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CONCAVE PYRAMIDS OF SECOND SORT - THE OCCURRENCE, TYPES, VARIATIONS

Marija Obradović [1]
Slobodan Mišić [2]
Branislav Popkonstantinović [3]

Abstract

Correspondingly to the method of generating the Concave Cupolae of second sort, the Concave Pyramids of second sort have the similar logic of origination, and their counterpart in regular faced convex pyramids (tetrahedron, Johnson's solids J1 and J2). The difference is that instead of onefold series of equilateral triangles in the lateral surface of the solid, there appear twofold series, forming deltahedral lateral surface with a common point, while bases are also regular polygons. This time, instead of the bases from n=3 to n=5, there are the basis from n=6 to n=9. The same lateral surface's net can be folded and creased in two different ways, which produces the two types of Concave Pyramids of second sort: with a major and with a minor solid height. Combining and joining so obtained solids by the correspondent bases, the concave (ortho) bipyramids of second sort emerge, which then may be elongated, gyroelongated, and conca-elongated, creating a distinctive family of diverse concave polyhedral structures.

Key words: *concave polyhedron, concave pyramid, deltahedra, lateral surface, regular polygonal base*

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1. INTRODUCTION

Concave Pyramids of second sort (***CP II***) are polyhedra which follow the method of generating Concave Cupolae of second sort (***CC II***) [3], using the same method of folding the plane net of double row of equilateral triangles, as shown in Fig. 1. Unlike ***CC***, the unit cell that forms the solid by its radial array now is a spatial pentahedral cell instead of hexahedral. The method of forming structures which (in their lateral surface) correspond to the polyhedra concerned in this paper, only without considering them as solids is elaborated in detail in [11]. There are given: the construction method, the geometric basis for setting a numerical algorithm with all the parameters and positions of the solids' vertices, as well as the graphic display of these forms, called in [11] "the core", for being just a part of the more complex solids, toroidal deltahedra. In this paper we consider their brief generation, the types of the solids and their variations, in order to encompass the possible concave solids with the predictable characteristics, which may occur based on ***CP II***.

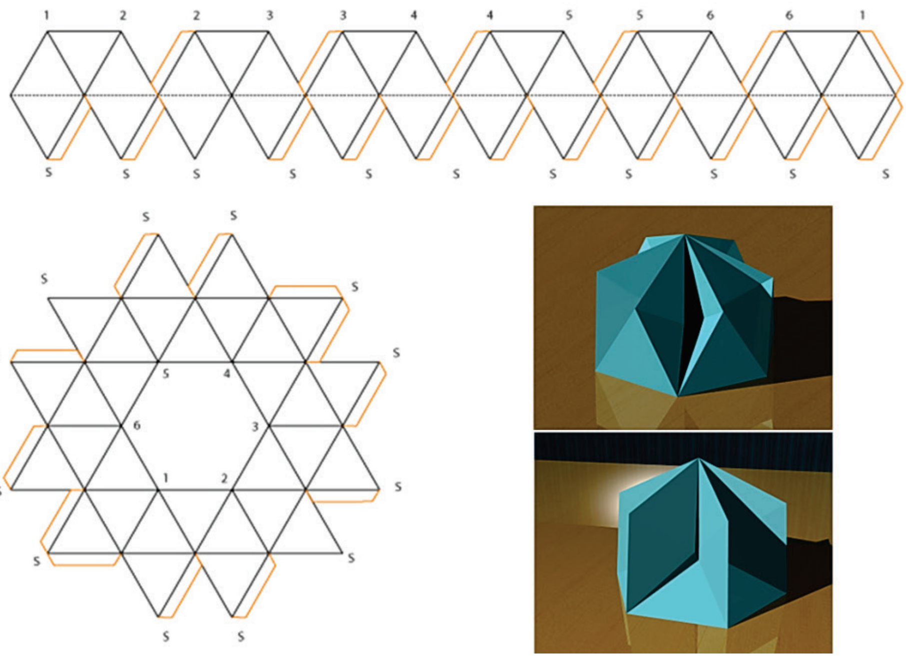


Figure 1. *Method of generating the Concave Pyramids by folding and creasing the plane net, obtaining two different types: CP-M, and CP-m*

Also, in order to establish the connection with the similarly obtained solids (Concave Cupolae), we named these polyhedra Concave Pyramids (of second sort), modeled after the familiar convex Pyramids, since they have triangular sides of the lateral surface converging at a single vertex in common, and also a polygonal base.

Note: In this paper, we have dealt only with ***CP II- (type) A***, with the number of unit cells equal to the number of the base polygon's sides, since it covers all the bases from ***n=6*** to ***n=9***, whether they are odd or even. The second type, ***CP II-B***, formed with the halved number of sides is possible only for the even bases, ***n=6***, ***n=8***, ***n=10***, so it will be subjected to the further research.

2. THE GENERATION OF CONCAVE PYRAMIDS

Concave Pyramid is a polyhedron formed over a regular polygonal base, starting from ***n=6*** to ***n=9***. As given in the Fig 1, by folding and creasing the plane net consisting of as many pentahedral cells (equilateral triangles arranged around the common vertex, named ***H***) as the sides in the base polygon, there can be obtained two types of the Concave Pyramids (alike the method of obtaining two types of Concave Cupolae of second sort).

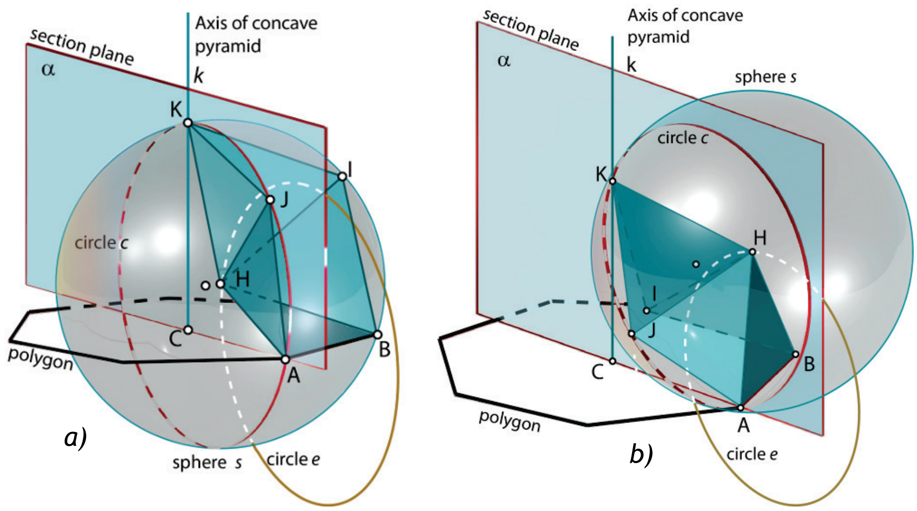


Figure 2*. a) The origin of the CP-M with the retracted central vertex H, b) the origin of CP-m with the extracted central vertex H*

The one is generated when the central vertex ***H*** of the unit pentahedral cell is retracted into the interior of the solid (Fig 2-a), which gives the major height of **CP** (***CP-n-M***). The other is generated when the central vertex **H** of is extracted to the exterior (Fig 2-b), giving the minor height of **CP** (***CP-n-m***).

Determination of the exact vertices' positions and all the linear and angular parameters needed for generation of ***CP II***, relies on the iterative procedure based on setting up spheres of radius ***R=a,*** where ***a=AB*** (the side of the base polygon). The sphere on which surface lie all the outer vertices of the unit pentahedral cell ***ABIJKH*** (marking is retained related to [11]) is set with the center in the vertex ***H***, due to the congruence of the cell's edges. The plane ***a*** which is determined by the vertex (***A***) of the base polygon and the axis (***k***) of the solid, which passes through the centroid ***C*** of the polygon, perpendicular to its plane, intersects the sphere ***s*** by the circle ***c***. The intersection point ***K*** of the circle ***c*** and the axis ***k*** gives the position of the vertex ***K***, the common vertex of all the unit cells in the ***CP II***.

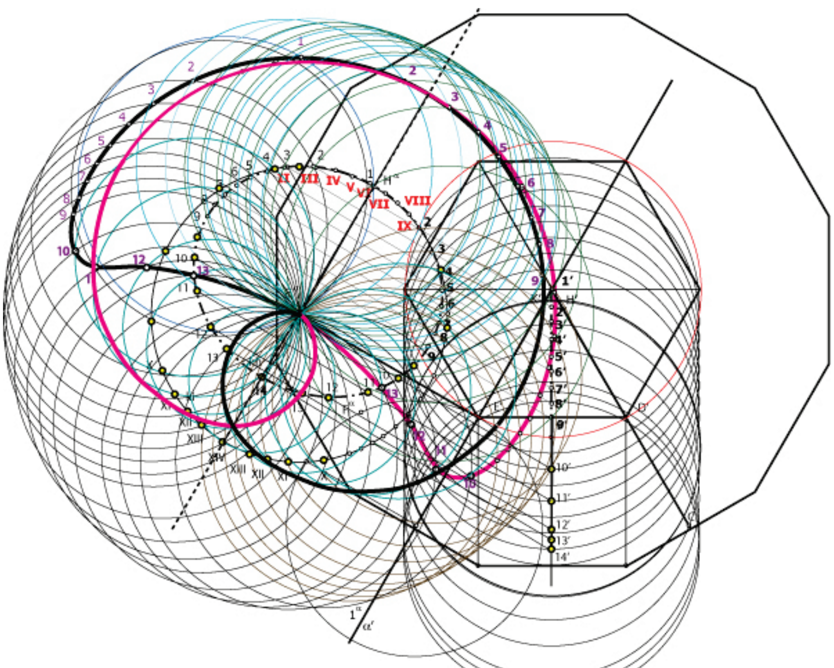
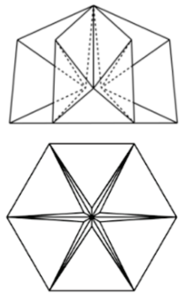
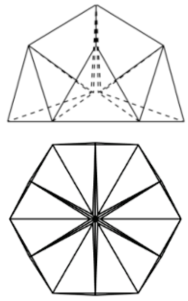
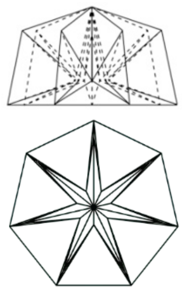
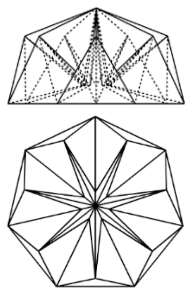
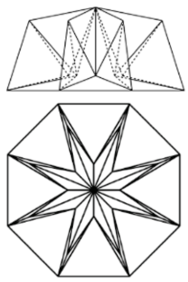
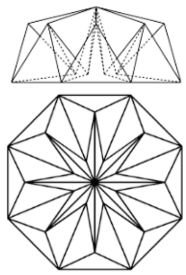
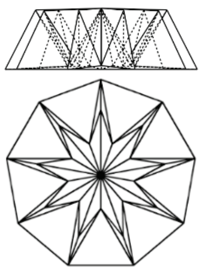
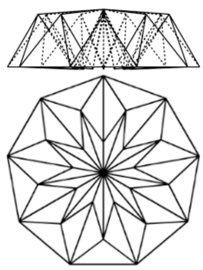


Figure 3*. The trajectory of the vertex K in the plane a*

Since the position of the vertex ***H*** is still vague, apart from the fact that it lies on its circle of rotation ***e*** of $r = \frac{a\sqrt{3}}{2}$ for the axis ***AB***, we may iterate the position of the sphere. Each possible position of the vertex ***K*** in the plane ***a***, will be located on the curve of the eight order, as shown in the Fig. 3, and explained in [11]. The curve – the trajectory of the vertex ***K*** - is a combination of two quartic curves: the bean curve and the Limaçon of Pascal. A half of each curve represents the position of the vertex ***K*** for a single continual movement of the chosen type of the unit cell: the pink one shows the position of the unit cell ***ABIJKH*** with retracted vertex ***H*** while mechanically moving around axis ***AB***, and the black one shows the movement of the unit cell ***ABCIJK***H with the extracted vertex ***K***. The axis ***k*** intersects these quartic curves at two pairs of real (and two pairs of imaginary) points, giving the four possible solutions for the position of the vertex ***K***, in symmetrical pairs regarding the plane (***1'***) of the base polygon. Two of them will give the solids of the major height (intersection with the bean curve), while the other two will give the solution for the solids with the minor height (intersection with the Limacon of Pascal). In this manner, it is possible to form two different ***CP II*** types for the polygonal bases ***n=6***, ***n=7***, ***n=8*** and ***n=9***. The fewer sides in the base polygon (***n<6***) will result with the intersection of the faces, which would be inconsistent with one of the main criteria for the formation of these solids, guided by the needs of the engineering profession. Also, the greater number of sides in base polygon (***n>9***) will result with the intersection of the lateral faces with the base, thus the solid with the requirements assigned could not be formed. Even in the case of ***CP-9-m***, there is occurrence of lateral sides' intersection with the base polygon's face, so this representative is discarded as unfit for a Concave Pyramid. However, the lateral surface itself can be used as a part of a polyhedral structure, if elongated (Fig. 5). The similar situation occurs with the decagonal base. The lateral surface may be formed, but in the case of ***CP-10-M***, the vertices ***I*** and ***J*** will be situated below the basic face plane, whereat the intersection of faces occurs, while for ***CP-10-m*** the vertex ***H*** will be set below the basic face plane, and the intersection od the faces occurs again. Hendecagonal base, and any base of ***n>10*** will not be supportable even for formation of the lateral surface, because there would be no intersection of the axis ***k*** with the octic trajectory curve.

In the **Table 1** we present the top and side views of the eight representatives of ***CP II***.

Table 1. *The top and the side view of CP II type A, n=6 to n=9*

mark	Retracted vertex H	Extracted vertex H	mark
CP-6-M F: 31 E: 48 V: 19			**CP-6-m** F: 31 E: 48 V: 19
CP-7-M F: 36 E: 60 V: 22			**CP-7-m** F: 36 E: 60 V: 22
CP-8-M F: 41 E: 68 V: 25			**CP-8-m** F: 41 E: 68 V: 25
CP-9-M F: 46 E: 76 V: 28			**CP-9-m** F: 46 E: 76 V: 28

3. THE VARIATIONS OF THE CP II - BIPYRAMIDS

An ***n***-gonal Concave Bipyramid (or dipyramid) is a concave polyhedron formed by joining an ***n***-gonal Concave Pyramid and its plane symmetrical image, base-to-base. Thereby we obtain only orto-bipyramids (***CbP-6***, ***CbP-7***, ***CbP-8*** and ***CbP-9***) as shown in Fig. 4, because there is an identical arrangement of faces over each side of the base polygon, due to the ***2n***-tuple radial symmetry of these polyhedra, i.e. gyro-bipyramids are not achievable.

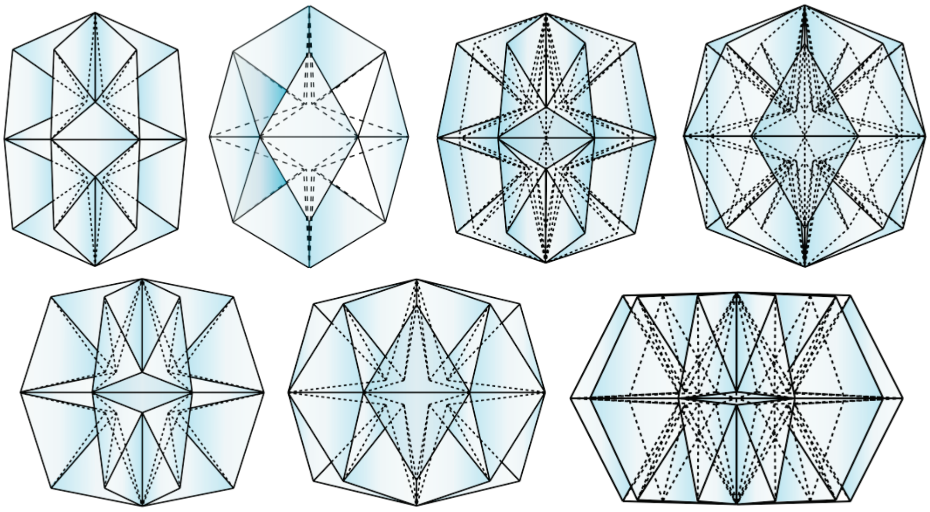


Figure 4. *Front views of Concave bipyramids, top row: CbP-6-M, CbP-6-m, CbP-7-M, CbP-7-m, bottom row: CbP-8-M, CbP-8-m, CbP-9-M*

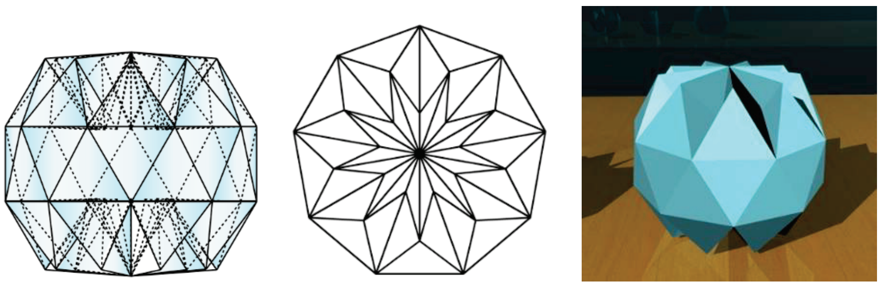


Figure 5. *Front view, top view and 3D model of Concave gyroelongated nonagonal Bipyramid CgebP-9-m*

Notice that all of these bipyramids (Fig. 6 and Fig. 7) will be also deltahedra, since their base polygons will be hidden in the interior of the solids. The last of ***CP II*** representatives, ***CP-9-m***, will not be able to form bipyramid, because its interior vertices ***H*** (the central vertices of the spatial pentahedral cells) will have negative height, related to the plane (***1'***) of the base polygon, so the intersection of the faces will occur. Nevertheless, there is a possibility of elongated bipiramids, or, in order to form a deltahedron, a gyroelongated nonagonal concave bipyramid, as the simplest case of deltahedral elongation (Fig. 5).

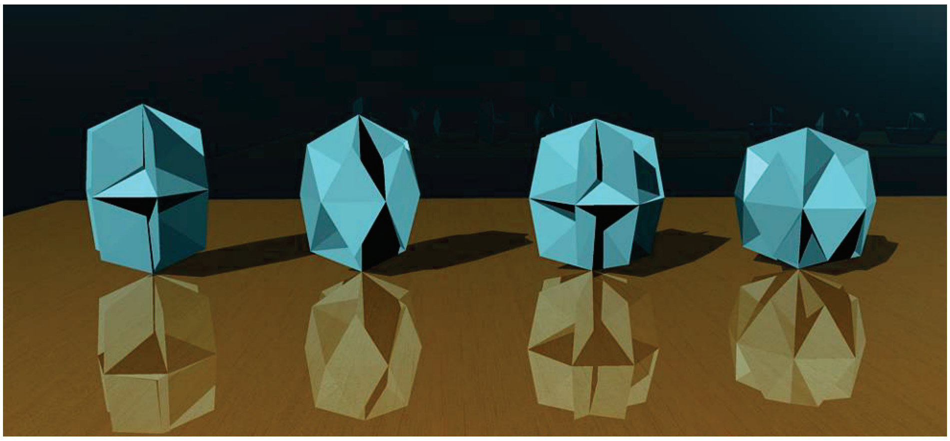


Figure 6. *Four representatives of CbP: CbP-6-M, CbP-6-m, CbP-7-M, CbP-7-m*

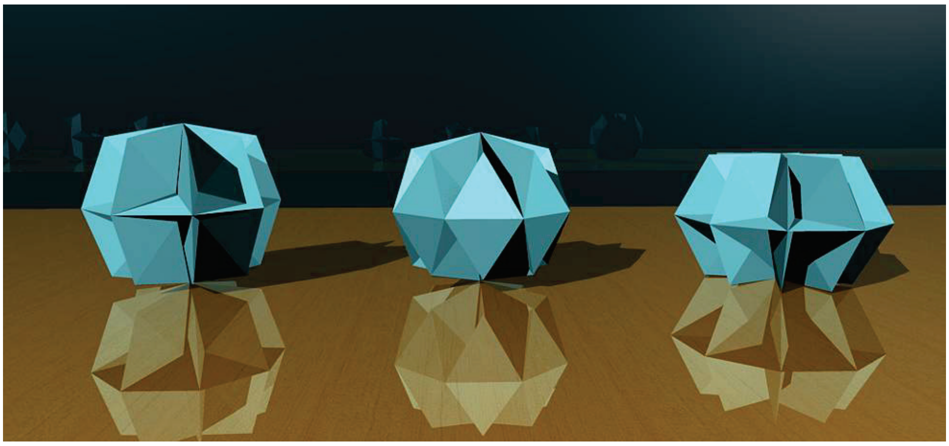


Figure 7. *Three representatives of CbP II: CbP-8-M, CbP-8-m, CbP-9-M*

4. ELONGATIONS

Using ***CP II*** as the basic building blocks, we can create multiple variations of concave polyhedra by adding the appropriate polyhedral extensions, such as: prisms, antiprisms or Concave Antiprisms of second sort (***CA II***) [10]. Thereby, in cases of gyroelongated and conca-elongated [7] bipyramids, we can obtain various deltahedral forms, appropriate for further consideration as feasible forms in architecture, suitable due to unification of its elements.

In Fig. 8 we show twelve representatives of possible variations just of the Octagonal Concave Bipyramid of second sort (***CbP-8 II***), from simple elongations by prisms, gyroelongations by antiprisms, to conca-elongations by Concave Antiprisms of second Sort (***CA II-M*** and ***CA II-m***) [7], [10]. In Fig. 9, 10 and 11, we show their rendered 3D models.

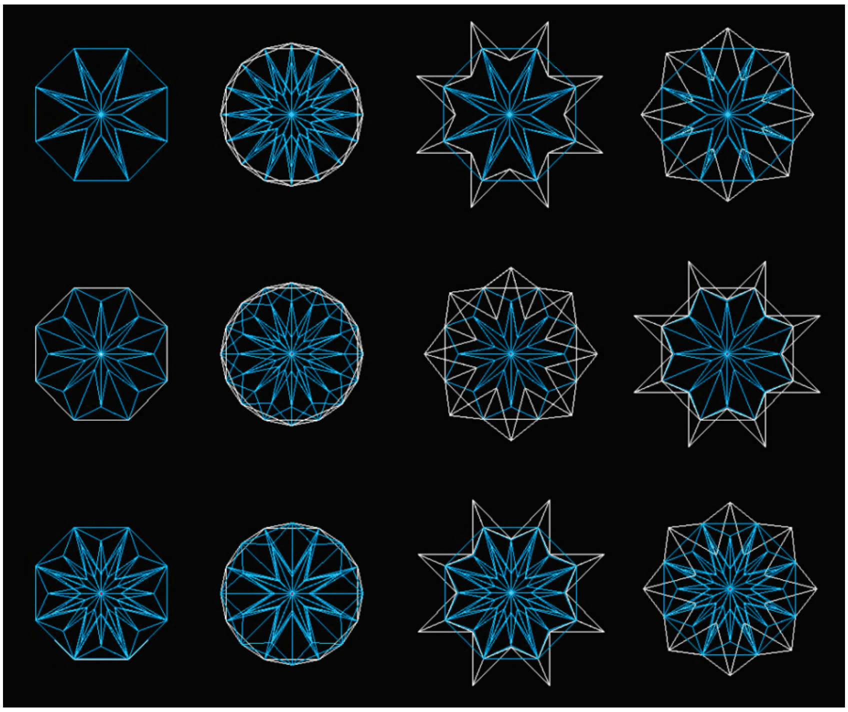


Figure 8. *The top view on 12 variations of elongated CbP-8 II:*
Top: CebP-8-M, CgebP-8-M, CceMbP-8M, CcembP-8-M (Fig. 9)
Middle: CebP-8-m, CgebP-8-m, CceMbP-8-m, CcemdP-8-m (Fig. 10)
Bottom: CebP-8-Mm, CgebP-8-Mm, CceMbP-8-Mm, CcembP-8-Mm (Fig. 11)

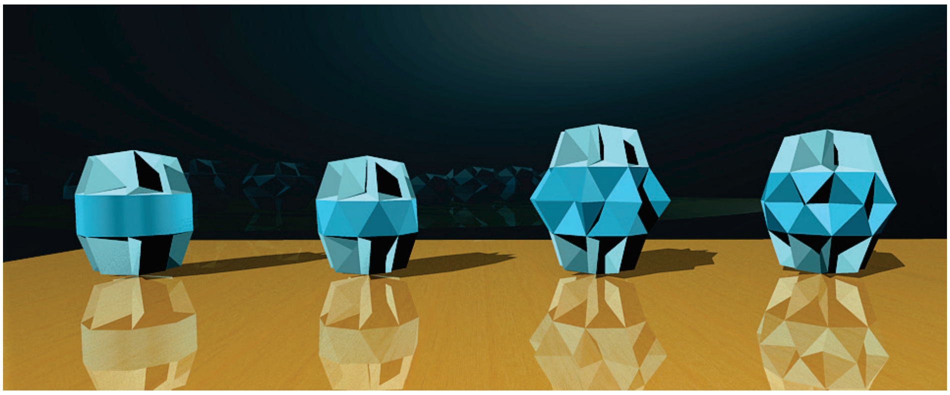


Figure 9. *Elongated octagonal Concave Bipyramids II – M(ajor height)*

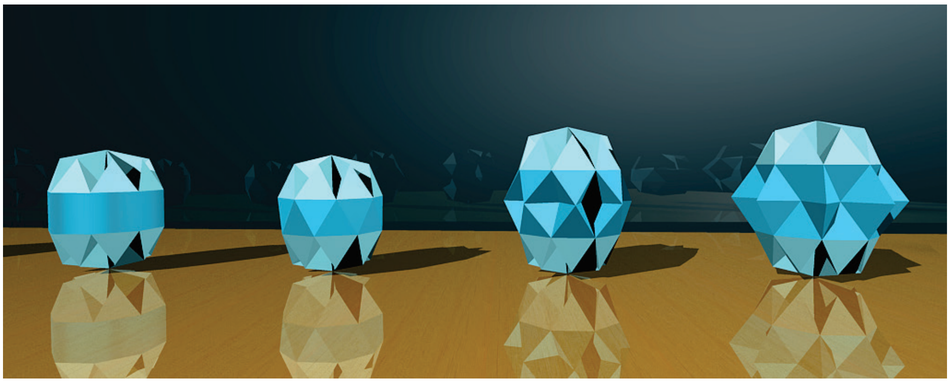


Figure 10. *Elongated octagonal Concave Bipyramids II – m(inor height)*

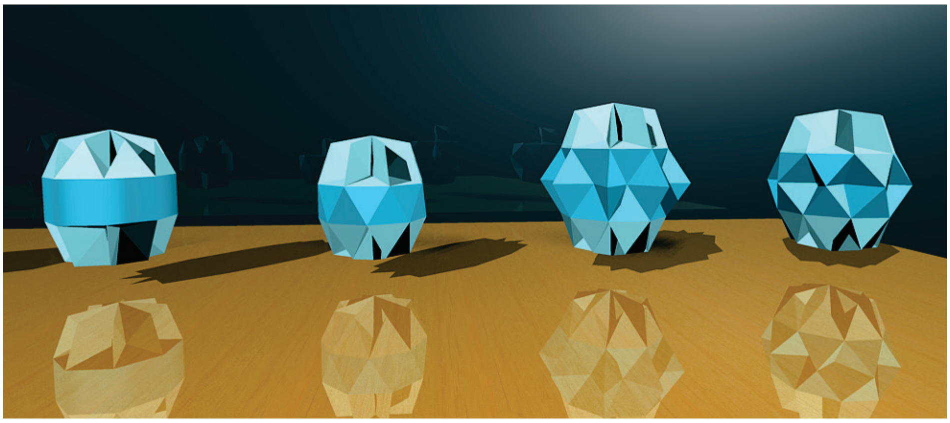


Figure 11. *Elongated octagonal Concave Bipyramids II –Mm (combinated)*

The **Table 2** presents possible variations of the concave polyhedra based on the geometry of the Concave Pyramids of second sort, from basic solids, elongated, gyroelongated and conca-elongated pyramids, to Concave Bipyramids and their elongations (even considering deltahedral structural shells of lateral surfaces for decagonal base).

***Table 2**. possible variations of CP-II, with bipyramids and elongations*

Type		n	6	7	8	9	10
Concave Pyramids Of second Sort	1	CP - n - M	✓	✓	✓	✓	-
	2	CP - n - m	✓	✓	✓	-	-
	3	CP - e - M	✓	✓	✓	✓	✓
	4	CP - g - m	✓	✓	✓	✓	✓
	5	CP - ceM - M	✓	✓	✓	✓	✓
	6	CP - ceM - m	✓	✓	✓	✓	✓
	7	CP - cem - M	✓	✓	✓	✓	✓
	8	CP - cem - m	✓	✓	✓	✓	✓
Concave Bipyramids Of second sort	9	CbP - n - M	✓	✓	✓	-	-
	10	CbP - n -m	✓	✓	✓	✓	-
	11	CbP - n - Mm	✓	✓	✓	✓	✓
Elongated Gyroelongated And Conca-elongated Bipyramids Of second sort	12	CebP - n - M	✓	✓	✓	✓	✓
	13	CebP - n - m	✓	✓	✓	✓	✓
	14	CebP -n -Mm	✓	✓	✓	✓	✓
	15	CgebP - n -M	✓	✓	✓	✓	✓
	16	CgebP - n -m	✓	✓	✓	✓	✓
	17	CgebP -n - Mm	✓	✓	✓	✓	✓
	18	CceMbP - n - M	✓	✓	✓	✓	✓
	19	CcembP -n -M	✓	✓	✓	✓	✓
	20	CceMbP -n - m	✓	✓	✓	✓	✓
	21	CcemdP -n -m	✓	✓	✓	✓	✓
	22	CceMbP - n - Mm	✓	✓	✓	✓	✓
	23	CcembP - n - Mm	✓	✓	✓	✓	✓

We can notice that 109 new concave polyhedral solids can be obtained, of which 72 will be deltahedra.

5. CONCLUSIONS

Using the method similar to one for the generation of ***CC II*** it is possible to obtain Concave Pyramids of second sort (***CP II***), seven of them, by whose variations it is possible to provide another 102 new concave polyhedra based on their geometry, 72 of which will be deltahedra. Due to unification of their building blocks, these polyhedra may be suitable for further consideration in terms of feasible forms for use in architectural practice.

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