagrid structural system and optimize its capacity. This should be the direction of future investigations.

Apart from clearly defined geometric surfaces, free forms also play an increasingly important role in modern architecture. These forms are most commonly obtained by using the geometry of NURBS. However, the construction of these structures presents a great problem.

Recent technological advances made it possible to mass-produce doubly curved panel boards that can wrap architectural free-form surfaces.

M. Eigensatz at all [12] created a computer algorithm for the production of large-scale architectural freeform surfaces obtained using NURBS geometry. The entire surface of a building is divided into panels, whose shape and size depend on the shape of the area to be wrapped. The shape and smoothness of the surface to be wrapped must not be distorted. Such panels only minimally alter the projected surface, and reduce the cost of building.

Freeform glass structures are commonly used in modern architecture. The area is divided into cylindrical (84%), flat (10%), and double curved (only 6%) panels, which means that 94% of the panels can be produced using an automated industrial process and retain the curvature of the surface designed by an architect [2].

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