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**Deflection control for reinforced recycled aggregate concrete beams: Experimental database and extension of the fib Model Code 2010 model**

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**Abstract:** Recycled aggregate concrete (RAC) has emerged as a viable solution for solving some of the environmental problems of concrete production. However, design guidelines for deflection control of reinforced RAC members have not yet been proposed. This study presents a comprehensive analysis of the applicability of the fib Model Code 2010 (MC2010) deflection control model to reinforced RAC beams. Three databases of long-term studies on natural aggregate concrete (NAC) and RAC beams were compiled and meta-analyses of deflection predictions by MC2010 were performed. First, the MC2010 deflection control model was tested against a large database of long-term tests on NAC beams. Second, a database of RAC and companion NAC beams was compiled and initial and long-term deflections were calculated using the MC2010 model. It was shown that deflections of RAC beams are significantly underestimated relative to NAC beams. Previously proposed modifications for MC2010 equations for shrinkage strain and creep coefficient were used, and new modifications for the modulus of elasticity and empirical coefficient $\beta$ were proposed. The improved MC2010 deflection control model on RAC beams was shown to have equal performance to that on companion NAC beams. The proposals presented in this paper can help engineers to more reliably perform deflection control of reinforced RAC members.
Deflection control for reinforced recycled aggregate concrete beams: Experimental database and extension of the fib Model Code 2010 model

Running Head: MC2010 deflection control for recycled aggregate concrete beams

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ABSTRACT

Recycled aggregate concrete (RAC) has emerged as a viable solution for solving some of the environmental problems of concrete production. However, design guidelines for deflection control of reinforced RAC members have not yet been proposed. This study presents a comprehensive analysis of the applicability of the fib Model Code 2010 (MC2010) deflection control model to reinforced RAC beams. Three databases of long-term studies on natural aggregate concrete (NAC) and RAC beams were compiled and meta-analyses of deflection predictions by MC2010 were performed. First, the MC2010 deflection control model was tested against a large database of long-term tests on NAC beams. Second, a database of RAC and companion NAC beams was compiled and initial and long-term deflections were calculated using the MC2010 model. It was shown that deflections of RAC beams are significantly underestimated relative to NAC beams. Previously proposed modifications for MC2010 equations for shrinkage strain and creep coefficient were used, and new modifications for the modulus of elasticity and empirical coefficient $\beta$ were proposed. The improved MC2010 deflection control model on RAC beams was shown to have equal performance to that on companion NAC beams. The proposals presented in this paper can help engineers to more reliably perform deflection control of reinforced RAC members.

Keywords:
Recycled aggregate concrete; reinforced concrete beam; deflection; database; Model Code 2010
1. Introduction

The focus of this paper is on deflection control of cracked reinforced concrete (RC) beams, used to check one of the most important Serviceability Limit States (SLS) in all modern design codes and guidelines. Although deflection control has gained importance over the past decades, it is still among the most complex limit states of RC structures to model.

This is largely because of the large number of influencing factors such as the geometrical properties of the member, moduli of elasticity of concrete and reinforcement, concrete tensile strength, area and distribution of reinforcement, load intensity and history, stiffness reduction caused by cracking and tension stiffening, member structural system, and moment redistribution in statically indeterminate systems caused by stiffness reduction, shrinkage, and creep. However, the research of factors influencing deflection has overtaken the advance in producing calculation and prediction models capable of using this attained knowledge. Hence, more attention must be given to the calculation models themselves.

In the area of deflection control, as in RC design in general, the fib Model Code 2010 (MC2010) is a globally leading document. Its current version is built upon decades of experience and tradition from the CEB-FIP Model Code 1978 and 1990. Today, the fib (International Federation for Structural Concrete) is in the process of producing a new version, the fib Model Code 2020. In order to maintain its status of an innovative and visionary document, the new Model Code 2020 should include design provisions for new materials, such as ‘green concretes.’

Green concrete is a wide group of sustainable alternatives to traditional cement concrete, typically produced using waste and/or recycled materials. Since concrete is globally produced in amounts greater than 20 billion tons annually, it causes significant impact on the environment. The first significant environmental impact of concrete is through the global annual production of cement which is responsible for 7–10% of all anthropogenic CO$_2$ emissions. The second significant impact of concrete is through its end-of-life. What remains after a concrete structure is demolished is construction and demolition waste (CDW): in the EU, approximately 850 million tons of CDW are generated annually, accounting for 46% of total waste generated, while concrete can constitute more than 40% of CDW.

One of the most investigated solutions for producing green concrete is recycling concrete waste in order to produce recycled concrete aggregate (RCA) for use in the production of recycled aggregate concrete...
Since concrete is composed of natural aggregates bound by hardened cement mortar, after crushing concrete waste, the produced RCA is composed of natural aggregate particles with a certain amount of ‘residual cement mortar’ attached. This mortar influences most of RCA properties: RCA usually has higher porosity, lower density, and greater water absorption compared with natural aggregates (NA). When RCA is produced, typically only coarse RCA is used (particles size >4 mm). So far, RCA has mostly been applied in non-structural applications; however, it is recognized that the potential of RCA can be maximally exploited only if it is used for producing structural RCA.

Overall, RCA has been extensively and comprehensively studied. Most of the research has focused on short-term mechanical and durability-related properties. Comprehensive literature reviews analysing these properties of RCA compared with companion natural aggregate concrete (NAC)—usually designed with the same effective water-cement \((w/c)_{\text{eff}}\) ratio \((w/c)\) ratio not taking into account additional water added for RCA absorption)—were published in recent years. Researchers have also studied ways of predicting RCA properties that can be incorporated into design codes. For the modulus of elasticity, Silva et al. provided a comprehensive literature review and suggested a predictive expression as an extension of Eurocode 2. Tošić et al. provided empirical equations for predicting the shrinkage strain and creep coefficient of RCA as an extension of MC2010.

Ultimate Limit State (ULS) behaviour of RCA structural members has also been extensively studied, from studies on beams and columns, to push-over and shaking-table tests on almost full-scale RCA frame structures. Studying the flexural and shear behaviour of reinforced RCA beams, Tošić et al. presented a meta-analysis of performed experiments and demonstrated that RCA beams could be reliably designed according to existing Eurocode 2 provisions. Having all these recommendations and guidelines is necessary for engineers to confidently and reliably design RCA structures. However, one aspect of RCA design is still lacking – SLS and deflection control, precisely the area in which RCA structural behaviour is expected to differ mostly from companion NAC behaviour.

There are only a small number of long-term tests on reinforced RCA beams. Unfortunately, most of the studies are published as conference proceedings and do not offer sufficient information for detailed analysis. The studies vary in RCA properties (with water absorption, \(w.a., 1.9–6.0\%\)), geometric properties of the beams (with 2000–3700-mm spans, 200–300-mm cross-section depths, 0.5–1.6% reinforcement ratios).
and duration of sustained load (118–1000 days). The authors generally find larger deflections and greater crack widths in RAC beams compared with companion NAC beams with an identical \((w/c)_{\text{eff}}\) ratio.\(^{32-35}\) Even though some authors\(^{33-35}\) have tested the applicability of existing deflection control models (ACI 318 and Eurocode 2)\(^{2,3}\), so far, this was only done on own experimental results – with a too small number of results for any definitive conclusion.

Therefore, the aim of this study is to perform a comprehensive meta-analysis of existing experimental results on the deflections of reinforced RAC beams and investigate the applicability of the MC2010 deflection control model to reinforced RAC beams. First, the performance of the MC2010 deflection control model was assessed on a large database of NAC beams, in order to verify its accuracy and precision. Second, a database of RAC and companion NAC beams was compiled and the relative performance of the MC2010 deflection control model was assessed on them. Finally, corrections of the MC2010 deflection control model for RAC beams were proposed in order to improve the model’s performance to be equivalent to that for companion NAC beams. The results of the study offer engineers a safe deflection control procedure for RAC members, thus completing all structural design aspects for RAC members.

2. Deflection control according to the fib Model Code 2010

2.1. Methodology of calculating deflections according to MC2010

The MC2010 approach to modelling deflections of RC members is based on the fact that there are two distinct states of an RC cross-section: state 1, i.e. the uncracked state, in which the full area of the concrete cross-section is effective; and state 2, i.e. the fully cracked state, in which concrete in tension is ignored – the cross-section is composed of reinforcement in tension and concrete in compression and is said to be fully cracked.\(^{36}\) Basic assumptions of deflection calculation are that (1) concrete in tension is ignored, (2) plane cross-sections are assumed to remain plane, (3) strains in concrete and reinforcement are assumed to be compatible, and (4) both materials are assumed to be ideally linear elastic.

The MC2010 deflection control model—just as the one in previous Model Codes— is founded on the hypothesis that ‘members that are expected to crack, but may not be fully cracked, will behave in an intermediate manner between the uncracked and fully cracked conditions.’\(^{11}\) In its most rigorous form, the model is based on the interpolation of curvatures calculated in states 1 and 2 at a number of sections along the member and on the subsequent calculation of deflections by numerical integration. The interpolation is...
performed using a distribution coefficient $\zeta$, taking into account the tension stiffening effect. For the case of bending without an axial force, $\zeta$ is defined as

$$\zeta = \begin{cases} 
1 - \beta \left( \frac{M_{cr}}{M} \right)^2 & \text{for } M \geq \sqrt{\beta} \cdot M_{cr} \\
0 & \text{for } M < \sqrt{\beta} \cdot M_{cr}
\end{cases}$$

where $\beta$ is a coefficient accounting for the influence of the duration of loading or repeated loading

$$\beta = 1.0 \quad \text{for single, short-term loading}$$

$$\beta = 0.5 \quad \text{for sustained or repeated loading}$$

Further in Equation (1), $M_{cr}$ is the cracking moment; and $M$ is the moment acting on the cross-section.

The cracking moment should be calculated as

$$M_{cr} = W_{i,1} \cdot f_{ctm}$$

where $W_{i,1}$ is the section modulus of the uncracked transformed section, taking into account the reinforcement contribution through the ratio of the steel and concrete moduli of elasticity $\alpha_e$; and $f_{ctm}$ is the concrete mean tensile strength. It should be noted from Equation (1) that the cracked zone of a member is not given by $M_{cr}$, but by $\sqrt{\beta} \cdot M_{cr}$. The idea behind the coefficient $\beta$ is to roughly reduce the cracking moment, or more precisely tensile strength, in order to take into account several phenomena such as the effects of restrained shrinkage, cracking caused by previous loading and creep. In this sense, $\beta = 1$ is only appropriate for first loading of a completely uncracked member. For long-term effects, $\beta = 0.5$ reduces the importance and effect of properly selecting the tensile strength by basically reducing $f_{ctm}$ by approximately $30\%$.37

Curvatures are interpolated using the following equation:

$$\left( \frac{1}{r} \right)_{eff} = \zeta \cdot \left( \frac{1}{r} \right)_1 + (1 - \zeta) \cdot \left( \frac{1}{r} \right)_2$$

where $(1/r)_{eff}$ is the effective/interpolated curvature, while $(1/r)_1$ and $(1/r)_2$ are curvatures in states 1 and 2, respectively. The curvatures in states 1 and 2 are composed of a component due to load $(1/r)_{load}$ and a component due to shrinkage $(1/r)_{cs}$ and are calculated as

$$\left( \frac{1}{r} \right)_n = \frac{M \cdot l^2}{E_{c,ef} \cdot I_{ln}} + \varepsilon_{cs}(t,t_s) \cdot \alpha_e \cdot \frac{S_{ln}}{I_{ln}} \quad n = 1, 2$$
where $I_{i,n}$ is the moment of inertia of the transformed section in state 1 or 2; $S_{i,n}$ is the first moment of area of the reinforcement about the transformed section’s centroid (in state 1 or 2); and $\varepsilon_{cs}(t,s)$ is the concrete shrinkage strain at time $t$ with drying initiation at time $t_s$. The effect of creep is taken into account using the effective modulus of elasticity $E_{c,ef}$:

$$E_{c,ef} = \frac{E_{cm}}{1 + \varphi(t,t_0)} \quad (6)$$

where $\varphi(t,t_0)$ is the creep coefficient of concrete loaded at time $t_0$, at time $t$, and $E_{cm}$ is the modulus of elasticity of concrete at 28 days. The effective modulus of elasticity also defines the modular ratio $\alpha_e = E_s/E_{c,ef}$ where $E_s$ is the modulus of elasticity of reinforcement (that may be taken as 200 GPa).

The shrinkage strain and creep coefficient in Equations (5) and (6) should be calculated using the MC2010 shrinkage and creep prediction models which will not be presented here in detail. Beside this rigorous approach, MC2010 also offers a simplified procedure for calculating deflections, stating that ‘in most cases, it will be acceptable to compute the deflections twice, assuming the whole member to be in the uncracked condition and in the fully cracked condition, and then interpolate.’ In other words, the distribution coefficient $\zeta$ is calculated only once, usually for the cross-section subjected to the maximum bending moment $M_{max}$:

$$\zeta_{simp.} = \begin{cases} 
1 - \beta \cdot \left( \frac{M_{cr}}{M_{max}} \right)^2 & \text{for } M_{max} \geq \sqrt{\beta} \cdot M_{cr} \\
0 & \text{for } M_{max} < \sqrt{\beta} \cdot M_{cr} 
\end{cases} \quad (7)$$

This distribution coefficient is then applied directly to interpolating deflections:

$$a_{simp.} = \zeta \cdot a_2 + (1 - \zeta) \cdot a_1 \quad (8)$$

In Equation (8), $a_1$ and $a_2$ are the deflections of the member in states 1 and 2, respectively:

$$a_n = K \cdot \frac{M_{max} \cdot l^2}{E_{c,ef} \cdot I_{ln} + \delta_{cs} \cdot \varepsilon_{cs}(t,t_s) \cdot \frac{S_{ln} \cdot l^2}{I_{ln} \cdot B}} \cdot \frac{S_{ln} \cdot l^2}{I_{ln} \cdot B} \cdot n = 1, 2 \quad (9)$$

where $K$ is a coefficient depending on the static system (0.104 for a simply supported beam under uniformly distributed loading and 0.107 for a simply supported beam in four point bending in thirds of the span), and $\delta_{cs}$ is a coefficient dependent on the member’s support conditions (=1 for a simply supported beam). This
simplified approach always provides conservative results, but rarely more than 10% greater than the ones obtained by the rigorous procedure.\textsuperscript{38}

2.2. Performance assessment of MC2010 deflection control on a large database of NAC beams

Before studying the applicability of the MC2010 deflection control model to reinforced RAC members, it was necessary to assess the model’s performance on ordinary RC (i.e. NAC) members. For this purpose, a large number of experimental results of long-term tests on NAC members were needed. The largest database of such tests was compiled by Espion in 1988.\textsuperscript{39} It contains 397 long-term results from 45 different research campaigns. Beside this database, only a few studies performed afterwards have been well-conceptualized and well-documented, e.g. the experimental programme of Gilbert and Nejadi from 2004.\textsuperscript{40} Since the database by Espion contains a large number of different research campaigns—ranging from simply supported to continuous beams, rectangular and T-shaped cross-sections, different load conditions, etc.—some criteria had to be applied in order to reduce the number of results to a smaller but more reliable database. Hence, the following criteria were applied:

- Studies carried out after 1945 (mostly because of construction technology and cement production);
- Simply supported RC beams with rectangular cross-sections;
- Deformed bars used as reinforcement;
- Four-point bending or uniformly distributed load tests (because of the similar shape of the bending moment diagram, most common in real members);
- The total imposed load caused cracking immediately after loading, i.e. beams were cracked throughout the entire experiment (this was considered most representative of realistic in-service behaviour of reinforced concrete members);
- The concrete compressive stress-to-strength at loading age ratio, $\sigma_c(t_0)/f_{cm}(t_0)$, was smaller than 0.6 immediately after loading (in order to enable the application of the MC2010 creep prediction model);
- Compressive strength between 20 and 50 MPa;
- Cross-section height greater than 100 mm;
- $L/d$ ratio smaller than 40; and
- Loading ($t_0$) earlier than 90 days.
After applying these criteria, 11 studies from Espion’s database were selected: Washa and Fluck (1952), P.C.A. (1950), Sattler (1956), Hajnal-Konyi (1963), Branson and Metz (1963), Pauw and Meyers (1964), Lutz et al. (1967), Jaccoud and Favre (1982), Bakoss et al. (1983), Van Nieuwenberg (1984), Clarke et al. (1988), together with the study by Gilbert and Nejadi (2004). In total, 12 research campaigns were selected, yielding 52 beams in total, each with an initial deflection, \(a_0\), and a long-term ‘final’ deflection, \(a_t\), corresponding to the end of experimental measurements, i.e. 104 data points. The database with all the gathered data is provided as Supporting Information to this paper.

It should be noted that, throughout this study, the term ‘final’ deflection refers to the deflection at the end of experimental measurements. The term is introduced for the sake of simplicity, because of different ages of final experimental measurements in different experiments. It does not refer to any type of final or ‘ultimate’ deflection, since no such concept exists; the absence of bound of basic creep (as reflected in the MC2010 model) means that, theoretically, deflections can increase indefinitely.

The ranges of the most important parameters in the database (labelled NAC-1) are given in Table 1, where \(b\) and \(d\) are the beam cross-section width and effective depth, respectively, \(L\) is the beam span, \(\rho_1\) and \(\rho_2\) are the tensile (bottom) and compressive (top) reinforcement ratios, respectively, \(t_0\) is the age at loading and \(t\) is the age at final measurement (measured from loading age), and \(M_{max}/M_c\) is the maximum applied moment-to-cracking moment ratio with \(M_c\) calculated using Equation (3). A relatively wide range of all parameter values can be seen, indicating good representativeness of the NAC-1 database.

The next step of the analysis was to calculate the deflections of all 52 beams (both initial and final). All mechanical and time-dependent properties necessary for this calculation (modulus of elasticity, tensile strength, shrinkage strain, and creep coefficient) were calculated using MC2010 expressions based on compressive strength. If aggregate type was not provided in the studies, for the purposes of calculating the modulus of elasticity, quartzite aggregates were assumed. In other words, the aggregate-dependent coefficient \(\alpha_E\) was taken as 1.0:

\[
E_{cm} = 21500 \cdot \alpha_E \cdot \left(\frac{f_{cm}}{10}\right)^{1/3}
\]  

Both the modulus of elasticity and tensile strength were calculated for each beam’s loading age, as \(E_{cm}(t_0)\) and \(f_{cm}(t_0)\), respectively. Unfortunately, for most studies, relative humidity (RH) and temperature—
necessary for calculating shrinkage strain and creep coefficient—were not provided, but were taken as values cited by Espion for each study.\textsuperscript{39}

Deflections were calculated using numerical integration of curvatures in 50 cross-sections across the length of each beam, using an Excel spreadsheet. For the beams with a $\frac{\sigma_c(t_0)}{f_{cm}(t_0)}$ ratio greater than 0.4 (there were 29 such cases)—the limit of linear creep in MC2010—long-term deflections were calculated by first dividing the cross-section into the part with $\frac{\sigma_c(t_0)}{f_{cm}(t_0)} < 0.4$ and $\frac{\sigma_c(t_0)}{f_{cm}(t_0)} > 0.4$ at time $t_0$, and subsequently applying the MC2010 linear creep coefficient $\varphi(t,t_0)$ and nonlinear creep coefficient $\varphi_c(t,t_0)$ to each part, respectively, in calculating the effective modulus, Equation (6). This approach has been applied successfully before and is demonstrated in more detail in the studies by Reybrouck et al. and Tošić et al.\textsuperscript{35,52}

After calculating all 104 deflections, a calculated-to-experimental deflection ratio, $a_{\text{calc}}/a_{\text{exp}}$, was determined for each value. The statistical descriptors (mean value $\mu$, standard deviation $\sigma$, and coefficient of variation CoV) are given in Table 2, for the entire database and separately for initial and final deflections. Very good agreement of calculated and measured values of deflections can be seen overall. The mean value of the $a_{\text{calc}}/a_{\text{exp}}$ ratio of 1.11 for the entire database is somewhat conservative, however, considering all of the assumptions made in the calculations (both for mechanical and time-dependent properties) and the scatter of results (CoV of 26.8%), the result is very good.

However, it is more interesting to analyse initial and final deflections separately since they are actually calculated using different models – the model for calculating final deflections includes shrinkage and creep, whereas the model for calculating initial deflections does not; furthermore, the value of the $\beta$ coefficient in Equation (2) is different. When looked at separately, an excellent performance of the MC2010 model can be seen for final deflections – a mean value of the $a_{\text{calc}}/a_{\text{exp}}$ ratio of 1.05 and a CoV of 15.1%. The performance of the model is worse for initial deflections (mean of 1.17 and CoV of 32.4%). However, precisely measuring ‘initial’ deflections can also be problematic and lead to errors. Because of this, and the fact that initial deflections are less important than long-term ones for RC structures, this result is also considered very good.

The performance of the model was also explored graphically, as shown in Figure 1 where the $a_{\text{calc}}/a_{\text{exp}}$ ratio was plotted against compressive strength, tensile reinforcement ratio, $L/d$ ratio and load level ($M_{\text{max}}/M_{\text{cr}}$ ratio), separately for initial and final deflections. From the figure, practically no significant correlation of the $a_{\text{calc}}/a_{\text{exp}}$ ratio with any of the analysed parameters can be seen, meaning that the model behaves equally well
over the entire range of parameter values in the NAC-1 database. There is a slight negative correlation of the $a_{\text{calc}}/a_{\text{exp}}$ ratio to the tensile reinforcement ratio, and the model is less precise for lower reinforcement ratios. Such reinforcement ratios are indicative of members loaded in service close to their cracking moment, and this is a case for which the model’s lower reliability is already known; nonetheless, the correlation is not significant.

For comparison purposes, deflections were also calculated using the simplified MC2010 method. As expected, the obtained results were more conservative compared with the rigorous method – the mean $a_{\text{calc}}/a_{\text{exp}}$ ratio for the simplified approach was 1.29 and 1.09 for initial and final deflections, respectively (compared with 1.17 and 1.05 for the rigorous method). Nonetheless, this is a good result for a simplified method, and is on the safe side, as should be the case with any simplification.

From the analysis in this section, it was concluded that the MC2010 deflection control model has a very good performance on NAC beams and does not require any modifications. Therefore, the unaltered version of the model was used in the subsequent analysis of RAC beams carried out in the following section.

3. Applicability of the fib Model Code 2010 deflection control to RAC members

3.1. Databases of RAC and companion NAC beams

Detailed databases of long-term tests on reinforced RAC and companion NAC beams were compiled. As stated in the Introduction, there are only a small number of long-term tests on RAC beams and, furthermore, most of the studies are published as conference proceedings and do not offer sufficient information for a detailed analysis.

The only studies that provide sufficiently detailed results of their research campaigns are those of Knaack and Kurama, Tošić et al., and Seara-Paz et al. In these three studies, 30 beams were studied in total: 10 NAC and 20 RAC beams. Knaack and Kurama tested:

- six NAC beams (UT-0-7, UT-0-28, UC-0-7, UC-0-28, CC-0-7, CC-0-28),
- six beams with 50% of RCA (i.e. RAC50: UT-50-7, UT-50-28, UC-50-7, UC-50-28, CC-50-7, CC-50-28), and
- six beams with 100% of RCA (i.e. RAC100: UT-100-7, UT-100-28, UC-100-7, UC-100-28, CC-100-7, CC-100-28) were studied.
The beams were divided according to whether they were loaded so as to crack immediately after loading, or to crack some time after loading (first letter in the label – U/C); whether they had only tensile, or both tensile and compressive reinforcement (second letter in the specimen label – T/C); RCA percentage (first number in the label – 0/50/100); and whether they were loaded after 7 or 28 days (last number in the label – 7/28).

In the study by Tošić et al.,35 two NAC (NAC7, NAC28) and two RAC100 beams (RAC7, RAC28) were tested by loading them after 7 and 28 days (as indicated by the number in the specimen name).

Seara-Paz et al.34 tested:

- two NAC beams (H50-0, H65-0),
- two beams with 20% of RCA (i.e. RAC20: H50-20, H65-20),
- two RAC50 beams (H50-50, H65-50), and
- two RAC100 beams (H50-100, H65-100).

The beams were divided according to the \((w/c)_{\text{eff}}\) ratio (0.50 and 0.65 for beams H50 and H65, respectively), and RCA percentage (indicated by the number in the specimen’s name).

All of the beams were simply supported and loaded in four-point bending. The RCA used in these studies was crushed concrete waste in the studies of Knaack and Kurama33 and Tošić et al.35, whereas the RCA in the study of Seara-Paz et al.34 was mostly concrete waste (85%) with approximately 10% of asphalt particles. The water absorption of RCA was in the range of 3.9–6.1%, indicating good to moderate quality.

However, for the purposes of this study, not all of these beams were considered in the analysis. First, one RAC beam from the study of Tošić et al.35 (RAC100 beam loaded after 7 days, RAC7) had a \(\sigma_c(t_0)/f_{cm}(t_0)\) ratio greater than 0.6 and was excluded since the MC2010 nonlinear creep coefficient could no longer be applied. Second, two RAC beams from the study of Knaack and Kurama33 did not report long-term deflection values, only initial deflections (beams CC-50-28 and UT-100-7); therefore, they were also excluded. Finally, the two RAC20 beams from the study of Seara-Paz et al.34 were excluded (beams H50-20 and H65-20) since this was the only study that investigated a RCA replacement percentage of 20% – the number of results was too small for analysis and these two beams were also excluded. In the end, this led to two new databases: a
'Companion-NAC’ database and a 'RAC’ database. These databases with all the gathered data are also provided as Supporting Information to this paper.

The Companion-NAC database contains 10 beams and 20 data points (10 initial and 10 final deflections). The ranges of the most important parameters in this database are given in Table 3. It can be seen that most of the parameters of the Companion-NAC beams also fall within the corresponding parameter ranges in the NAC-1 database (with somewhat smaller cross-sections and higher compressive strengths). However, one significant difference is the load level ($M_{\text{max}}/M_{\text{cr}}$) which has values lower than 1.0 in the Companion-NAC database, signifying uncracked beams. This is due to four beams tested by Knaack and Kurama$^{33}$ which were designed not to crack immediately after loading, but to crack over time, i.e. these initial deflections are in the uncracked state, whereas their final deflections are in the cracked state. They were kept in the database since this is a situation that can occur in practice (e.g. RC slabs loaded close to their cracking load) and the MC2010 model’s performance should also be assessed in such cases.

The RAC database contains 15 beams and 30 data points (15 initial and 15 final deflections). The ranges of the most important parameters in this database are also given in Table 3. The geometric properties and reinforcement ratios of RAC beams are the same as in companion NAC beams. RAC compressive strength is slightly lower, as expected of RAC and NAC produced with the same ($w/c_{\text{eff}}$) ratios. There were also seven RAC beams from the study of Knaack and Kurama$^{33}$ which were designed not to crack immediately after loading, but to crack over time (four RAC50 and three RAC100 beams).

A direct comparison of deflections between RAC and companion NAC beams is not straightforward since different studies used different variables (load level, $\sigma_{(t_0)}/f_{\text{cm}}(t_0)$ ratio, etc.), and RAC and companion NAC did not have identical mechanical properties. Nonetheless, generally, RAC beams had slightly larger deflections than companion NAC beams and this difference tended to increase over time. For the 30 RAC beams in the database, a ratio of RAC-to-companion NAC beam deflections, $a_{\text{RAC}}/a_{\text{NAC}}$, was calculated and the results are shown in Table 4. It can be seen that, overall, deflections of RAC beams are 14% larger than those of companion NAC beams with significant scatter. When divided into initial and final deflections, an increasing trend of deflection ‘divergence’ can be seen – the ratio increases from an average of 1.09 to 1.19.
Although the number of experimental results in the databases is not so large, at this time, these are the most reliable and usable results. The databases still allow a meaningful analysis of the MC2010 deflection control model and this was carried out as the next step in the study.

3.2. Performance of the MC2010 deflection control on companion NAC and RAC beams

Following the same procedure described in Section 2.2, deflections were calculated for RAC and companion NAC beams. In this step, all RAC properties were calculated from compressive strength using default MC2010 expressions, i.e., assuming that expressions for NAC are valid (even for shrinkage and creep). Since the NA type was known in these three studies, appropriate $\alpha_E$ coefficients were used in Equation (10). Again, as in Section 2.2, the calculated-to-experimental deflection ratio, $a_{\text{calc}}/a_{\text{exp}}$, was determined using the rigorous MC2010 method.

For the companion NAC beams, statistical descriptors of the $a_{\text{calc}}/a_{\text{exp}}$ ratio are given in Table 5. The results are very similar to those of the NAC-1 database, with slightly lower CoVs (as expected from a smaller number of studies) and a larger mean $a_{\text{calc}}/a_{\text{exp}}$ ratio for initial deflections (1.33 compared with 1.17 for the NAC-1 database). This can be explained by the presence of the initially uncracked beams tested by Knaack and Kurama,\(^3\) two out of four of which were wrongly predicted by the MC2010 model to be cracked – leading to much higher calculated initial deflections compared with measured ones. At the same time, the NAC-1 database does not contain such beams. Once this is taken into account, the performance of the MC2010 deflection control model can be considered the same on both NAC databases, as expected. Again, the simplified method was also tested and again led to more conservative results – the mean $a_{\text{calc}}/a_{\text{exp}}$ ratio for the simplified approach was 1.46 and 1.12 for initial and final deflections (compared with 1.33 and 1.01 for the rigorous method).

For the RAC beams, statistical descriptors of the $a_{\text{calc}}/a_{\text{exp}}$ ratio are also provided in Table 5. Here, it can clearly be seen that the MC2010 deflection control model significantly underestimates RAC deflections. For initial deflections, even though the $a_{\text{calc}}/a_{\text{exp}}$ ratio is greater than 1.0, this is not a conservative result since the corresponding value for the companion NAC beams was 1.33 (three of the seven initially uncracked beams tested by Knaack and Kurama\(^3\) were also wrongly predicted as cracked). However, the greatest underestimation is in the final deflections – the mean $a_{\text{calc}}/a_{\text{exp}}$ ratio is only 0.77. In both the initial and final deflections, there are no significant differences relative to RCA content: for initial deflections the mean
a_{calc}/a_{exp} ratio values for RAC50 and RAC100 beams are 1.14 and 1.08, respectively, whereas for final
deflections they are 0.78 and 0.76, respectively. Therefore, treating all RAC beams as one database is
justified. The simplified method also provides similar results – a mean a_{calc}/a_{exp} ratio of 1.22 and 0.86 for
initial and final deflections.

A graphical comparison of the $a_{calc}/a_{exp}$ ratio for RAC and companion NAC beams, relative to
compressive strength, $L/d$ ratio, tensile reinforcement ratio, and load level, is shown in Figure 2. First, the
underestimation of RAC deflections, in absolute terms and relative to companion NAC, can clearly be seen for
both the initial and final deflections, especially taking into consideration that experimental RAC beam
deflections are generally larger than those of companion NAC beams (Table 4). Second, as in the case of the
NAC-1 database, no significant correlation of the $a_{calc}/a_{exp}$ ratio with any of the analysed parameters can be seen, i.e. the model behaves similarly over the entire range of parameter values.

Considering the above analysis, it is clear that RAC cannot be treated the same as NAC in all aspects of
the MC2010 deflection control model (predicting mechanical, and time-dependent properties, as well as
calculating deflections) and corrections must be applied. In the following section, specific extensions of the
MC2010 model are proposed in order to enable its applicability to RAC deflection control.

4. Improvement of the fib Model Code 2010 deflection control for RAC members

4.1. Corrections for predicting RAC mechanical and long-term properties

The underestimation of RAC deflections, in absolute terms and relative to companion NAC beams, can
have two causes. The first one is inadequate MC2010 equations for predicting the mechanical and time-
dependent properties of RAC (modulus of elasticity, tensile strength, shrinkage strain, and creep coefficient).
The second one is an inadequate deflection control method itself, i.e. some inadequacy of Equation (1) for
RAC beams. Finally, a combination of both causes is also possible.

It is already known that there are significant differences in mechanical and time-dependent properties
between RAC and NAC and that default MC2010 equations for predicting these properties cannot be directly
used for RAC. Therefore, the first step was to investigate whether changing only these expressions will lead to
equal performance of MC2010 deflection control on RAC and companion NAC beams.
Since it was previously shown that the tensile strength of RAC can be successfully predicted using Eurocode 2 equations (identical to MC2010), the equation for $f_{cm}$ was left unchanged. For deflections, especially initial deflections, the modulus of elasticity is of the greatest importance. The MC2010 equation for $E_{cm}$ in Equation (10) already allows an adjustment for aggregate type through the $\alpha_E$ coefficient (1.2 for basalt, 1.0 for quartzite, 0.9 for limestone, and 0.7 for sandstone aggregates). Silva et al. previously showed, through a large meta-analysis, that $E_{cm}$ for RAC is conservatively predicted if $\alpha_E$ is taken as 0.7 (i.e. as for sandstone aggregates). Since this is a conservative proposal, in this study, the following equation was used to calculate the $\alpha_E$ coefficient in Equation (10):

$$\alpha_E = 1.0 - 0.3 \cdot \frac{RCA\%}{100}$$ (11)

where $RCA\%$ is the percentage of coarse RCA in RAC. Equation (11) yields $\alpha_E = 0.85$ for RAC50 and 0.70 for RAC100, in line with the conclusions of Silva et al.¹⁸

For the shrinkage strain and creep coefficient, Tošić et al.¹⁹,²⁰ proposed an extension of the MC2010 models for RAC by performing meta-analyses of previously published experimental results. The authors proposed correction coefficients to be applied as global scaling factors of shrinkage strain and creep coefficient calculated according to MC2010.

$$\varepsilon_{cs,RAC}(t,t_s) = \xi_{cs,RAC} \cdot \varepsilon_{cs}(t,t_s)$$ (12)

$$\phi_{RAC}(t,t_0) = \xi_{cc,RAC} \cdot \phi(t,t_0)$$ (13)

The correction coefficients are dependent on RAC compressive strength and RCA percentage:

$$\xi_{cs,RAC} = \left( \frac{RCA\%}{f_{cm}} \right)^{0.30} \geq 1.0$$ (14)

$$\xi_{cc,RAC} = 1.12 \cdot \left( \frac{RCA\%}{f_{cm}} \right)^{0.15} \geq 1.0$$ (15)

After recalculating RAC properties, deflections were again calculated for RAC beams using the rigorous method. The statistical descriptors of the new $a_{calc}/a_{exp}$ ratios are given in rows 2–4 of Table 6. Applying these corrections improved the model’s performance, with practically no cost in terms of CoV. However, the mean values of the $a_{calc}/a_{exp}$ ratio remain lower than those of the companion NAC beams (Table 5), both for the initial and final deflections.
The adopted corrections for the modulus of elasticity, shrinkage strain, and creep coefficient were derived from large databases of experimental results at the material level\textsuperscript{18–20}. Although it might be possible to improve these expressions in the future, according to current results, they are an adequate solution, and since they are determined at the material level, no results at the structural level can be used to improve their adequacy. In this paper, any remaining difference between the deflection control model’s performance on RAC and companion NAC beams is \textit{hypothesized} to be due to differences in structural behaviour. This hypothesis can only be tested in tensions stiffening experiments, which are very scarce for RAC\textsuperscript{54}, i.e., at the moment, no definite conclusion can be made.

Hence, in this study, tension stiffening was presumed to be different in RAC and in companion NAC beams. The general approach of interpolating curvatures (or deflections), using the distribution coefficient $\zeta$ in Equation (1), remains valid since it has a strong physical meaning. However, the empirical coefficient $\beta$, defined by Equation (2), is not adequate for RAC beams. In the following section, besides the previously presented corrections for $E_{cm}$, $\varepsilon_{cs}$, and $\phi$, a correction of the coefficient $\beta$ will be presented.

4.2. Corrections for RAC deflection control

As shown earlier, for adequate RAC deflection control, it is not enough to correct only the mechanical and time-dependent properties. The deflection model itself must be improved. For this purpose, the empirical coefficient $\beta$ is replaced by the new coefficient $\beta_{RAC}$ for RAC:

$$
\beta_{RAC} = 0.75 \quad \text{for single, short-term loading}
$$

$$
\beta_{RAC} = 0.25 \quad \text{for sustained or repeated loading}
$$

In other words, the $\beta$ coefficient is reduced from 1.0 to 0.75 for calculating initial deflections and from 0.5 to 0.25 for calculating long-term deflections. As explained in Section 2.1, $\sqrt{\beta}$ actually represents a reduction of the cracking moment in Equation (1). Hence, this proposal for $\beta_{RAC}$ actually reduces the cracking moment by approximately 15\% for single, short-term loading, and by 50\% for sustained or repeated loading. Both reductions are aligned with experimental results: (1) for initial deflections, studies on flexural strength of RAC beams have reported lower cracking moments compared with companion NAC beams\textsuperscript{55} (due to the presence of two interfacial transition zones between aggregate and mortar in RAC); and (2) for final deflections, a larger reduction of the cracking moment is in line with larger shrinkage of RAC.
Using Equation (16) (and all previously presented corrections), RAC deflections were recalculated using the MC2010 rigorous method and the new $a_{\text{calc}}/a_{\text{exp}}$ ratios are given in rows 5–7 of Table 6. The choice of values for the $\beta_{\text{RAC}}$ coefficient was such that the mean $a_{\text{calc}}/a_{\text{exp}}$ ratios for RAC beams are made identical to the ones for companion NAC beams, for both initial and final deflections. Even the CoVs are almost identical with the only difference being a slightly larger CoV for RAC final deflections.

Graphically, the results are shown in Figure 3, through a comparison of the $a_{\text{calc}}/a_{\text{exp}}$ ratio for ‘corrected’ RAC (labelled ‘RAC+’ in the figure) and companion NAC beams, relative to compressive strength, $L/d$ ratio, tensile reinforcement ratio, and load level. Now, it is clear that the ‘clouds’ of points for NAC and RAC coincide completely. This demonstrates the equality of performance of the ‘corrected’ MC2010 deflection control model on RAC beams and the ‘original’ MC2010 deflection control model on NAC beams. The simplified model was also tested, and as expected, it yielded conservative results, similar to the ones for companion NAC beams – the mean $a_{\text{calc}}/a_{\text{exp}}$ ratio for the simplified approach was 1.46 and 1.26 for initial and final deflections, respectively (compared with 1.46 and 1.12 for the companion NAC beams).

With the corrections presented in this paper, RAC members can reliably be designed for SLS. Together with the already demonstrated design of RAC members in terms of ULS, this completes all necessary structural design aspects for reinforced RAC members.

5. Conclusions

This study presented a comprehensive analysis of the applicability of the fib Model Code 2010 deflection control model to reinforced RAC beams. For this purpose, three databases of long-term studies on NAC and RAC beams were compiled and meta-analyses of deflection predictions by MC2010 were performed. The following conclusions were drawn from this study:

- Very good performance of the MC2010 deflection control model (rigorous method of numerical integration of curvatures) was demonstrated in terms of predicting initial and long-term deflections from a database of 52 NAC beams. This included both equations for predicting mechanical and time-dependent properties, as well as the deflection control model itself. The mean value of the calculated-to-experimental deflection ratio, $a_{\text{calc}}/a_{\text{exp}}$, was calculated as 1.17 and 1.05 for initial and final deflections, respectively.
• Only three long-term experimental campaigns of RAC beams were found with reliable and sufficient data for a meta-analysis. A database of RAC and companion NAC beams was compiled. The companion NAC database comprised 10 beams, whereas the RAC database comprised 15 beams (7 RAC50 and 8 RAC 100 beams).

• The performance of the MC2010 deflection control model on companion NAC beams was found to be similar to that of the larger NAC database – the mean value of the $a_{\text{calc}}/a_{\text{exp}}$ ratio was 1.33 and 1.01 for initial and final deflections, respectively. However, when using the default expressions of the MC2010 model for RAC beams, deflections are significantly underestimated compared with companion NAC beams – the mean value of the $a_{\text{calc}}/a_{\text{exp}}$ ratio was 1.11 and 0.77 for initial and final deflections, respectively.

• If modifications of MC2010 equations for the modulus of elasticity, shrinkage strain, and creep coefficient for RAC are applied, deflection predictions improve but still remain lower than those of the companion NAC beams. Therefore, the deflection control model itself must be modified.

• When the empirical coefficient $\beta$ (used for calculating the tension stiffening distribution coefficient $\zeta$) is modified to 0.75 for single, short-term loading and 0.25 for sustained or repeated loading, the ‘corrected’ MC2010 deflection control model has equal performance on RAC beams to that of the original model on companion NAC beams. This is true for both the rigorous method of integrating curvatures and the simplified method of directly interpolating deflections (which provides sufficiently conservative results).

With the corrections presented in this paper, more reliable deflection control of RAC members is possible. Nonetheless, the study has some limitations: although they contain the best available results, the companion NAC and RAC databases are still small; only simply supported beams were analysed; only rectangular beam cross-sections were analysed; a narrow range of load levels was analysed; and the duration of the available experiments is relatively short. In order to verify the modifications proposed in this study, more long-term tests on RAC and companion NAC beams will be needed, broadening the scope of the databases. The ones used in this study are provided as Supporting Information, enabling other researchers to expand them in the future.

Acknowledgements
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Notations

- $\alpha_e$: modular ratio of steel and concrete
- $\alpha_{E}$: coefficient that takes into account the effect of the aggregate type on the modulus of elasticity of concrete
- $\beta$: coefficient accounting for the influence of the duration of loading or repeated loading
- $\beta_{RAC}$: $\beta$ coefficient for RAC
- $\delta_{cs}$: shrinkage deflection coefficient dependent on the static system
- $\varepsilon_{cs}$: concrete shrinkage strain
- $\varepsilon_{cs,RAC}$: shrinkage strain of RAC
- $\zeta$: distribution coefficient for interpolating deformation variables (curvature, deflections, etc.)
- $\mu$: mean value
- $\xi_{cc,RAC}$: correction coefficient for RAC creep coefficient
- $\xi_{cs,RAC}$: correction coefficient for RAC shrinkage strain
- $\sigma$: standard deviation
- $\sigma_c$: concrete compressive stress
- $\varphi$: concrete creep coefficient
- $\varphi_{RAC}$: creep coefficient of RAC
- $\varphi_0$: non-linear concrete creep coefficient
- $(1/r)_i$: curvature in state 1 or 2
- $(1/r)_{eff}$: effective curvature
- $a_0$: initial deflections
- $a_{calc}$: calculated deflections
- $a_{exp}$: experimentally measured deflections
- $a_i$: deflections in state 1 or 2
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{\text{simp.}})</td>
<td>deflections calculated using the simplified MC2010 method</td>
</tr>
<tr>
<td>(a_t)</td>
<td>long-term deflections</td>
</tr>
<tr>
<td>(\text{CoV})</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>(E_{c,\text{ef}})</td>
<td>concrete effective modulus</td>
</tr>
<tr>
<td>(E_{cm})</td>
<td>concrete modulus of elasticity</td>
</tr>
<tr>
<td>(f_{cm})</td>
<td>concrete mean tensile strength</td>
</tr>
<tr>
<td>(I_{i,n})</td>
<td>moment of inertia of the transformed section in state 1 or 2</td>
</tr>
<tr>
<td>(K)</td>
<td>bending deflection coefficient dependent on the static system</td>
</tr>
<tr>
<td>(M)</td>
<td>moment acting on an RC cross-section</td>
</tr>
<tr>
<td>(M_{\text{cr}})</td>
<td>cracking moment</td>
</tr>
<tr>
<td>(M_{\text{max}})</td>
<td>maximum bending moment acting on RC member</td>
</tr>
<tr>
<td>(\text{RCA}%)</td>
<td>percentage of coarse RCA in RAC</td>
</tr>
<tr>
<td>(S_{i,n})</td>
<td>first moment of area of the reinforcement about the transformed section’s centroid in state 1 or 2</td>
</tr>
<tr>
<td>(t)</td>
<td>time</td>
</tr>
<tr>
<td>(t_0)</td>
<td>loading age</td>
</tr>
<tr>
<td>(t_s)</td>
<td>end of curing</td>
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<tr>
<td>((w/c)_{\text{eff}})</td>
<td>effective water-cement ratio</td>
</tr>
<tr>
<td>(W_{1,1})</td>
<td>section modulus of an uncracked transformed RC section</td>
</tr>
</tbody>
</table>

**References**


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5. Pecić, N. Improved method for deflection control of reinforced concrete structures. (Faculty of Civil Engineering, Belgrade University, 2012).


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List of figures:

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Figure 2. Relationship between the $a_{calc}/a_{exp}$ ratio and a) compressive strength, b) span-effective depth ratio, c) tensile reinforcement ratio, and d) load level, for the Companion-NAC and RAC databases

Figure 3. Relationship between the $a_{calc}/a_{exp}$ ratio and a) compressive strength, b) span-effective depth ratio, c) tensile reinforcement ratio, and d) load level, for companion NAC and 'RAC+' beams
### Table 1. Range of parameters in the NAC-1 database

<table>
<thead>
<tr>
<th>Database</th>
<th>No. of beams</th>
<th>No. of deflections</th>
<th>$b$ (mm)</th>
<th>$d$ (mm)</th>
<th>$f_{\text{cm}}$ (MPa)</th>
<th>$L$ (mm)</th>
<th>$L/d$ (–)</th>
<th>$\rho_1$ (%)</th>
<th>$\rho_2$ (%)</th>
<th>$t_0$ (days)</th>
<th>$t$ (days)</th>
<th>$M_{\text{max}}/M_{\text{cr}}$ (–)</th>
<th>$\sigma_{c}(t_0)/f_{\text{cm}}(t_0)$ (–)</th>
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</thead>
<tbody>
<tr>
<td>NAC-1</td>
<td>52</td>
<td>104</td>
<td>100–750</td>
<td>95–300</td>
<td>21.5–39.6</td>
<td>1829–6400</td>
<td>10.7–40.9</td>
<td>0.44–2.64</td>
<td>0.00–1.67</td>
<td>14–53</td>
<td>60–1734</td>
<td>1.12–4.08</td>
<td>0.20–0.58</td>
</tr>
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</table>
Table 2. Statistical descriptors of the $a_{\text{calc}}/a_{\text{exp}}$ ratio for the NAC-1 database

<table>
<thead>
<tr>
<th>Database</th>
<th>Deflections</th>
<th>$n$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>CoV (%)</th>
</tr>
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<tr>
<td>All</td>
<td>104</td>
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<td>0.30</td>
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<td>26.8</td>
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<td>NAC-1</td>
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<td>52</td>
<td>1.05</td>
<td>0.16</td>
<td>15.1</td>
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Table 3. Range of parameters in the Companion-NAC and RAC databases

<table>
<thead>
<tr>
<th>Database</th>
<th>No. of beams</th>
<th>No. of deflections</th>
<th>RCA (%)</th>
<th>b (mm)</th>
<th>d (mm)</th>
<th>$f_{cm}$ (MPa)</th>
<th>L (mm)</th>
<th>L/d (–)</th>
<th>$\rho_1$ (%)</th>
<th>$\rho_2$ (%)</th>
<th>$t_0$ (days)</th>
<th>$t$ (days)</th>
<th>$M_{max}/M_{cr}$</th>
<th>$\sigma_c(t_0)/f_{cm(t_0)}$</th>
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<tr>
<td>Companion-NAC</td>
<td>10</td>
<td>20</td>
<td>0</td>
<td>150–200</td>
<td>169–249</td>
<td>30.5–60.7</td>
<td>3200–3700</td>
<td>13.7–18.9</td>
<td>0.58–1.32</td>
<td>0.00–0.47</td>
<td>7–42</td>
<td>119–1000</td>
<td>0.81–3.35</td>
<td>0.10–0.58</td>
</tr>
<tr>
<td>RAC</td>
<td>15</td>
<td>30</td>
<td>50, 100</td>
<td>28.1–51.8</td>
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<td></td>
<td></td>
<td></td>
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Table 4. Statistical descriptors of the $a_{RAC}/a_{NAC}$ ratio for RAC and companion NAC measured beam deflections

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<th>$a_{RAC}/a_{NAC}$ ratio</th>
<th>Deflections</th>
<th>$n$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>CoV (%)</th>
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</thead>
<tbody>
<tr>
<td>RAC- Companion NAC</td>
<td>All</td>
<td>30</td>
<td>1.14</td>
<td>0.32</td>
<td>27.6</td>
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<tr>
<td></td>
<td>Initial</td>
<td>15</td>
<td>1.09</td>
<td>0.34</td>
<td>31.4</td>
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<td>Final</td>
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<td>1.19</td>
<td>0.29</td>
<td>24.3</td>
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Table 5. Statistical descriptors of the $\alpha_{\text{calc}}/\alpha_{\text{exp}}$ ratio for the Companion-NAC and RAC databases

<table>
<thead>
<tr>
<th>Database</th>
<th>Deflections</th>
<th>$n$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>CoV (%)</th>
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<tr>
<td>Companion-NAC</td>
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<td>20</td>
<td>1.17</td>
<td>0.26</td>
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<tr>
<td></td>
<td>Initial</td>
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<td>1.33</td>
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<td>1.01</td>
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<tr>
<td>RAC</td>
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<td>0.28</td>
<td>29.4</td>
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<td>0.24</td>
<td>21.8</td>
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<td>0.77</td>
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Table 6. Statistical descriptors of the corrected $a_{\text{calc}}/a_{\text{exp}}$ ratios for the RAC database

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<tr>
<th>Database</th>
<th>Corrections</th>
<th>Deflections</th>
<th>$n$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>CoV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAC</td>
<td>$E_{c_{\text{cm}}}, \sigma_{c_{\text{cs}}}, \phi$</td>
<td>All</td>
<td>30</td>
<td>1.06</td>
<td>0.29</td>
<td>26.9</td>
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<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>15</td>
<td>1.27</td>
<td>0.24</td>
<td>19.4</td>
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<td></td>
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<td>Final</td>
<td>15</td>
<td>0.90</td>
<td>0.24</td>
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<td>$E_{c_{\text{cm}}}, \sigma_{c_{\text{cs}}}, \phi$, $\beta$</td>
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<td>1.17</td>
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<td>1.32</td>
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<td>15</td>
<td>1.02</td>
<td>0.21</td>
<td>20.6</td>
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Figure 1. Relationship between the $a_{calc}/a_{exp}$ ratio and a) compressive strength, b) span-effective depth ratio, c) tensile reinforcement ratio, and d) load level, for the NAC-1 database.

160x101mm (300 x 300 DPI)
Figure 2. Relationship between the $a_{\text{calc}}/a_{\exp}$ ratio and a) compressive strength, b) span-effective depth ratio, c) tensile reinforcement ratio, and d) load level, for the Companion-NAC and RAC databases
Figure 3. Relationship between the $a_{calc}/a_{exp}$ ratio and a) compressive strength, b) span-effective depth ratio, c) tensile reinforcement ratio, and d) load level, for companion NAC and ‘RAC+’ beams.
Case and extension of the fib Model Code 2010 model

Belgrade, Serbia

CERIS, Department of Civil Engineering, Architecture and Georesources, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001, Lisbon, Portugal
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*Notes:*
- RCA: Reinforced Concrete Association
- B: Beam
- L: Load
- M: Moment
- K: Kelvin
- RH: Relative Humidity
- T: Temperature
References:

2 Espion, B. (1988). Long-Term Sustained Loading Test on Reinforced Concrete Beams. Bru:
8 Washa, G. W., & Fluck, P. G. (1952). Effect of compressive reinforcement on the plastic flo
For Review Only