# One Example of Cascaded Reservoirs Hydropower System Modelling for Master Plan Analysis

D. Ivetić*, D. Prodanović*, M. Milašinović** and T. Dašić*

* Institute for Hydraulic and Environmental Engineering, Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, Belgrade, Serbia (E-mail: *divetic@hikom.grf.bg.ac.rs*; *eprodano@hikom.grf.bg.ac.rs*; *mtina@grf.bg.ac.rs*)
** Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, Belgrade, Serbia (E-mail: *milosolimilos@gmail.com*)

**Abstract**

Amongst numerous types of hydropower plant dispositions, cascaded reservoirs with small reservoir volumes stand out for its complexity and difficulties in modelling and control. These systems also tend to be expensive with questionable profitability. That's why it is important to obtain accurate assessment of the hydraulic parameters and energy production in early stages of the project. Goal of this paper is to develop the model which is easy to establish and will solve the dynamical hydraulics of the system with sufficient accuracy for Master plan analysis. Model is developed inside Simulink environment. Diffusive wave hydraulic equations are discretized on the staggered computational grid inside reservoir elements. Reservoirs themselves are approximated as prismatic straight channels. The model can be used to test different automatic operating rules for the dam gates, and predict more accurately some important parameters like annual energy production, flood wave transformation etc. Presented approach was applied on the Zapadna Morava River case study, where observed hydrographs were used for testing. Computed values of annual energy production were compared to the ones from the original master plan, showing significant improvement in accuracy. Model also showed that the case study system of reservoirs, has poor capability to dampen flood waves, and that it can safely pass through 10% and 1% flood waves.

**Keywords**

Cascaded reservoirs hydropower system; Modelling; Energy production; Master plan analysis; Flood protection

## INTRODUCTION

Since one of the goals of the modern societies is to shift the majority of energy production from fossil sources to renewable energy sources, new sites for hydropower exploitation earlier not attractive for exploitation are the subject of the interest for many researchers around the world. Amongst numerous types of hydropower plant dispositions, those with cascaded reservoirs of rather small reservoir volume stand out for its complexity and difficulties in modelling and control since it is a complex nonlinear system (Mahmoud, 2004). Usually these systems are expensive and profitability of the project can be questionable. Numerous researchers have addressed the issue of cascade reservoirs system modelling along with the design of supervisory control and optimization system (Sohlberg, 2002; Linke, 2010; Sahin, 2010).

In master plan analysis, standard practice is to use the simplest hydraulic computation to assess the energy production and check the hydraulic parameters. In most cases this will involve the flow duration curve and simple hydraulic of the flow control objects (gates, turbines, etc). Such approach, lacking in accurate system dynamic presentation, will not allow the accurate assessment of interaction between cascaded reservoirs nor the operation modes of hydropower itself. The other possibility is to develop the "heavy duty" hydraulic model which would take into the account numerous interconnections between the reservoirs themselves as well as between the system and its natural surroundings. A number of profiles with detailed geometry has to be entered to define the river flow and hydraulic of reservoir. This would be a complicated, but also interesting task, from the researcher's point of view. Major issue would be the time needed to develop the model and the computational time needed for the simulations. Goal of this paper is to develop the model which is easy to establish and will solve the dynamical hydraulics of the system with sufficient accuracy for the master planning phase. The model can be used to test different automatic operating rules for the dam gates, and predict more accurately some important parameters like annual energy production, flood wave transformation etc. in the early stages of project.

The dynamical hydraulic model of the reservoirs interconnection, using diffusive wave mathematical model (Miller & Cunge, 1975), is presented in this paper. Reservoirs themselves are approximated as prismatic straight channels. Different types of weirs, or gates acting as weirs, turbines and control rules can be implemented in the model. Case study of the Zapadna Morava river cascaded reservoirs (ZMRCR) system (Systema Rinova Uno, 2014) has been used to test the presented model. Flap gate operating rules were developed in order to achieve maximal energy production as well as to allow the system to react adequately on the incoming flood wave.  Real observed hydrographs were used for model testing based on which the energy production estimates were obtained, as well as the estimate of the flood protection role of the ZMRCR system.

## METHODS

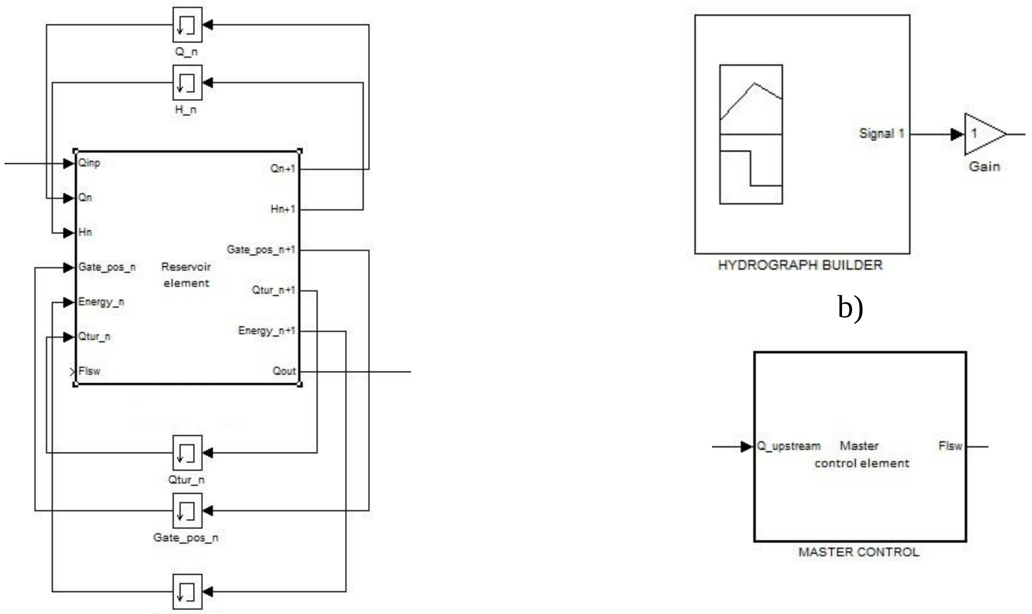
Cascaded Reservoir Hydropower systems (CRHS) are specific and complex in many ways. Model of cascaded reservoirs includes nonlinear input and output parameters, as well as a nonlinear flow rates and dynamical hydraulic heads. Every single reservoir in the system is connected with the rest of the reservoirs, forming specific relationships which can be accounted for in a model depending on its complexity. This type of modelling task can be successfully performed inside Simulink environment The Simulink allows the needed flexibility for dynamical modelling, with the possibility to easily program new functions or elements to be implemented in the model. In the following chapters, brief explanation of the presented modelling approach is given, as well as for the governing hydraulic equations along with the single reservoir numerical model.

### The Simulink Cascaded reservoirs model

CRHS can be decomposed and represented as a group of Simulink elements in large number of ways. The way decomposition is done clearly depends on the amount of details needed to successfully fulfil the modelling goal.  To make the things as simple as possible, only few types of main functional elements need to be defined. Elements of same type, as it will be shown, may differ from each other in some details (e.g. different geometry of the reservoirs). First type of the elements used and the most important ones are the reservoir elements (Fig. 1a), second are hydrograph builders (Fig. 1b) and third are master control elements (Fig. 1c). Apart from these, different auxiliary elements are used (like memory, sum, rate transition blocks), but since they are used to support the computation they are not worth mentioning.

#### *Reservoir elements*

Reservoir elements are user-defined functions, with the task to compute the dynamical heads and discharges in every time step. Since cascaded reservoirs hydropower systems are built by damming the river in specific places, usually they are run-of-river type or with small or moderate storage. In this modelling approach, each reservoir is approximated with a prismatic channel. Cross sections of the channels are complex trapezoids (Fig. 2). If data about the actual river cross-sections are available, then it can be used to define more accurately the dimensions of the channel cross section. It is important to obtain an accurate approximation of the actual reservoir storage volume, by calibrating the length of the channel.



a)

b)

c)

**Figure 1**. Main elements used in the CRHS Simulink model: a) Reservoir element, b) Hydrograph builder and c) Master control element

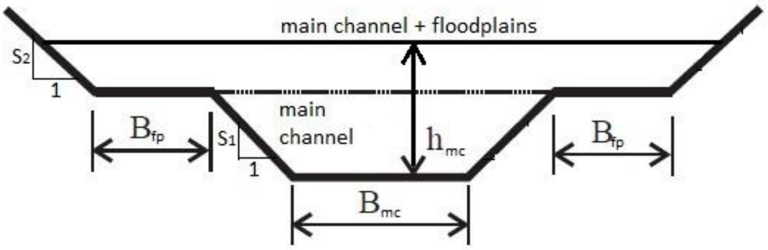


**Figure 2**. Approximated reservoir cross section

Inside this element, dam gates are also modelled. Different types of gates can be implemented (flap, radial, vertical etc.) and for each of them, adequate discharge coefficient curve needs to be used. Gate movement is controlled by the algorithm defined by a user. Someone prefers using relatively simple control loops, which define the movement command (raise, lower or hold current position) depending on the computed water level in front of the dam. On the other side different variations of the PID, MPC or any other controls could be used (Siebenthal 2005). It should be noted that when using PID controls, calibration can be quite difficult since output needs to be discrete (three options available) since the gates are moving at the finite speed, not analogue like in most cases of PID controller exploitation.

Turbines implemented in the model, will start operating as soon as the minimal operating discharge is available in the cross section located in front of the dam. For each turbine, flow can be determined, based on which the actual efficiency coefficient at the present moment can be derived using the manufacturer's efficiency curve. Finally, using this data the actual energy production in each time step for the $i$-th turbine ($E_i$) is calculated using the following formula:

$$E_i = \Delta t \rho g \eta_i^{tur} \eta^{gen} \eta^{tr} Q_i H^{net} \qquad (1)$$

where $\Delta t$ is the simulation time step, $\rho$ is water density, $g$ is gravitational acceleration, $\eta_i^{tur}$ is the $i$-th turbine efficiency, $\eta^{gen}$ and $\eta^{tr}$ are the generator and transformer efficiencies, $Q_i$ is the $i$-th turbine discharge and $H_{net}$ is the net falling height.

*Hydrograph builders*

Hydrograph builder elements are made out of predefined *Signal Builder* and *Gain* elements (Fig. 1b). For the purpose of the model testing, it is to be expected that a significant number of test scenarios will be examined. Test scenarios will most probably differ in the shape and value of the input signals to the model, which are the main stem and tributary river hydrographs. Ideal case is when the real observed hourly (at least) hydrographs for the analysed river and its influencing tributaries are available for the user.

Depending on the goal of the analysis, different periods of the year can be of interest for the user; for example if the flood control is analysed then the high-water period hydrographs are of interested, for energy production whole year would be of interest etc. Since the flood events like 100-year, 1000-year or Probable Maximum Flood (PMF) are a statistic category (rarely there are recorded hydrographs of these events), user can scale up through *Gain* element some observed flooding event to obtain rough approximation of these less probable events.

*Master control elements*

When using the model with high-water hydrograph scenarios, user needs to define the rules when the turbines are going to stop producing the energy and spillway gates start opening due to the incoming flood event. These rules should be defined inside the master-control element (Fig. 1c). Through this single element, user can override local gate algorithms and control both turbine and spillway gate operation in all of the dams and hydropower plants in the cascaded system. The definition of the master control rules is a case-specific task. Topology, river size, dam type, gate type etc. influence the choice of rules. Sometimes the spillway gate controlling algorithm is efficient in the gate manipulations even through flood events, so the extra master control element is not necessary.

It should be noted again, that spillway gates in most cases move at a slow rate. This information must be taken into the account when defining the operating algorithm. If the set of conditions is defined that will initiate the flood protection procedure, once the conditions are met, there should be enough time left for the procedure to be conducted. Insufficient time, might allow the flood wave to reach the reservoir before the procedure has been completed which will cause overtopping of the dam. If needed this element can control also the storage volume depletion through turbines of each reservoir for the flood way absorption.

## Mathematical model

The motion of the water in one dimension is modelled by a set of nonlinear partial differential equations, also known as Saint Venant equations (Abbot, 1989). These equations are derived from the conservation of the mass and momentum and certain assumptions. Full derivation of these equations along with the used assumptions can be found throughout the hydraulic literature. In this paper certain simplification has been made on the dynamic equation, where the term describing the change of the velocity head in space is neglected. It is expected that this term will have the smallest influence on the results. This way diffusive wave mathematical model is obtained:

$$\frac{\partial h}{\partial t}+\frac{1}{B}\frac{\partial Q}{\partial x}=0 \tag{2}$$

$$\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{\partial h}{\partial x}+I_d+\frac{n^2Q^2}{A^2R^{4/3}}=0 \tag{3}$$

Where $h$ is the hydraulic head, $B$ is the wetted width, $A$ is wet cross section, $I_d$ is river bed inclination, $n$ is the Manning friction coefficient and $R$ is the hydraulic radius. Diffusive wave allows the simulation of water flow under the mild backwater effect.

### Numerical model

Solution of the nonlinear Saint Venant’s equations has to be computed with some of the available numerical methods due to the absence of a closed form solution. Usually Finite Difference Methods are used to discretise the PDE in both space and time. In this paper discretised equations are solved on a staggered computational grid by an explicit scheme. Explicit scheme implies that the in each space and time step, CFL condition has to be satisfied. This condition dictates the size of the time step, usually imposing relatively small values. Staggered grid is used to obtain formally same accuracy as an order higher methods with twice less space steps. This way certain speedup of the algorithm can be obtained with minimal effects on the quality of the results. Spatial grid points in which discharge is defined are alternately replaced with the grid points in which the head is computed. Staggered grid with the continuum and dynamic equation finite difference stencils is presented in Figure 3.

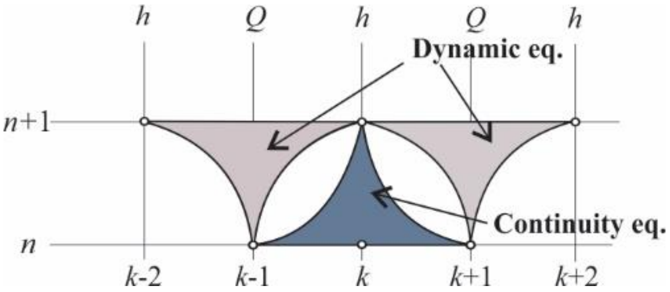


**Figure 3**. Finite difference scheme for diffusive model equations on staggered computational grid

### CRHS model performance indicators

In order to assess the CRHS performance in terms of energy production, computed values of annual energy production will be compared to the ones obtained by using the flow duration curves. This is the usual manner how estimates of energy production are obtained in the master plan stage of the project.

On the other hand CRHS flood protection role assessment was obtained through two indicators: *attenuation coefficient* defined as the ratio of the input and output hydrograph peak discharge; and *number of dams overtopped* during the flooding event.

## CASE STUDY: ZAPADNA MORAVA RIVER CASCADED RESERVOIRS SYSTEM

ZMRCR system project has been reactivated in 2008 as a part of the initiative to increase the utilization of the hydropower potential of the Republic of Serbia. Since early ideas dating back to the 1961, conceptual design has evolved from five small hydropower plants to latest design by “Sistema rinova uno” with ten small hydropower plants. The project is currently at hold, although data from the conceptual design has been used for this case study. River reach enveloped by this design is around 75 km, starting from the city of Kraljevo and ending at the town of Varvarin. Google map screenshot with the position of the dams is shown in Figure 4. Total river bed height difference between these two points is approximately 50 m. At this stage recommended hydropower plants were standardized in terms of type, number and installed flow rates of turbines. Each plant should be equipped with three Kaplan bulb aggregates of the same characteristics with the total operating flow rate of 180 $m^3/s$. Apart from the hydropower plant characteristics, all of the dams are equipped with five bottom hinged flap spillway gates, used for the regulation of the reservoir heads as well as for the flood water evacuation. Each of the hydropower plants is categorized as a run-of-river, small hydropower plant (SHP). All the necessary data for the analysis presented was available from the documentation.

**Figure 4**. Disposition of the ZMRCR dams and hydropower plant positions

## ZMRCR model

Using the available data, ZMRCR system was modelled in Simulink environment. Reservoirs were approximated as straight channels, with the complex cross sections as shown in Figure 2. Data for reservoir approximation is presented in table 1, including the main channel and floodplains width, side slopes, maximal falling height, gate height, total spillway width, river bed inclination, length of the channels and space step size. Apart from the main Zapadna Morava River, three tributary rivers were integrated into the model: Gruža, Rasina and Vrnjačka reka. At the points of confluence continuum equation must be satisfied.

**Table 1**. Cross section and approximated reservoir data for ZMRCR system

| no. SHP | Name | Cross section data | | | Approximated reservoir data | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $B_{mc}$ | $B_{fp}$ | $S_1 = S_2$ | $H_{max}$ | $H_{gate}$ | $B_{sp}$ | Id | Length | dx |
| - | - | (m) | (m) | (-) | (m) | (m) | (m) | (‰) | (m) | (m) |
| 1 | Vitanovac | 19 | 33.0 | 2.00 | 4.10 | 6.35 | 80 | 0.7 | 4800 | 600 |
| 2 | Vraneši | 15 | 34.0 | 2.00 | 4.10 | 6.35 | 80 | 0.7 | 2500 | 250 |
| 3 | Stubal | 10 | 16.0 | 2.00 | 5.85 | 8.3 | 80 | 0.7 | 9000 | 500 |
| 4 | Grabovac | 14 | 35.0 | 2.00 | 5.85 | 8.3 | 80 | 0.7 | 4200 | 300 |
| 5 | Medveđa | 15 | 25.0 | 2.00 | 5.85 | 8.3 | 80 | 0.7 | 11200 | 700 |
| 6 | Počekovina | 12 | 25.0 | 2.00 | 4.10 | 6.35 | 80 | 0.7 | 7200 | 600 |
| 7 | Selište | 12 | 25.0 | 2.00 | 4.10 | 6.35 | 80 | 0.7 | 6000 | 600 |
| 8 | Globoder | 16 | 30.0 | 2.00 | 4.50 | 6.35 | 80 | 0.7 | 8400 | 600 |
| 9 | Kukljin | 18 | 32.0 | 2.00 | 4.10 | 6.35 | 80 | 0.7 | 5000 | 500 |
| 10 | Bošnjane | 18 | 40.0 | 2.00 | 5.85 | 8.3 | 80 | 0.7 | 5500 | 550 |

In each reservoir element, flap spillway gates were modelled using the discharge coefficient curve available in the USGS handbook (Rantz, 1982). Spillway gates manipulation was controlled by the simple automated algorithm with the task to keep the falling height for the turbines as high as possible in order to maximize energy production. This algorithm is determining the movement of the gate, based on the computed water level in front of the dam, and predefined optimal range. If the level is lower than the bottom boundary of the range, gate is rising, if it is higher than top boundary, gate is lowering, and if the level is inside the range gate is not moving. In order to prevent frequent gate movements, range size is 0.2 m. In real-life application, algorithm would use the measured water level. Controlling the gates, in this manner only, is not sufficient, due to its highly local character and independence from the rest of the cascaded reservoir system. In addition higher level controls are defined inside master control element. It is usually in the high-water or flood events, when the strong need for the cooperation between all of the dams is emphasized. In the presented example, based on the data of the incoming hydrograph into the highest reservoir, flood procedure can be initiated in order to open the spillway gates and shutdown turbines. For procedure to start, flow must exceed the rate of 600 $m^3/s$ (approximately six times the average flow). It should be noted that if the hydrographs with the time step of an hour or less are available, extra condition should be imposed in terms of the gradient of discharge change.

### Test scenarios

ZMRCR system model was used in this research for determining the maximum angular velocity of the spillway gates, prediction of energy production and for defining the flood protection role of the presented system in terms of its capability to dampen the flood wave and safely pass it through the spillway gates without dam overtopping. Angular velocity was established using fictive hydrographs and was later verified with the observed ones. Annual energy production estimation was performed using the observed daily hydrographs for five years from the hydrological station Jasika, located inside the river reach of the ZMRCR system (Fig. 5). Finally flood protection role of the ZMRCR system was analysed using the observed flood event from March 2006 (Fig. 6) which matches the 10-year flood event. This flood event was scaled up, so the peak discharges match the ones for the 100-year and 1000-year. Time step size was fixed to 2s for all of the simulations.

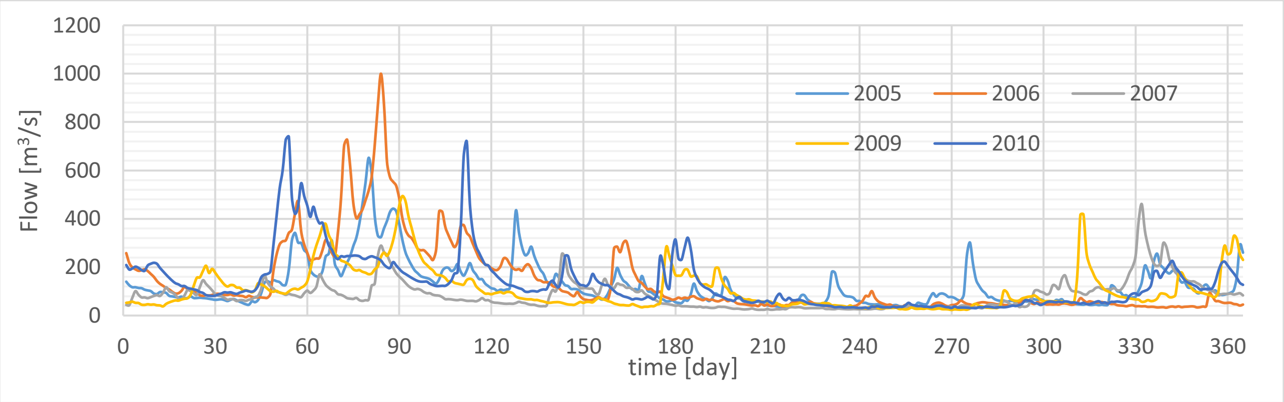


**Figure 5**. Observed daily hydrographs on the Zapadna Morava River for 2005, 2006, 2007, 2009 and 2010; Data acquired from hydrological station Jasika

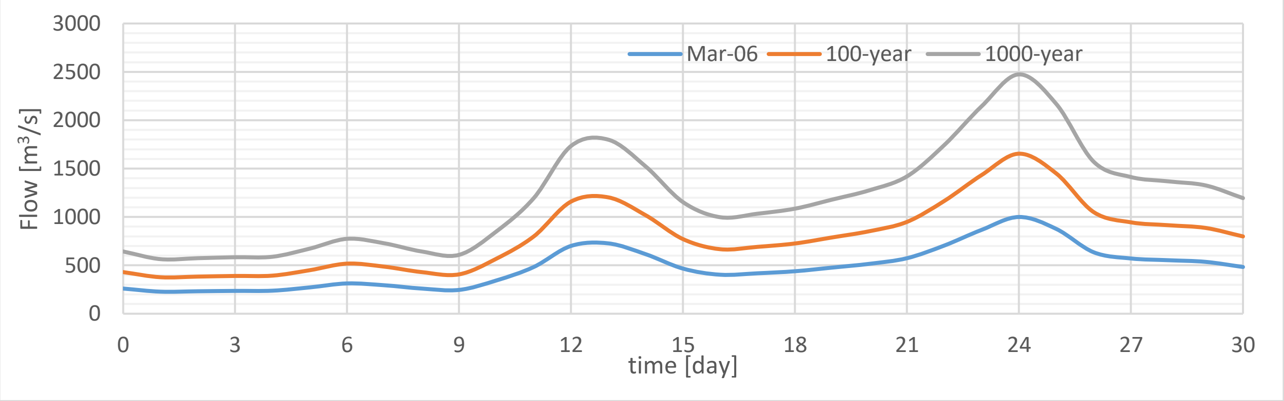


**Figure 6**. Original observed daily hydrographs for the flooding event in March 2006 and scaled up hydrograph matching the discharges for 100-year and 1000-year waters

## RESULTS AND DISCUSSION

Maximal angular velocity ($\omega_{max}$) of the flap gates was determined, by monitoring the exiting hydrographs from the reservoirs. By visual inspection it was concluded that with the value of $\omega_{max}$ = 4.5deg/min, downstream wave induction from spillway gate lowering will be prevented. Annual energy production estimates were computed using the observed hydrographs (Fig. 5), in both the traditional manner by utilizing the flow duration curves and the presented approach. Comparison for the selected five years are presented in the table 2.

**Table 2**. Annual energy production estimation computed by the presented model and using the flow duration curve for years 2005, 2006, 2007, 2009 and 2010

| Year | | 2005 | 2006 | 2007 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Model | $E_{mod}$ [GWh] | 367.37 | 316.59 | 320.03 | 338.36 | 363.11 |
| Flow Duration Curve | $E_{dur\_c}$ [GWh] | 378.46 | 329.15 | 270.37 | 328.86 | 385.75 |

It can be seen that for some of the years (2007 and 2009) model estimates higher energy production, while for some (2005, 2006 and 2010) lower energy production than the traditional way. Results were satisfying since the higher energy production matches the low-water years, which shows that the model has better capability to exploit the available inflow to the ZMRCR system. On the other hand lower energy production was obtained in the high-water years, where flooding events prevented the energy production for some period in the model. In total it can be expected that energy production estimate obtained in this manner is more accurate than using the duration curve.

Values of flood protection indicators of the ZMRCR system were computed for observed flooding event from March 2006 and scaled up flooding events with peak discharges matching the 100-year and 1000-year (Table 3). ZMRCR shows very poor capability to dampen the peak discharges of the flooding event, which is expected since all of the reservoirs have relatively small storage volume. As well as in the master design project (Sistema Rinova Uno), testing had shown that ZMRCR system is capable of safely passing through 10-year and 100-year water while 1000-year flood causes overtopping of three dams.

Computational time needed for the annual energy production computation was around 4 hours, while for monthly flood events was not more than 20 minutes on i7 intel processor.

**Table 3**. Flood protection indicators computed for the observed, 100-year and 100-year flooding event

| Flood event | Observed, 10-year | 100-year | 1000-year |
|---|---|---|---|
| Attenuation coefficient | 0.95 | 0.99 | 1.00 |
| Number of dams overtopped | 0 | 0 | 3 |

## CONCLUSIONS

Modelling of cascaded reservoir hydropower systems performance can be a difficult task. Apart from that, they are expensive systems, so the profitability of these projects needs to be assessed in detail. In early stages of the project, like master plan and concept design, sufficiently accurate estimates of the energy production, flood protection role etc. are needed. In this paper, a new easy to establish, modelling approach is presented, intended for these project stages. Idea was to make things as simple as possible while retaining all the necessary components of the real life system.

Diffusive wave mathematical model was used to describe one-dimensional water movement, and the equations were solved by finite difference method on a staggered computational grid. Simulink environment was used for the modelling, where three types of main elements were defined. Inside reservoir elements, reservoirs are approximated as prismatic, straight channels. Spillway gates, turbines as well as the local control system are also defined inside this elements. Hydrograph builder elements are used to create different testing scenarios, while master control elements are used for overriding the local gate and turbine controls in some specific cases like flooding events. Presented approach was applied on the Zapadna Morava River Cascaded reservoir system. First fictitious hydrographs were used to determine the angular velocity of the flap gates. Observed hydrographs for five different years from the local hydrological station Jasika, were used for the annual energy production estimation and the flood protection role definition. Results of energy production computations were compared with the ones from the existing master plan, showing deviations of up to 15%. This is a major difference, which could me make a significant impact on decision makers planning the CRHS project. Apart from this, it was shown through attenuation coefficient that ZMRCR system has a very small capability to dampen the peaks of the flood waves. Also it proved to be capable of passing through the 10% and 1% flood waves, while for the 0.1% model had predicted overtopping of three dams.

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