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A STOCHASTIC MODEL FOR SERIES OF SINGLE AND AGGREGATED OVER THRESHOLD FLOOD CHARACTERISTICS VALUES

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ABSTRACT

Stochastic structure of extreme hydrological events can be analyzed using characteristic flood values from partial hydrographs obtained by introducing base flow cut values. The process encompasses detection of a discrete probability distribution of a number of events in a chosen time interval and continuous distribution of the exceedance values (peaks). This article presents a stochastic model for the analysis of the base flow exceedance volumes and accompanied cycle times between the ends of sucessive events. The model is recurrent in nature, based on Markov's discrete model principles and the assumptions about the form of the process intensity functions. The number of occurrence discrete distributions are discussed according to chosen forms of the time and volume intensity functions. The continuous distributions of the exceedance characteristic values are modelled for the base series of values and recurrently for their aggregation. The distribution of the maximum exceedance volume in chosen time interval is formulated. The article presents an application of the suggested procedures on the mean daily flows hydrographs from the Bezdan gauging station on the Danube river.

1. INTRODUCTION

There are number of factors that influence flood occurrence. Most of them are inter-dependent. Due to their random nature, floods are usually analysed using stochastic models. The most widespread approach in their estimation is based on annual maximum series (AMS) of flood discharges. A value of an interest is usually a peak discharge value, but it may also be a volume of flood wave or its duration. Another approach is the peak over threshold method (POT). As there might be a number of flood occurrences within a year, only those ones whose peaks exceed a given threshold level are used to define flood characteristics in the POT. These floods form a partial duration series.

The work presented in this article uses the theoretical background of the POT method to explore characteristics of parts of the flood waves which exceed certain threshold i.e. over threshold flood hydrohraphs or partial duration series. Datasets of flood characteristics are derived from the daily mean flow data. In addition to the basic datasets (single or raw values) of the considered flood characteristic (a peak discharge, a flood duration, a flood volume, a number of flood occurrences within a specified interval, a time duration between the two floods, etc.), datasets derived through the aggregation of two or more consecutive members of the basic series are also considered. Members of the derived data-sets are also random variables. Together with the corresponding raw data they are termed the flood characteristics.

The main hypothesis of this work is that all relevant information about the floods and their structure are inherent in the values of the flood characteristics that exceed a given threshold, i.e. in the partial duration series of the flood characteristics. The analysed flood characteristics are excess flood volumes and cycle durations (the time between completion of the two consecutive excess volumes) along with the associated event times (the interruptions).

The methodological approach to stochastic modelling of the flood characteristic includes the following analyses:

a) a number of occurrences in a time interval,

b) a cycle duration or a time between two, three or more consecutive events,

c) a number of occurrences in an interval measured in the units of the characteristic variable,

d) the value of the flood characteristic in a single event or its cumulative value in two, three or more consecutive events,

e) the maximum flood characteristic value in a time interval.

The proposed recurrent models for distributions of the flood characteristics and their aggregates rest on Markov's discrete stochastic processes theory both for the time and characteristic value intermissions, as well as on the assumption that the two types of intermittence are independent. The models are based on the occurrence intensity function with the shape corresponding to the Weibull distribution for the starting over threshold value series and the parameter of the distribution for the number of their occurrences.

Basic theories that are used in preparation of this investigations are from ex-Yugoslavian researchers P. Todorovic and E. Zelenhasic (*Todorovic* 1970, 1978; *Todorovic and Zelenhasic*, 1970). On the same basis Vukmirovic (1975) studied the river bed load movement, Despotovic (1996) explored heavy rains and Plavsic (2004) analysed the flood risk. Theoretical prerequisites and examples can be found in various text books about andvanced statistical aproaches in hydrology. For this work we used publications in Serbian (*Vukmirovic*, 1990; *Zelenhasic*, 1997; *Zelenhasic and Ruski*, 1991). Works about a over threshold flood volumes as random processes are rare. The methodlogy presented here is a part of the PhD thesis of Pavlovic (2013) where detailed references can be found.

The data used to check the validity of the posed hypothesis and the applied methodology are the mean daily flow series for the Bezdan gauging station on the Danube River in Serbia. These data refer to a 79-years long period, from 1931 to 2009.

2. OVER THRESHOLD FLOOD CHARACTERISTICS

As stated in the introduction, the work presented in the article explore characteristics of the parts of the flood waves which exceed a certain threshold i.e. characteristic values which represent over threshold parts of hydrographs during flood events. Establishing a proper threshold (base flow) one can assume that characteristic values are inter-independent and can represent extreme events (following the POT theory). A datasets of flood characteristics are derived from the daily mean flow data/hydrographs as they are commonly available from hydrometheorological services. Figure 1 depicts the origin of the characteristics values and their meaning.

The datasets of this primary values form the basic series. But if one is interested in behaviour of mulitiple consecutive values (i.e. the volume of two or more neighbouring over threshold flood events) the series of aggregates are formed. One of the possible methods, used in this work, is displayed in Figure 2. A distribution of both basic and series of aggregates of over thereshold flood volumes and corresponding cycle durations are the objects of the investigation.

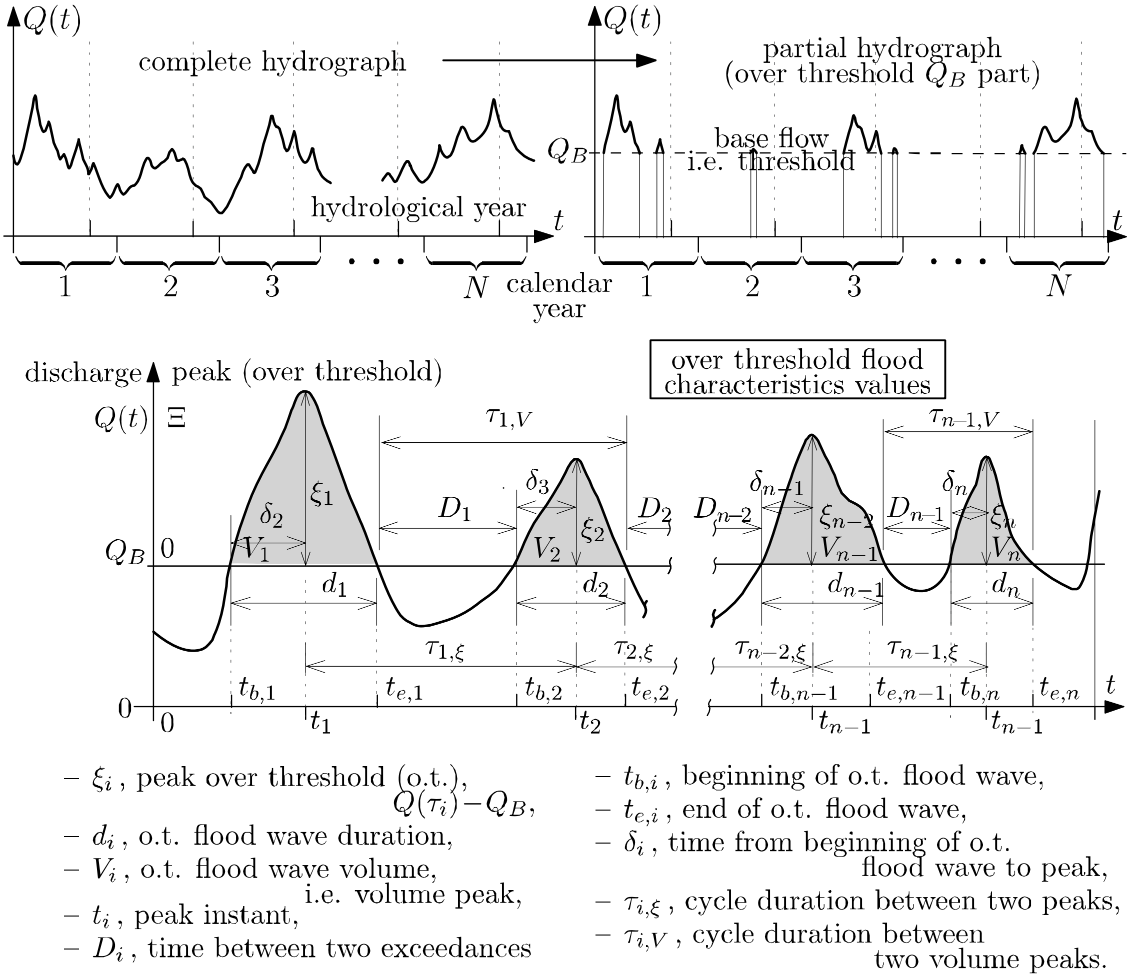

Figure 1. Origin of the characteristic values and their meaning. From top left to top right – a complete hydrograph transformed by a threshold base flow to a partial hydrograph. Bottom – over threshold flood characteristics values described in the picture and by symbol and meaning.

3. STOCHASTIC MODEL BASICS

The main hypothesis applied in the stochastic model presented is that all relevant informations about the nature of floods and their structure are inherent in the values of the flood characteristics that exceed a given threshold, i.e. in the partial duration series. The analysed flood characteristics are the excess flood volumes *V* and the flood cycle durations τ (the time between the completion of the two consecutive excess volumes) along with the associated event times t_e (the interruptions) (see Fig.1). This informations lead to conclusions about probability distributions of exceedance characteristics, both for the base series and for series of aggregates.

The methodological approach supposes that a random process for over threshold flood characteristics has to be defined as follows in Equation 1 and in Fig.3:

$$\chi(t) = \chi_t = \sup_{\substack{t_\nu \leq t \\ \nu = 1,\ldots,\eta_t}} \{X(t_\nu), t \geq 0\} \tag{1}$$

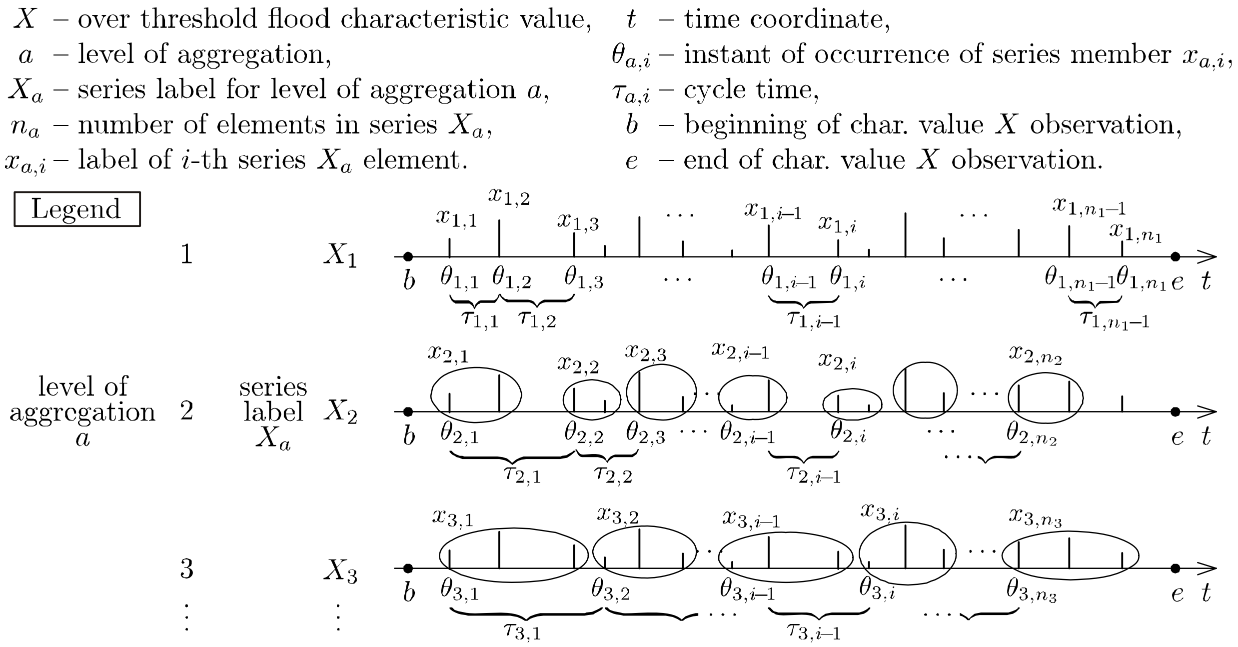

Figure 2. Graphical review of series of aggregated flood characteristics values creation.

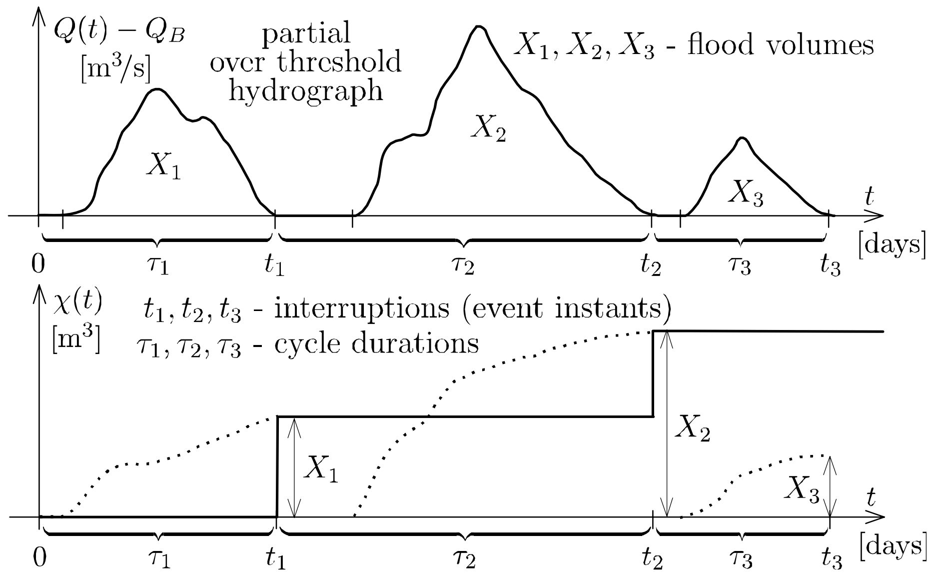

Figure 3. Graphical representation of random process of over threshold flood characteristics values – example for the over threshold flood volumes.

The time of interruption t_i is the instant of the realisation of the complete single overthreshold flood volume. The cycle time τ_i is the period between two successive interrptions. For the aggregates, the interruption is the instant of the completion of the last flood volume in the group which forms the aggregate.

The proposed stochastic model which describes over threshold values (here the flood volumes and the cycle durations) rests on Markov's discrete stochastic processes theory both for the time and characteristic value interruptions (*Vukmirovic*, 1975, 1990), respectively represented by its dicrete distribution mass function $p_v(t)$ and $p_n(x)$; the index is the number of interuptions and the argument is the domain – time or volume. There are two key postulates about the both two types of the interruptions; the only possible events are that none or only one interruption can be achived in *dt,* the time increment. An additional assumption is that two previosly mentioned types of interruptions (in time and volume domain) are independent.

The result of the preassumptions is of essential importance of the two process functions: time and volume process intensity functions, respectively labeled $\lambda(t,\nu)$ and $\kappa(x,n)$. The symbol ν stands for the number of interuptions in the time interval *t*, and symbol *n* for the interruptions in the volume domain x. The process intensity functions are the limit values of the probability of occurence of one event (process interruption) in the time increment *dt*. The process has the general form (2) with the initial conditions (3) [1]

$$p'_\nu(t) = \lambda(t,\nu-1)\,p_{\nu-1}(t) - \lambda(t,\nu)\,p_\nu(t)\,, \quad \nu \geq 1$$
$$p'_0(t) = -\lambda(t,0)\,p_0(t) \tag{2}$$

$$t=0 \quad \Rightarrow \quad p_0(0)=1\,,$$
$$t=0 \quad \Rightarrow \quad p_\nu(0)=0\,, \quad \forall\,\nu \geq 1\,. \tag{3}$$

For a solution of the system (2) the next events and their probabilities have to be studied:

a) η_t, the ν number of interruptions in *t* time interval, and its discrete distribution $p_\nu(t)$, calculated for base series,
b) τ_i cycle duration or a time between two (i=1), or three or more consecutive interruptions/events in time (*i*=2,...) and its continuous distribution $G(\tau_i)$,
c) μ_x, the *n* number of interruptions in *x* volume value domain intervals and discrete distribution $p_n(x)$, calculated for base series,
d) X_i over threshold flood volumes in single event (*i*=1) or its cumulative value over one, two, three or more consecutive events (*i*=2,3,...) and continuous distribution $H(x_i)$.
e) $F_t(x)$, the distribution of the maximum flood characteristic value in a time interval *t*.

Basic datasets/series of over threshold cycle durations and flood volumes are labeled with τ or τ_1 and *x* or x_1 respectively, aggregates of sequences of two consecutive base dataset members are τ_2 i x_2, and aggregates of *i* members with τ_i and x_i.

Figure 4 depicts a graphichal scheme of analysed events and its distributions and releations between them which follows from stochastic process and that will be explained. Dot-outlined rectangles encompasses the essence of the outcomes of the contents of this work about overthreshold flood cycle times and volumes. Bold arrows ephasise how a classic POT method employs part of the steps which has to be conducted to give the answer about $F_{t=1}(x)$, the distribution function of maximum value of random variable X over the time *t* equals 1 year.

In following section the specific solutions of system (2) will be presented in the form for the interrupts in time domain; the solutions for number of interrupts in the volume domain is equivalent, only different symbols have to be used. In the following section, the distribution of over threshold flood volumes is elaborated in its reccurent form, starting from a basic series and a reccurently for a series of aggregates.

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[1] The system is written for time domain interrupts. Changing ν to *n*, and *t* to *x*, emerges the system which is valid for overthreshold volumes.

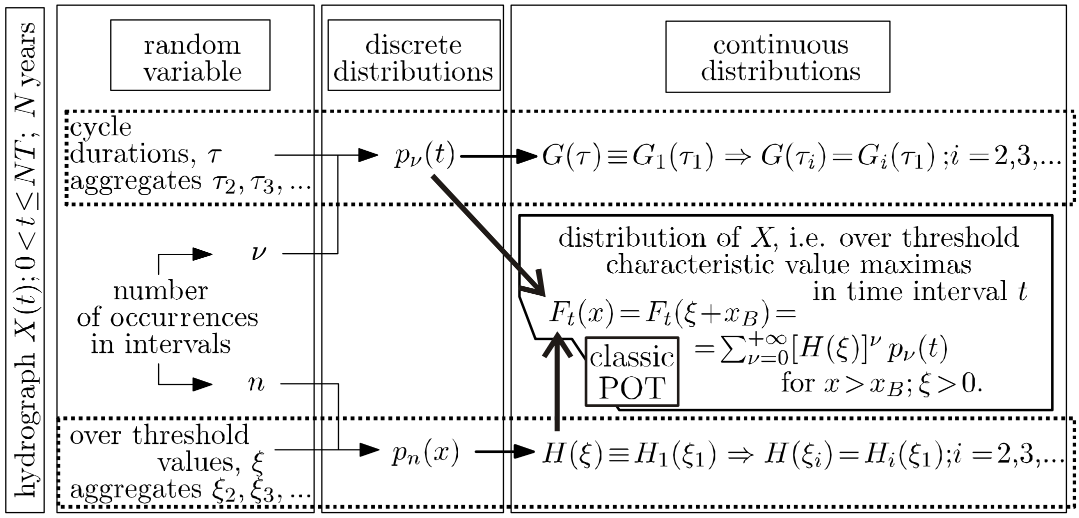

Figure 4. The graphical scheme of the analysis of the single and aggregated over threshold flood characteristic values – t cycle durations and ξ characteristic values.

4. ANALYSIS OF DISTRIBUTION OF NUMBER OF EVENTS IN TIME AND VOLUME DOMAIN

The solution of the system (2) depends of the form of the process intensity functions $\lambda(t,\nu)$ and $\kappa(x,n)$. The chosen forms of these functions leads to a solution in a discrete probability mass functions $p_\nu(t)$ – the probability that there are n process interrupts (or events) in the time interval t and $p_n(x)$ – the probability that there are n interrupts over the interval x measured in the volume domain.

Figure 5. presents a graphical scheme how the number of interrupts can be analysed from the observed datasets regardless of the nature of the over threshold flood characteristic value. Essentialy, the cumulative sum of the value has to be made, and the cumulative sum of the appropriate discretisation step is used as “the counting sieve”.

To fit a empirical distribution, from the experiences in the application of the POT method, several models for the intensity functions are presumed, as presented in Table 1.

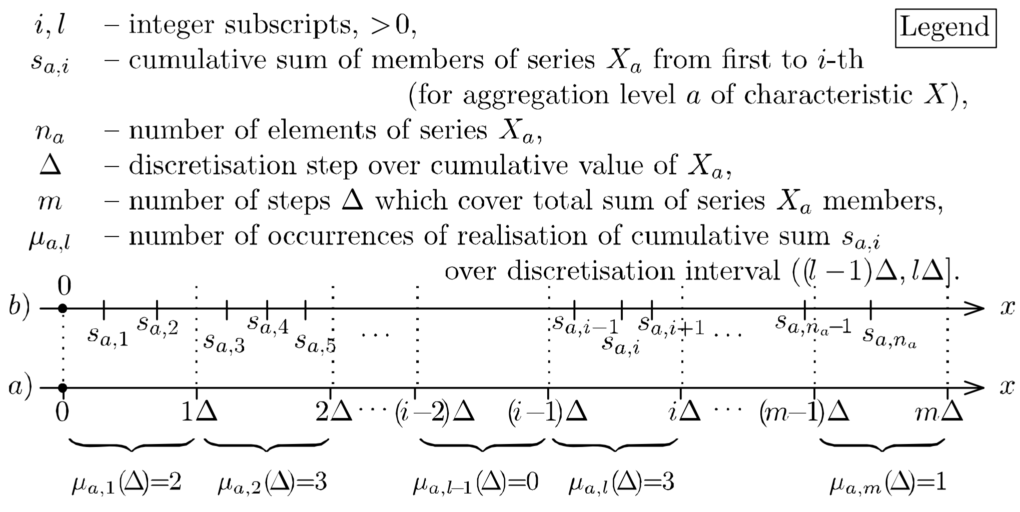

Figure 5. The graphical scheme of the empirical analysis of discrete mass distribution function from the observed datasets.

Table 1. The models of the process intensity functions (expressed for the time intensity).

type	process intensity function	discrete distribution (d.d.) $p_\nu(t) = P\{\eta_t = \nu\}$	name of d.d. $p_\nu(t)$
1	$\lambda_0 = \text{const.}$	$e^{-\lambda_0 t} \frac{[\lambda_0 t]^\nu}{\nu!}$	Poisson distribution with constant parameter
2	$\lambda(t)$	$e^{-\Lambda(t)} \frac{[\Lambda(t)]^\nu}{\nu!}$	Poisson distribution with variable parameter
3	$\lambda(t) \left(1 + \frac{\nu}{a}\right)$	$\frac{\Gamma(\nu+a)}{\Gamma(\nu+1)\,\Gamma(a)} \left(1 - e^{-\Lambda(t)/a}\right)^\nu e^{-\Lambda(t)}$	negative Binomial distribution
4	$\lambda(t) \left(1 - \frac{\nu}{b}\right)$	$\frac{\Gamma(b+1)}{\Gamma(\nu+1)\,\Gamma(b+1-\nu)} e^{-\Lambda(t)} \left(e^{\Lambda(t)/b} - 1\right)^\nu$	Binomial distribution

It is obvious that the chosen models are simple in the form, with disjunct influence of the time t and the number of interrupts ν (for the ease of solving the general system (2)). This means that the form $\lambda(t,\nu)$ is transformed to $\lambda(t,\nu)= \lambda(t)\ \lambda_*(\nu)$. Types 1 and 2 are independent of the number of the interrupts while types 3 and 4 are dependent. Value $\Lambda(t)$ is the integral of time dependent part of intensity function over the time interval $[0,t]$, i.e. $\Lambda(t) = \int_0^t \lambda(s)\, ds$ and further on will be used as a convenient representation of the intensity function (as intensity function will not be directly modelled from datasets).

For the analysis of the number of interrupts over volume domain $p_n(x)$, the process intensity function has label $\kappa(x,n)$. The hypothesis are the same as for $\lambda(t,\nu)$. Table 1 will be valid, only the changes in the labels has to be done; $\kappa(x)$, t and $K(x) = \int_0^x \kappa(s)\, ds$ instead of $\lambda(t)$, ν and $\Lambda(t)$ respectively (the x is the volume interval in which the number of interrupts in the volume domain is observed).

5. Analysis of distributions of over threshold flood volumes and cycle times for base and aggregats series

The distribution function of the over threshold flood volumes of n consecutive events can be defined in form:

$$H_n(x) = P\{X_n \le x\} \tag{4}$$

and its conection with the discrete distribution of n the number of interrupts in the volume domain:

$$p_n(x) = P\{X_n \le x\} - P\{X_{n-1} \le x\} \tag{5}$$

According to the equation (5) (*Todorovic*, 1970; *Vukmirovic*, 1990; *Pavlovic*, 2013), the distribution function of n consecutive over threshold flood volumes and according density function are:

$$H_n(x) = 1 - \sum_{i=0}^{n-1} p_n(x), \quad h_n(x) = \varkappa(x, n-1)\, p_{n-1}(x) \tag{6}$$

For the singe over threshold event (n=1, or for base series), formula (6) gives,

$$H_1(x) = 1 - e^{-\mathcal{K}(x)}, \quad \mathcal{K}(x) = \int_0^s \varkappa(s) ds \tag{7}$$

From Equation (7) and the forms of the discrete distributions from the last column of Table 1 a conclusion can be derived that the distribution function for the over threshold volumes of base series is independent of the type of discrete distribution of the interrupts - $p_n(x)$. The form of H_1 is similar to the form of several continuous distribution functions. For modelling of H_1, a single parameter exponential and two parameters Weibull and Pareto distributions are chosen. Modelling H_1 is the implicit way to describe the volume dependent part $\kappa(x)$ of the process intensity function via $K(x)$ (its integral over the interval x). It is assumed that the intensity function consists of two independent parts $\kappa(x,n) = \kappa(x)\, \kappa_*(n)$, as shown in Table 1.

In Table 2, different forms of the distribution function $H_n(x)$ of n consecutive over threshold flood volumes are shown as a function assumed form of the process volume intensity function $\kappa(x,n)$. Due to nature of H_1, the $H_n(x)$ is given in recurent form (see the similarity in eq. (7) and 2nd column of Table 2.). Note that in last two types of $H_n(x)$, which corresponds to negative binomial and binomial discrete distributions of interrupts in volume domain, participates the discrete distribution parameters a and b. This is the principal result of the assumed nature of the very stochastic process.

Table 2. The $H_n(x)$, distribution functions of n consecutive over threshold flood volumes as a function of the form of the process volume intensity function $\kappa(x,n)$.

type	$\varkappa(x,n)$	$H_n(x)$ [$H_1(x)$ always according to eq. (7)]
1	$\varkappa_0 = \text{const.}$	$1 - e^{-\varkappa_0 t} \sum_{i=0}^{n-1} \frac{(\varkappa_0 t)^i}{i!}$
2	$\varkappa(x)$	$1 - e^{-\mathcal{K}(x)} \sum_{i=0}^{\nu-1} \frac{\mathcal{K}(x)^i}{i!}$
3	$\varkappa(x)\left(1+\frac{n}{a}\right)$	$H_{n-1} - \frac{e^{-\mathcal{K}}}{(n-1)!} \left\{\prod_{i=1}^{n-1}(a+i-1)\right\}(1-e^{-\mathcal{K}/a})^{n-1},\ n=2,3,...$
4	$\varkappa(x)\left(1-\frac{n}{b}\right)$	$H_{n-1} - \frac{e^{-\mathcal{K}}}{(n-1)!} \left\{\prod_{i=1}^{n-1}(b-i+1)\right\}(e^{\mathcal{K}/b}-1)^{n-1},\ n=2,3,...$

The same form results are for the cycle time durations. In all the expressions labels H, $\kappa(x,n)$, $\kappa(x)$, $\kappa_*(n)$ and $K(x)$ has to be replaced with G, $\lambda(t,\nu)$, $\lambda(t)$, $\lambda_*(\nu)$ and $\Lambda(t)$.

6. DISTRIBUTIONS OF MAXIMUM OVER THRESHOLD FLOOD VOLUMES AND CYCLE TIMES FOR BASE SERIES

Distribution function of the maximum over threshold flood volumes in time interval t (i.e. (0,t]), using the process definition (1), is given as eq. (8)

$$F_t(x) = P_t\{X \leqslant x\} = P_t\{\chi(t) \leqslant x\} = p_0(t) + \sum_{\nu=1}^{+\infty} [H(x)]^{\nu} p_{\nu}(t) \tag{8}$$

The elements of the equation are discrete distribution of number of interrupts in the time domain $p_v(t)$ and the distribution function of the base series of the over threshold flood volumes $H(x)=H_1(x)$ (defined by $p_n(x)$ discrete distribution of number of interrupts in the volume domain). The classic POT method (that results in matches AMF method for analysis of maximum annual river flows) assumes that t=1 year which means that $p_v(t)$ is discrete distribution of number of over threshold flow peaks in one year.

The same form (8) is valid for the over threshold cycle durations changing label H with G, while x is not the volumes but cycle times i.e. $X\equiv\tau$ and x equals specific time value. The G is the distribution function of the over threshold cycle durations for its base series.

7. TESTING THE METHODOLOGY FOR STOCHASTIC MODEL

The methodology for stochastic modelling of series of single and aggregated over threshold flood characteristics values was tested against mean daily flow data from the Bezdan gauging station on the entrance of the Danube River to the north of Serbia. The data refer to a 79-years long record, from 1931 to 2009. The analysed flood characteristics are: over threshold discharge flood volumes and cycle durations.

Analyses are conducted for the numerous base flows (or thresholds) ranging from 2500 m^3/s to 5500 m^3/s. Treshold limit is posed by the minimum number of 25 over threshold flood waves, supposed to be the smallest reasonable data sample for statistical analysis. Figure 6 presents the modelled H_n (x) and empirical distribution function of base and aggregate (aggregation levels 2 and 3) series for the base flow of 5100 m^3/s (top) and $p_n(x)$, discrete distribution functions for the number of interrupts in volume domain of base series (bottom). Base dataset (n=1) is modelled by Weibull distribution function which outperforms other two functions (exponential and Pareto) for majority of base flows). Form of $H_{n=\{2,3\}}(x)$ is according to the use of the Poisson distribution for $p_n(x)$, the number of the interrupts in volume domain for base series (type 2 in Table 2). Base and level 2 aggregates passed the Kolmogorov-Smirnov goodness of fit test (p values are 87% and 11%) while aggregates of level 3 didn't pass (p=2%). One common problem is noticed regarding the huge range of over threshold flood volumes values which leads to high assymetry in data. Here no attempts are made to correct it somehow and datasets are used as they are.

A discrete function of the number of time interrupts $p_v(t)$ follows the negative binomial distribution, but not with the constant parameter a value over different time steps. The distribution function $G(\tau)$ for the cycle durations of base series also follows the Weibull distribution for the majority of base flows. Figure 7 is similar to fig.6 except it shows appropriate diagrams for G fammily ($G_{n=\{1,2,3\}}(t)$). The base flow is again 5100 m^3/s. Due to the inconstant parameter a, the assumption of its constancy introdouced in solving the system (1) is compromised and consequently assumed ease of the use of the model. A modified method is used to solve the problem; appropriate value for parameter a of the discrete function if obtained by fitting G_2 to empirical distribution in way that the Cramer-von Mises goodness of fit test gives the best possible p value. For some threshold values, an abnormaly high values of parameter a (couple of tens of thousands) was calculated by previous procedure. This behaviour may have mathematical legitimacy, but it physical meaning is doubtful, as it has to represents the value expressed in days.

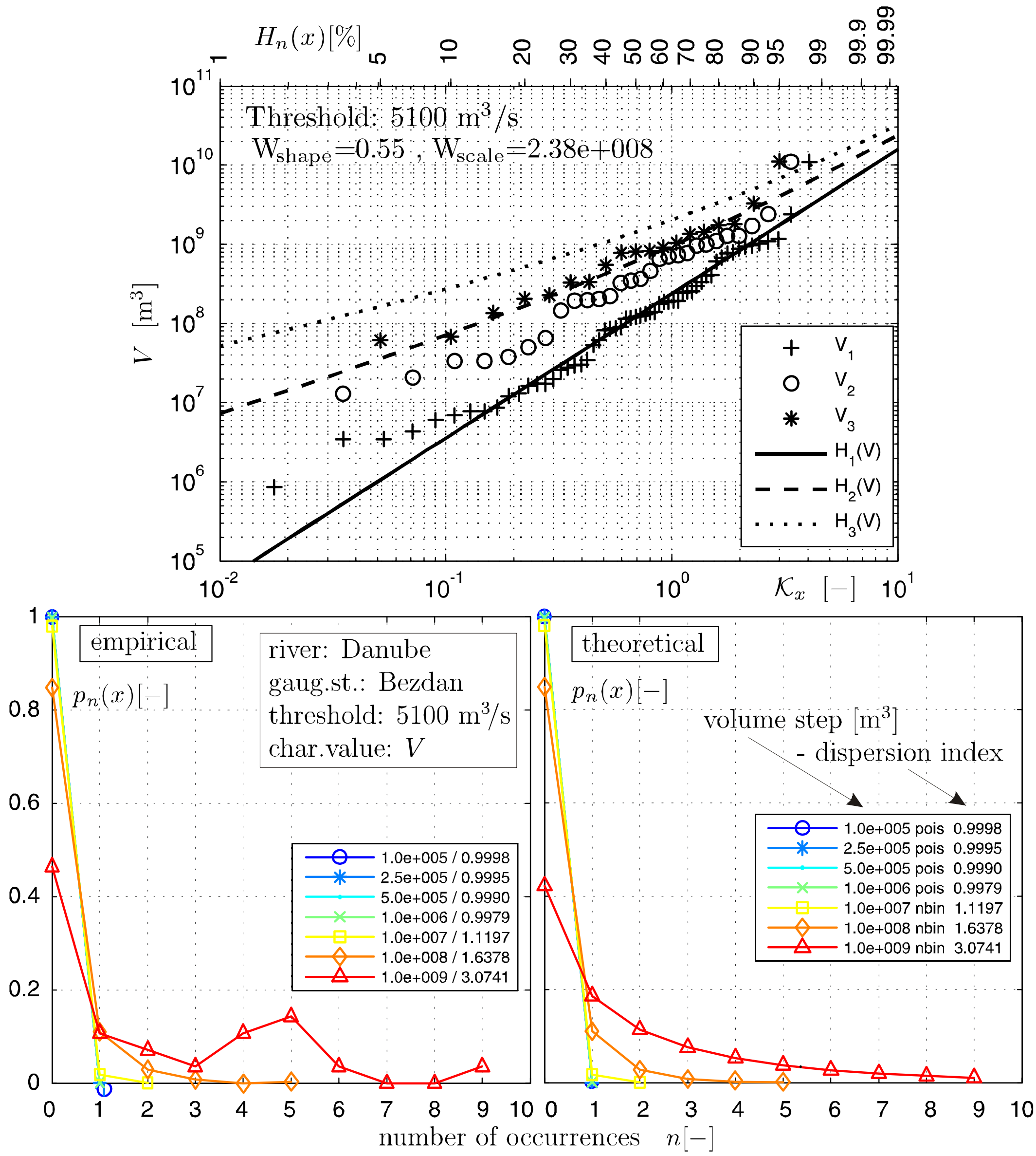

Figure 6. Base flow 5100 m^3/s. (Top) Distribution functions $H_{n=\{1,2,3\}}(x)$ - lines theoretical by stochastic model, markers – empirical. (Bottom) discrete distributions of the number of process interrupts in volume domain $p_n(x)$ – for various volume steps and corresponding dispersion index as theoretical distribution type indicator.

The previously displayed results and the experience gained through research, lead to the conclusion that stochastic model that is shown has some imperfections but can be used, having in mind its limitations and possible drawbacks.

8. CONCLUSIONS

The article presents a stochastic model for the analysis of the base flow exceedance volumes and accompanied cycle times between the ends of the sucessive exceedence events. Substantialy it is Markov's discrete stochastic processe both for the time and characteristic value intermissions/interrupts, with the assumption that the two types of intermittence are independent. The solution of the model's equation system depends on the shape of the process intensity functions, time intensity $\lambda(t,\nu)$ and volume intensity $\kappa(x,n)$.

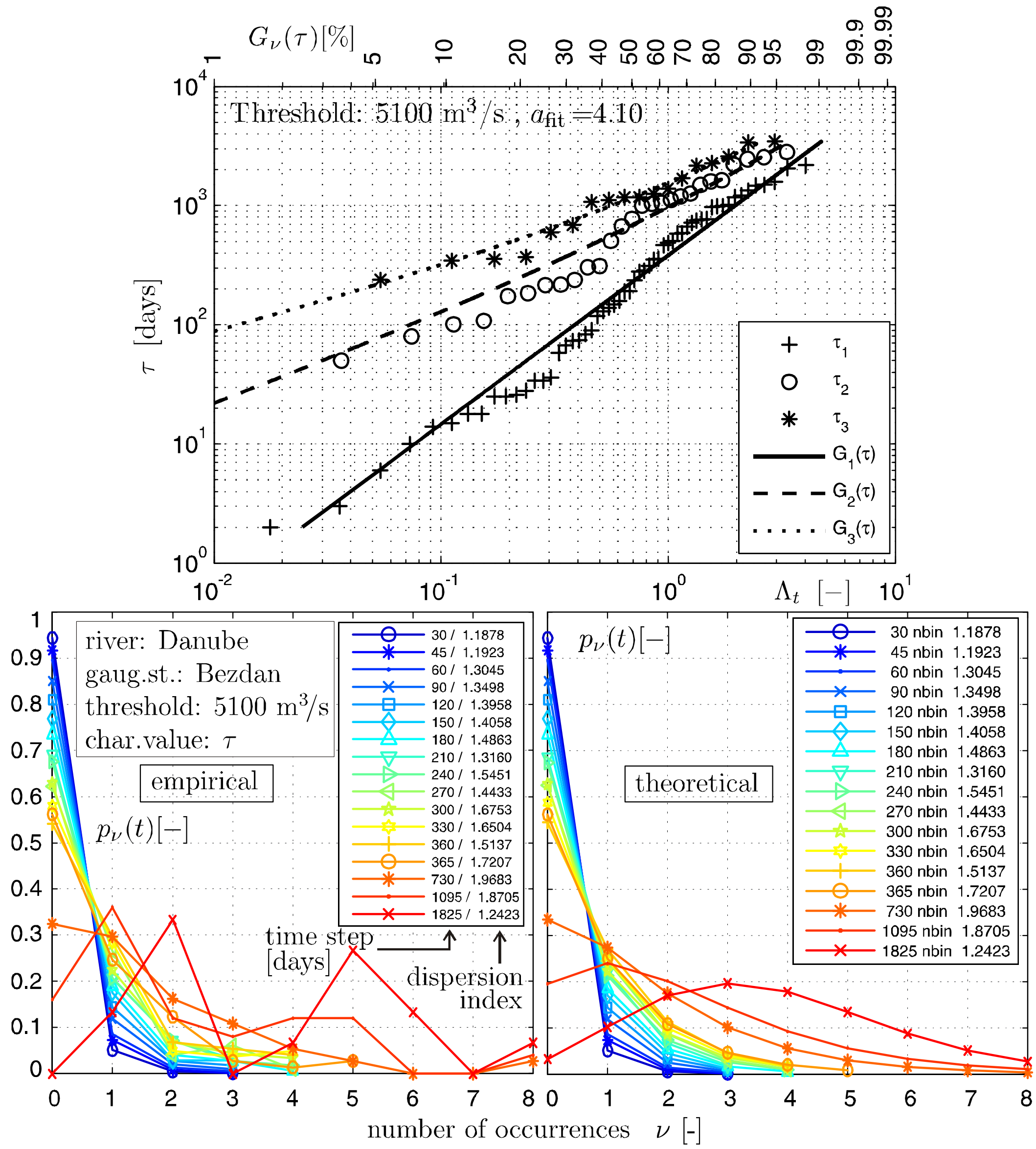

Figure 7. Base flow 5100 m^3/s. (Top) Distribution functions $G_{\nu=(1,2,3)}(x)$ - lines theoretical by stochastic model, markers – empirical. Fit p-values are 57%, 50% 94% respectively. (Bottom) Discrete distributions of the number of process interrupts in time domain $p_\nu(t)$ – for various time steps and corresponding dispersion index as theoretical distribution type indicator.

Their assumed shape imply the shape of a discrete distribution of the number of occurrences (process iterruptions) – binomial, Poisson or negative binomial. The previous distribution leads to the continuous distributions of flood volumes $H_n(x)$ and cycle times $G_\nu(t)$.

One of main conclusions is that the distributions of the base series, $G_1(t)$ and $H_1(x)$, are independent of the discrete distribution of process interuptions – their shape can fit to exponential, Weibull or Pareto distributions. The continuous distributions of the aggregated flood characteristics (ν,n=2,3,...) are expressed in recurrent form to $G_1(t)$ and $H_1(x)$.

The stochastic recurrent model is applied on mean daily hydrographs data from the gauging station Bezdan, the Danube River entrance in Serbia, on numerous threshold values-base flows. The results are that the number of process interruptions follows a negative binomial discrete distribution and that over threshold base cycle times and volumes can be suitably described by a Weibull distribution.

The methodology for calculation of recurrence distribution model for aggregated overthreshold values is modified due to the impossibility for the direct application of theoretical hyphothesis for the setup of $G_v(t)$ and $H_n(x)$ – that the discrete distribution parameter can be directly used as the parameter of the recurrence continuous distribution model for over threshold values. This modification is convenient for good goodness of fit of aggregated values to model, but can compromise the methodology in the part of the fit in discrete distribution in process interuptions (both in time and overthreshold value domain). Substantially, that can lead to doubts about the true nature of the process intensity functions. Although the model is imperfect, the conclusion is that it can be used having in mind its limitations.

REFERENCES

Todorović, P. (1970). On some problems involving random number of random variables, *Annals of Mathematical Statistics*, 1970, vol 41, no 3, pp 1059-1063.

Todorović, P. (1978). Stochastics model for floods, *Water Resources Research*, vol 14, no 2, pp 345-356.

Todorović, P., Zelenhasić, E. (1970). A Stochastics model for flood analysis, *Water Resources Research*, vol 6, no 6, pp 1641-1648.

Vukmirović, V. (1975). Analiza kretanja vučenog nanosa pomoću slučajnih procesa, doktorska disertacija, Građevinski fakultet Univerziteta u Beogradu, Beograd. (The Bed Load Movement Analysis by Random Processes, PhD Dissertation, Faculty of Civil Engineering, Univ.of Belgrade, Sebia, in Serbian)

Despotović, J. (1996). Analiza jakih kiša kao uzročnika poplava pomoću slučajnih procesa, doktorska disertacija, Građevinski fakultet Univerziteta u Beogradu, Beograd. (Analysis of Heavy Rains as a Cause of Floods by Random Processes, PhD Dissertation, Faculty of Civil Engineering, Univ.of Belgrade, Sebia, *in Serbian*)

Plavšić, J. (2004). Analiza rizika od poplava pomoću prekidnih slučajnih procesa, doktorska disertacija, Građevinski fakultet Univerziteta u Beogradu, Beograd. (Flood Risk Analysis by Random Processes, PhD Dissertation, Faculty of Civil Engineering, Univ.of Belgrade, Sebia, *in Serbian*)

Vukmirović, V. (1990). *Analiza verovatnoće pojave hidroloških veličina*, Građevinski fakultet Univerziteta u Beogradu i Naučna knjiga, Beograd. (*Analysis of Hydrological Values Distributions*, *university textbook, in Serbian*)

Zelenhasić, E. (1997). *Stohastička hidrologija*, Pan-merkur, Novi Sad.(*Stochastic Hydrology*, *university textbook, in Serbian*)

Zelenhasić, E., Ruski, M. (1991). *Inženjerska hidrologija*, Naučna knjiga, Beograd. (*Engineering Hydrology*, *university textbook, in Serbian*)

Pavlović, D. (1913). Modeliranje stohastičke strukture karakteristika velikih voda dobijenih iz serija pikova iznad praga, doktorska disertacija, Građevinski fakultet Univerziteta u Beogradu, Beograd. (Modelling of Stochastic Structure of Flood Characteristics Derived From Peaks Over Threshold Series, PhD Dissertation, Faculty of Civil Engineering, Univ.of Belgrade, Sebia, *in Serbian*)